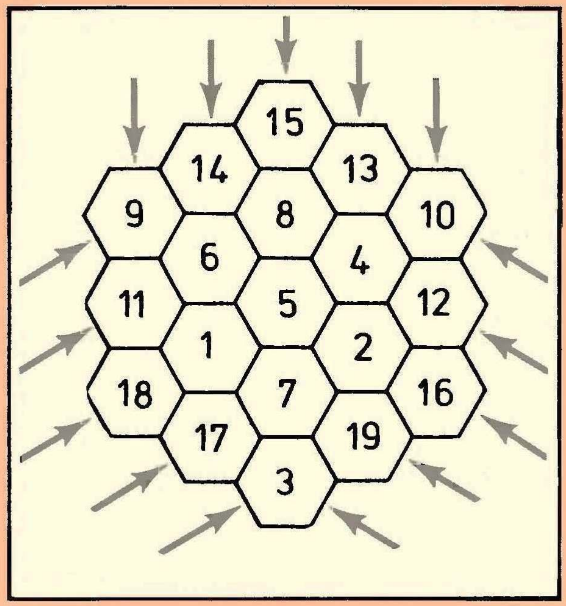

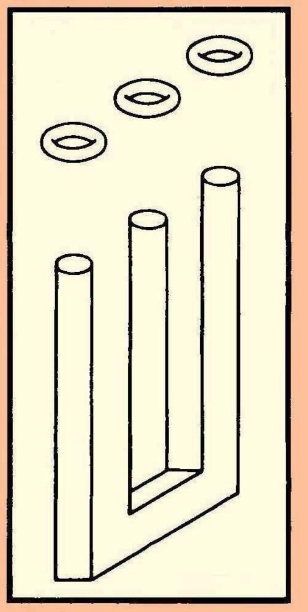

MATH PUZZLES AND GAMES

Michael Holt

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by Michael Holt

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INTRODUCTION

A word of advice on solving puzzles. If you really get stuck, don't give up—or you'll spoil the fun. Put the sticky puzzle aside, and perhaps the next day a new line of attack may suddenly strike you. Or you can try to solve an easier puzzle similar to the sticky one. Or again you can guess trial answers just to see if they make sense. Who knows, you might be lucky and hit on the right answer. Of course, a pure fluke is not as satisfying as working the puzzle out logically.

As a last resort you can always look up the answer, but only glance over the first few lines. This may give you the clue you need without giving the game away. As you will see, I have written very complete answers to the harder puzzles or to those needing several steps to solve.

But most of the puzzles call for straightforward arithmetic or simple reasoning. Almost all of them need no algebra to speak of; the couple that do can be solved, as I show, by trial and error or cunning.

The puzzles are not graded but are simply grouped by type; however, within each type you will find the easier ones at the start and the harder ones at the end. Practically all the puzzles are mathematical, but I have included a few word puzzles where they have an unusual slant or are totally different from crosswords, acrostics, and the general run of word puzzles.

As a slight departure from some puzzle books, I have included some easy-to-do mathematical tricks. Most of them I have performed myself either on stage or on television in Britain. Follow the instructions carefully, and with a little practice you should be able to amaze an audience with your mathematical powers. I should add that all the tricks work automatically: the magic is in the mathematics and does not require any sleight-of-hand skill on your part. I have ended the book with a group of puzzles about physics—that is, problems you can solve by experiment, if you prefer, about monkeys on ropes, moving belts, magnets, and the like.

In this collection of puzzles you will find, I hope you will agree, a happy mixture of puzzles of all kinds suitable for the entire family. Some are easy, some hard; some are versions of classical puzzles, and a few are truly brand-new ones. I find that today the Russians are the best puzzle-makers, although nobody has outdone the colorful story puzzles of America's greatest puzzlist, Sam Loyd, or his life-long British rival, Henry Dudeney. Both men worked toward the end of the nineteenth century. To them and to ***Moscow Puzzles: Three Hundred Fifty-nine Mathematical Recreations*** (Scribners), I am particularly indebted. Where I have adapted a Loyd or Dudeney puzzle, I have invariably had to simplify it for a younger—and more modern—audience. For some puzzle ideas I have drawn from mathematical research in education.

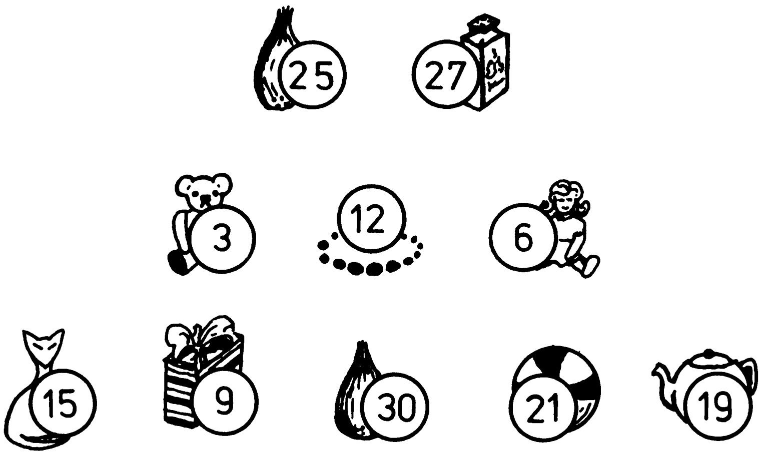
But far and away the most original "idea-men" are the children I have known, including my own son and daughter. To all of them I owe a lot.

1. Number Problems

Here is a wide selection of problems and puzzles that you can solve with only the simplest arithmetic; a few you could do by algebra, but as the answers show, this is not essential. The main problem is to wrest the actual "sum" from the words of the puzzle. The sum itself should present little or no difficulty—especially if you have a pocket calculator handy.

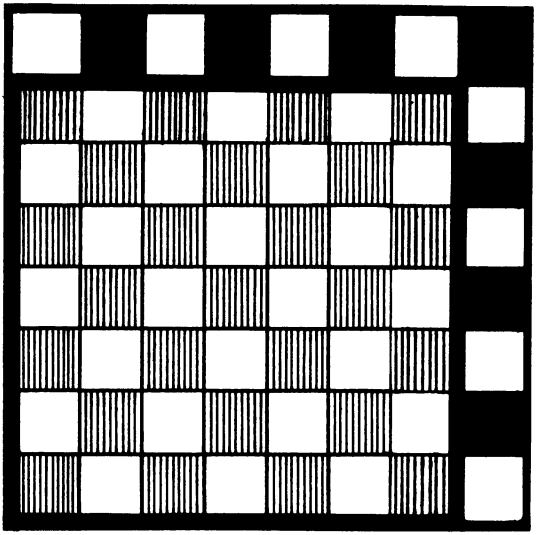
All the Fun of the Fair

Our picture shows one of the booths at a fair. You can have as many throws as you like, but you must score exactly 50. How do you do so? That is, what toys will you be able to win?



Chessboard Problem

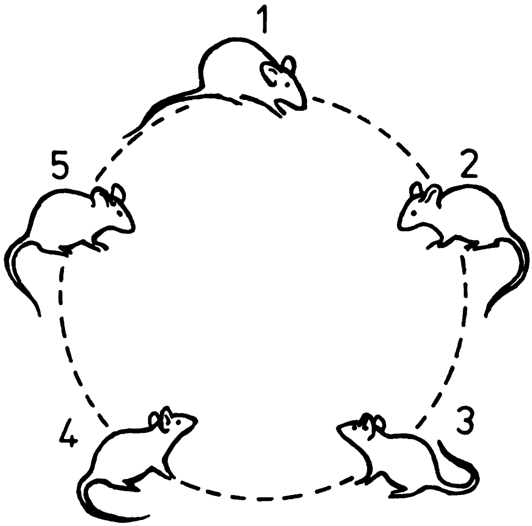
How many squares are there on an eight-by-eight chessboard? For a start the answer's *not* 64. There are all the bigger squares (multiples of the smaller ones) to take into account. Here is a method that may ease the problem and reduce it from sheer impossibility to simple slog! Think of how many seven-by-seven squares there are. Draw one on an eight-by-eight board. As you see, you can slide it up one square or across one square. That's two positions up and two positions across, allowing it 2 X 2 different positions. Thus there are 2 X 2 different seven-by-seven squares on the board. Now what about the six-by-six squares? Draw a six-by-six square on your chessboard picture. You will find you can slide it up two places and across two places. So, including the original position, that gives . . . well, how many positions in all? Got the hang of it? (Don't forget there is just *one* eight-by-eight square.)



Cat and Mice

Puss had taken a catnap. He dreamed that there were five mice sitting in a circle around him, four gray mice and one white mouse. In his dream Master says: "Go on, Puss, you can eat them up. But you can only eat each fifth mouse, going around in a clockwise direction. The last mouse you eat must be the white mouse."

HINT: Number the mice in a circle from 1 to 5. Pretend that Puss starts on the mouse at position 1. At what number must the white mouse be?



A Question of Ages

Two youngsters are chatting about their ages. Sam is just three times as old as May. But in two years' time he will only be twice as old as she is. How old is each youngster?

Another Question of Ages

In a film the actor Charles Coburn plays an elderly "uncle" character who is accused of marrying a girl when he is three times her age. He wittily replies: "Ah, but in twenty years' time I shall only be twice her age!" How old is the "uncle" and the girl?

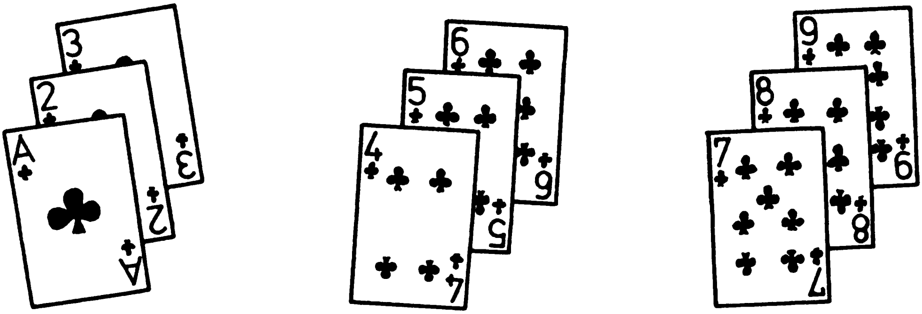
Teen-Age Problem

Sam is four years younger than Jo. But in five years' time Jo will be twice Sam's age now. How old are they now?

CLUE: One of them is a teen-ager.

15 Shuffle

Move just one card to a new pile so that the sum of each pile is 15.



Birthday Paradox

A famous Italian composer died shortly after his eighteenth birthday–at the ripe old age of 76 years! How on earth could that be? Don't worry who the composer was; he wrote the opera *The Barber of Seville.* He was born in February 1792.

Word Sums

In word sums or letter sums letters stand for digits. What you have to do is find the digits that fit the sums. Trial and error and some sound reasoning will be needed. Here's a simple one to set you off:

```
   T W O
 + T W O
 -------
 F I V E
```

Each letter O stands for 4. There are several solutions possible.

Easy as ABC?

In these two sums, each letter stands for a different digit. What are the digits?

```
   A A A        A A A
   B B B        D D D
 + C C C      + E E E
 -------      -------
 F G H I      F G H I
```

To start off, remember that the biggest number you can get from adding three single-digit numbers is three 9s, or 27. Since *B* and *C* must be different from A, then the most they add up to is $9 + 8 + 7 = 24$. But the "carry" number must be different in the tens place and also in the hundreds place. So $A + B + C$ must equal 19. You're on your own.

The Missing Dollar

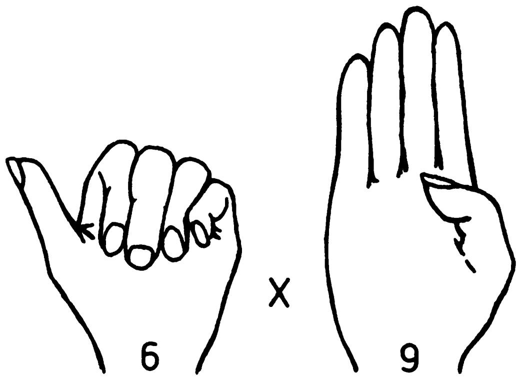
Three college girls want to buy a $30 radio they see in the local store. They each chip in $10 and buy it. The manager tells the saleslady she should have sold it for $25, since it was a sale. The very next day when the girls are passing the store, the saleslady beckons them in. She explains that she has made a mistake over the price and she owes each of them $1. She has taken $5 from the till, and out of it she gives each girl $1—so each paid $9, not $10. The saleslady secretly pockets the other $2. The girls have paid three lots of $9, or $27 in all, and the saleslady has made $2 on the sly. But $27 and $2 is $29. So where has the other dollar gone?

Merchants' Finger Figuring

In the Middle Ages merchants used to multiply on their fingers. They used this finger figuring only for numbers between 5 and 10. You can use it if ever your memory of your multiplaction tables fails you.

This is how the merchants multiplied 6 by 9. First show 9 on one hand by putting up five fingers, closing your fist, and then lifting four more fingers one by one: 6-7-8-9. So you have four fingers up. To show 6 on the other hand, put up five fingers, close your fist, then lift one finger, your thumb; you've put up six fingers in all.

You multiply the numbers like this. Add the fingers sticking up and multiply those bent down. Using the picture, this means you add 1 (the thumb) to 4 (the four fingers) to give 5; this is the number of tens, or 50. Multiply the bent fingers: $4 \times 1 = 4$ ones = 4. Total the two answers: $50 + 4 = 54$, which is 6×9. Try it out on other multiplications. There is, of course, no answer given.



You Can't Take It (All) with You

Suppose a millionaire offered you all the dollar bills you could take away with you, but you would have to count them nonstop! You can take what you count until you stop. We'll say you count at one bill a second. How many do you think you could really take?

Peasants' Multiplying

The oldest dodge for multiplying is based only on doubling and halving. This method was certainly used by the Egyptians. Here is how to multiply 35 by 5.

First halve the left side each time and double the number on the right, like this:

35 X	5
17	10
8	20
4	40
2	80
1	160

Happily, ignore remainders. So you write half of 35 as 17, not as 17½. This is why the method is so easy to use. Go on halving till you get to 1 on the left.

The next step is to cross off all the *even* numbers on the left side together with their partners on the right:

35 X	5
17	10
~~8~~	~~20~~
~~4~~	~~40~~
~~2~~	~~80~~
1	160

Finally, add up the numbers not crossed out on the right side: 5 + 10 + 160 = 175. This, as you can work out, is 35 X 5.

To get to know the method, try swapping the numbers over: that is, try 5 X 35. Obviously, this will give bigger numbers to double, which makes it harder for you. Try a few really easy ones, such as 13 X 10. The answer (130) is obvious, and the doubling is simple. Or try 13 X 1: even simpler!

There are no answers to this puzzle. The method is all based on changing numbers into computer (binary) numbering. In it you do not count up to 10, as we usually do, but only up to 2. To change a number to binary, you divide by 2 again and again and note down the remainders. We won't turn this into a lecture; we'll simply leave it at that.

Tear 'n' Stack

Here is a puzzle to demonstrate the power of doubling. Take a sheet of paper and tear it across the middle. Put the 2 halves together and tear them in half to get 4 pieces of paper. Stack them and tear in half to get 8 pieces. Stack and tear again to get 16 pieces. Continue doing this 47 times. You can't, of course, as you can quickly discover. How high a stack would you get if you could? As high as your table? the roof of a house? the top of the Empire State Building? nearly to the moon? Say a sheet is one thousandth of an inch (known as a thou) thick; so a stack of 1,000 sheets will be one inch high.

Grains of Wheat

Long ago in India the grand vizier Sissa Ben Dahir invented the game of chess for King Shirham. The king was so delighted with the game, he offered the grand vizier any boon he desired. The clever vizier seemed very modest in his desire, saying to the king: "Majesty, give me a grain of wheat to put on the first square of the chessboard, and two grains to put on the second square, and four grains to put on the third, and eight grains to put on the fourth. And so, O King, doubling the number of grains of wheat on each square that follows, give me enough wheat to cover all sixty-four squares of the board."

What a fool! the King thought to himself. Then he said aloud: "Your boon is granted, Grand Vizier."

But easier said than done! The king sent for a sack of wheat to be brought to the throne. When the grains were counted out, the sack was emptied before they had counted out the twentieth square. Can you say, then, roughly how many grains of wheat were in a sack? Choose from:

100,000	1,000,000	10,000

How many grains of wheat were needed to cover the chessboard in the grand vizier's way? Choose from:

about 18 million million million grains
today's annual world wheat production for two thousand years
18,446,744,073,709,551,615 grains
the number of grains of sand on the beach at Coney Island

A Sweet Problem

Can you put ten sugar lumps into three cups so there is an *odd* number of lumps in each cup?

Stock Taking

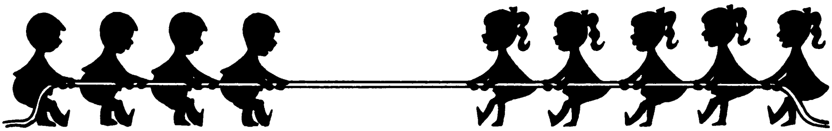
A cunning farmer, asked how many sheep he had, replied: "A third of my sheep are in the barn. A fifth are out to pasture. Three times the difference of these two numbers are newborn. And one is my daughter's pet. But there's less than twenty in all." How many sheep had the farmer?

Slobodian Coin Puzzle

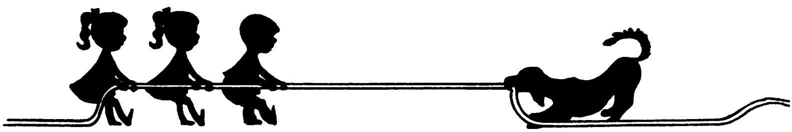
In Slobodia they only have two coins, a one-slob coin and a two-slob coin. How many ways can a Slobodian pay a sum of one slob? Simple. One way. How many ways for a sum of two slobs? Two ways—with two one-slob coins or with one two-slob coin. Can you tell how many ways a Slobodian can pay a sum of six slobs?

Tug of War

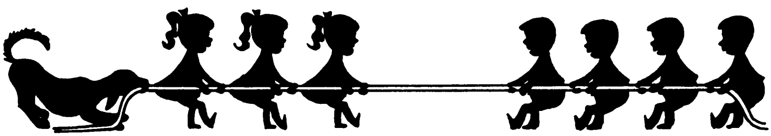
In a tug of war four boys can tug as hard as five girls.



And two girls and one boy can tug as hard as one dog.



The dog and three girls tug against four boys.



Which side will win the last tug of war?

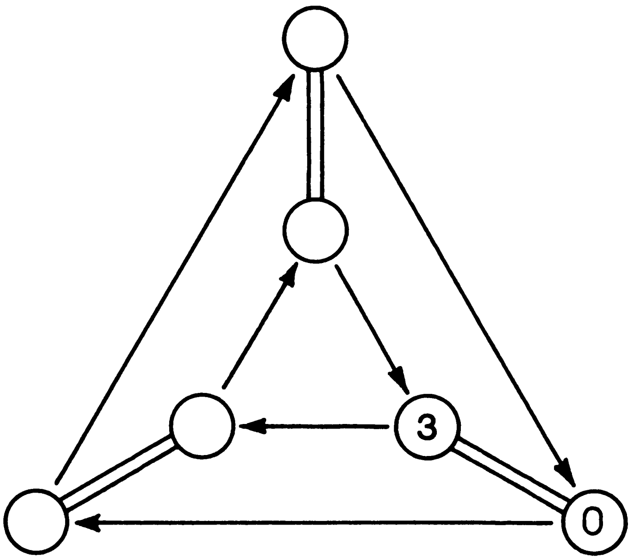
Check-out Check

In the store Gus said to the check-out person: "I bought two sticks of chewing gum at ten cents and three bars of chocolate. But I can't remember how much the chocolate cost." "That'll be $2.20," said the checker. Gus said: "But you've made a mistake!" The checker checked again and agreed with Gus. How did Gus spot the error?

CLUE: It's all to do with dividing one number into another.

Puzzle Triangles

Six discs are joined by one-way arrows and two-way links, as shown in our picture. Each arrow means you add 4 to the number the arrow begins on and then divide the result by 6 and keep the remainder and put *that* in the disc the arrow points to. The puzzle is to give each disc its number, from 0 up to 6. For example, you begin at 0, already marked. Add 4 and the result is 4; divide by 6 and keep the remainder, 4, and put that in the disc the arrow points to. Also, on the inner triangle, beginning on 3 (already marked), the disc pointed to is 1 because (3 + 4)/6 leaves a remainder of 1. The difference between the numbers on the discs at either end of each two-way link is 3. See if you can fill in all the discs.



Nice Work If You Can Get It!

A college girl applied for a job advertised in her local newspaper. The boss, who interviewed her, had a mathematical turn of mind. He offered her two pay rates: a straight ten dollars a day; or one cent the first day, two cents the second day, four cents the third day, eight cents the fourth day, and so on, the pay doubling each day. The girl chose the second pay rate. The boss hired her. Why?

CLUE: He *actually* paid her a different rate!

12 Days' Gifts

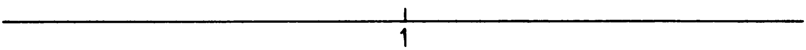
On the twelfth day of Christmas my true love sent to me
12 drummers drumming, 11 pipers piping,
10 lords aleaping, 9 ladies dancing,
8 maids amilking, 7 swans aswimming,
6 geese alaying, 5 gold rings,
4 calling birds, 3 French hens,
2 turtledoves, and a partridge in a pear tree.

How many things did my true love send to me?

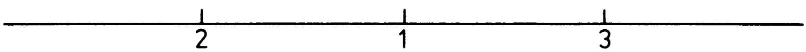
Dividing-the-Line Code

Here is a really new way of coding a message. It depends on a special way of putting our counting numbers in order. The method actually hinges on the way binary numbers (as used in computers) work. But you need not bother about that.

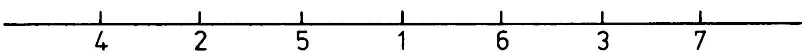
First draw a long line on a sheet of paper. Mark a point near its middle and call it 1.



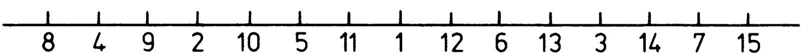
Mark the two parts 2 and 3:



Then mark the four parts thus created 4, 5, 6, and 7:



Now mark the parts so created 8, 9, 10, up to 15.



Now divide the line once more so the last number is 31.

Now for the code. Suppose you want to put this little message into code: MEET ME AT THE HAUNTED HOUSE. Number off the letters in the order they come in the message:

1 2 3 4 5 6 7 8
M E E T M E A T

You see we run the words together, for it is usually possible to unravel words in a message once you have all the letters, provided you have enough words (as the eight letters show!). Having numbered your letters serially, now write them above your prepared number line. So the first eight letters come out like this:

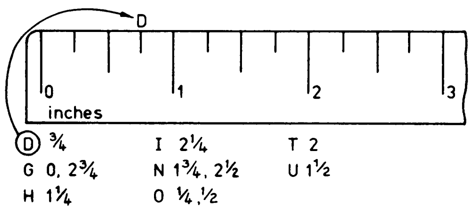
T T E M M E E A
8 4 2 5 1 6 3 7

I have given you a line that will take only 31-letter messages. If you want to send a longer message, then all you have to do is divide the parts of the line once more and put in the numbers above the marks, from 32 up to 63.

Can you see the pattern of each new set of numbers you write in? Just to remind you, the number of parts the line is divided into is part of the following series: 2, 4, 8 . . . With your coded message you can send the key number, here 16; this tells the person who receives the message how many parts he must divide his line into to decode the message. But it doesn't matter if you don't put in the 16. Can you see why?

Holiday Message

Copy this picture of a ruler. On it write each letter above its correct mark shown in the box. The letter *D* has been done for you. Some of the letters appear twice and so go above two marks. Can you find the holiday message?

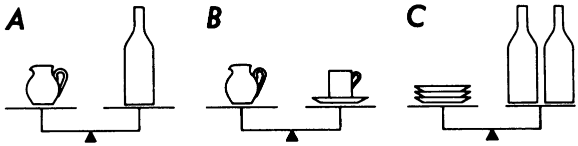


Bottle and Cork

A bottle costs three cents more than a cork. (Both are returnable!) Together they cost five cents. How much does each cost? Too easy? Then try the next puzzle.

Juggling and Balancing

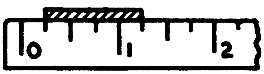
In picture *A* the jug on the left pan balances the bottle on the right pan. In *B* the jug alone balances a mug and a plate. In *C* three of these plates balance two bottles. How many mugs will balance a jug?



Hidden Animal

The idea in this code is to find the hidden animal. What you do is work out each little problem. The answer will be a number. You change this number to its letter in the alphabet, 1 to *A*, 2 to *B*, 3 to *C*, and so on. The completed letters spell a well-known animal. For example, suppose the problem was: *How many in a dozen?* The answer is 12. The twelfth letter of the alphabet is *L*. So you would write *L*.

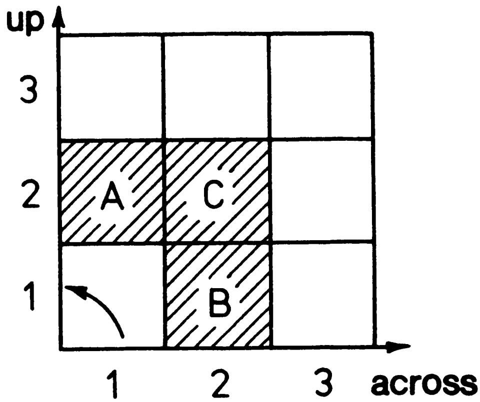
Copy and fill in a table like this:

	Number	*Letter*
How many square feet in a rug three feet by four feet?		
How many dimes in half a dollar?		
Half the number of days that are in September		
4 squared equals . . .		
How long is the grey rod? 		
2 X 3 X 3 =		
This number times itself is 16:		

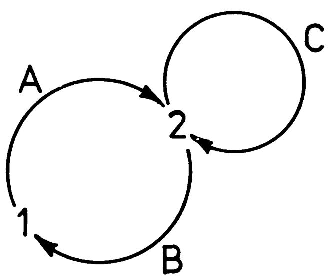
CLUE: It cannot change its spots.

Pictures by Numbers

You know the game of Battleships, I expect, where you name squares in a grid by calling two numbers. For instance, in the bit of grid shown here there are three shaded squares: *A*, *B*, and *C*. To name a square, you call the *across* number first, then the *up* number. So for square *A* you call 1, 2; for *B*, 2, 1. The order of the numbers makes all the difference! The little arrow in the corner of the grid may remind you of this order. Square *C* is 2, 2.

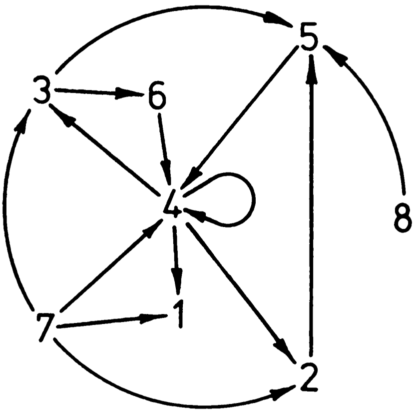


As in Battleships, we can shade in a picture on a grid by calling out number pairs. For fun we can code the number pairs by using a cunning device, a mapping. This is how we put the numbers for the shaded squares into code:



The *1*-to-*2* arrow means you shade the square *A* (1, 2), *2*-to-*1* means you shade square *B* (2, 1), and the arrow looping back on *2* means shade square *C* (2, 2).

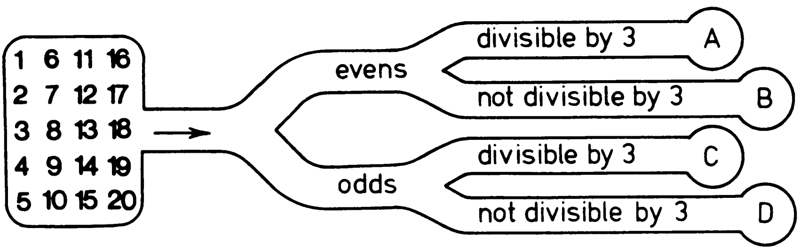
All set? Then try your hand at drawing the picture from the mapping shown here. You'll need an eight-by-eight grid to draw it on.



CLUE: It's one of these pets: a cat, a dog, or a hamster.

Number Tracks

Here's a novel number puzzle. First you need to copy the tracks if this book is not yours. Then jot down where each of the numbers 1 to 20 end up—at *A, B, C,* or *D*—after you have sent them along their tracks. For example, 1 must take the lower odds track at the first fork because it is an odd number; 2, an even number, takes the upper track. At the next fork 1 takes the lower track because it is not divisible by 3. So 1 ends up in *D;* 2 ends up in *B,* 3 in *C,* and 6 in *A.* See if you can sort out the other numbers waiting at the starting gate. Spot anything special about all the numbers that end up in *A?*



Number Fo*u*rte!

Think of a number—any number will do. Write it out in full as a word. Count the number of letters in the word and write down that number too as a word; again count the number of letters and spell that number out, and so on. Go on as far as you can, but we bet you will always finish up with the word *four.* Here is an example:

FIFTY-THREE ⟶	TEN ⟶	THREE ⟶	FIVE ⟶	FOUR
(ten letters)	(three letters)	(five letters)	(four letters)	(four letters)

The chain of numbers ends at four and stays there. You can bet a friend that whatever number he thinks of, provided it is spelled properly, the chain will always end up at FOUR. Try a few numbers yourself first to see.

Solve It in Your Head

If you've done equations, you can surely solve these with pencil and paper:

$$7x + 3y = 27$$
$$3x + 7y = 23$$

But can you solve them together in your head? You might multiply the first equation by 3 and the second by 7 in your head. Better still, use another, simpler method.

HINT: Add the equations; then take them away.

Movie Times

At his neighborhood movie theater George checks on the time of the films showing that day. He sees that some of the numerals have fallen off the display board:

Carry On UFOs	2:30	5:30	
The Reel Life	3:30		8:30
The Impossible Triangle	3:45		

Can you say when the second showing of *The Reel Life* begins? When does the film *The Impossible Triangle* end?

Time, Please

Tell me quickly: What time is it when it's 60 minutes to 2?

Picture and Frame

A portrait of your favorite TV star costs half a dollar more than the frame to go around it. Together they cost two dollars. How much does each cost?

Idle Ivan and the Devil

An old Russian tale tells how Idle Ivan was fooling about by a river. He sighed to himself: "Everybody tells me to get a job or go to the devil. But I don't 'xpect even he could help me get rich."

No sooner had he said this than the devil himself was standing before him. "You want to make money, Ivan?" asked the devil. Ivan nodded, lazily. "Then," the devil went on, "you see that bridge over there? All you have to do is cross it. And every time you do, the money in your pocket will double." Ivan was about to make for the bridge when the devil stopped him. "One moment," said the devil foxily. "Seeing that I'm so generous, I think you ought to give me a little for my pains. Will you give me eight rubles every time you cross the bridge?"

Idle Ivan readily agreed. He crossed the bridge and put his hand in his pocket. As if by magic the money *had* doubled! He lobbed eight rubles over the river to the devil and crossed again. Again his money doubled; he paid another eight rubles to the devil and crossed a third time. Once more his money doubled. But when he counted it, he found he had only eight rubles in his pocket, which he threw to the devil, leaving him with no money in his pocket to double.

The devil laughed and vanished.

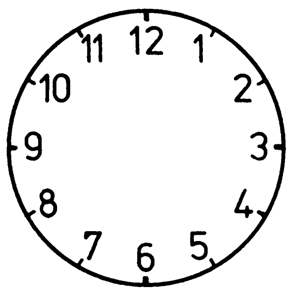
PUZZLE: How much money did Idle Ivan start with in his pocket?

Happy Landing

Gary takes off from La Guardia Airport, New York, at 10:00 in the morning. In Anchorage, Alaska, the time is exactly 5:00 in the morning. Gary's flight takes just seven hours. Can you tell what time he should land in Anchorage by *Anchorage time?*

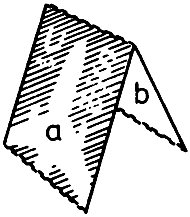
Cracked-Clock Problem

How can you split the clockface in two so that the sum of the numbers on the two halves are the same?

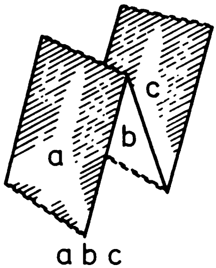


Stamp-Strip Puzzle

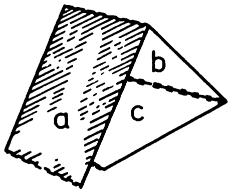
Here is a pretty puzzle to try next time you buy a strip of stamps. No reason why you shouldn't use a strip of ordinary paper. Look at the strip of two stamps, which the artist has labeled *a* and *b*. The puzzle is: How many ways of folding the stamps are there if you don't break the strip? Well, obviously, just one, *ab*. Now take a strip of three stamps. How many different ways of folding the stamps are there now? As the picture shows, there are two ways, *abc* and *acb*. How many different ways are there of folding a strip of four stamps? To start you off, we show you the first two ways, *abcd* and *abdc*. Look at the pictures and see if you can write down the other three ways.



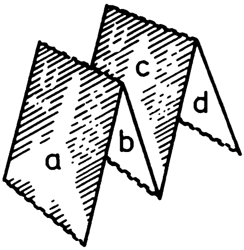
a b



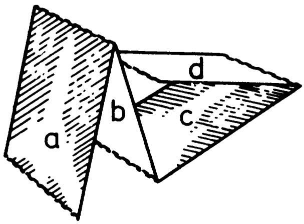
a b c



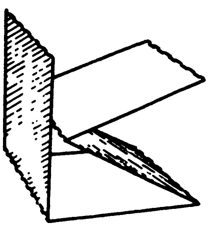
a c b

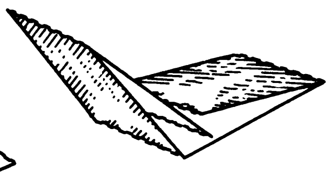


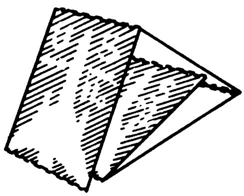
a b c d



a b d c







The Smallest Flock

A farmer had a very large flock of sheep. He discovered something very odd about their number: When he counted them in 2s he had 1 left over; he also had 1 left over when he counted them in 3s, 4s, 5s, and so on up to 10s. The problem is: What was the *smallest* size of flock he must have had?

HINT: If this is too hard, pretend that he counted the flock in 2s, 3s, and 4s only, 1 over each time. What is the smallest size flock then? It is not 2 X 3 X 4 + 1, or 25, because that is not the *smallest* flock possible. Remember 2 goes into 4, so if the flock is divisible by 4 with 1 left over, it is also divisible by 2 with 1 left over. So the smallest size of flock is 3 X 4 + 1, or 13. If you like to think of it another way, remember that the dividing by 2 is covered by the dividing by 4.

Letter Frame-up

A well-known (almost!) saying is hidden in this letter frame. See if you can read it. Begin at one of the letters. Reading every other letter, go twice around the frame. What is the saying?

```
L S I M N U G C
R             S
L             H
E             T
O             S
H             O
R             P
T             N
A A D G E E E
```

The Farmer's Will

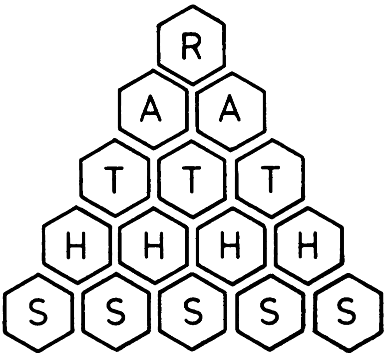
A farmer died leaving in his will 17 fine horses to his three children, Ann, Bob, and Charlie. To Ann he left half his horses, to Bob a third of them, and to Charlie a ninth of them. How on earth did they share out the horses?

The Checkers Match

Five children enter for a checkers match. Each one has to play every other one. How many games must they play?

The Spelling Bee

The picture shows where the rare Spelling bee lives: in a letter comb. It was, as you recall, named for the famous Dr. Spelling, the noted "bee-all-and-end-all-ologist." How many ways can the bee move down the honeycomb from cell *R* and spell out the word *rat?* As Dr. Spelling discovered, once started, he—the bee, not the noted bee- . . . -ologist—will never go back up again. Not hard, you'll agree, to see there are just four ways. So can you say how many ways the bee can spell out Lewis Carroll's word *rath,* and then *raths?* You should find a number pattern to your answers. You'll find the word *raths* in Lewis Carroll's poem "Jabberwocky," in which occurs this line: "And the mome raths outgrabe." Carroll tells us the word *mome* comes from *solemome,* or *solemn,* and means "grave"; *rath* is a kind of land turtle; and *grabe* means "squeaked." See if you can translate the line into *our* English, yours and mine.



Number Oddity

Make a number out of all the digits 1 to 9 leaving out 8: 12,345,679. Now multiply it first by *any* single-digit number—5, say—and then multiply the product by 9. You should get your single-digit number back ninefold.

12,345,679	Now complete:	12,345,679
X 5		X 7
61,728,395		
X 9		
555,555,555		X 9

Check that it works for the other single-digit numbers. Why does it work?

Six 1s are 24?

Write six 1s and three plus signs in a row in such a way that they add up to 24.

Pairing Puzzle

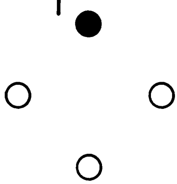
How do you pair off these numbers so that the sum of each of the four pairs adds up to the same number?

1 2 3 4 5 6 7 8

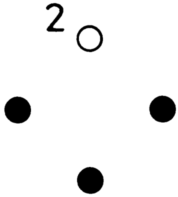
Diamonds of Marbles

Ned was playing solitaire one day. He put some black and some white marbles in four of the hollows to form a diamond. He started making different patterns with them. He began with one black bead at the top corner and three white in the other corners, as shown in the picture. Another pattern was with the black marble at the bottom of the diamond; another was four black marbles, one in each corner of the diamond. How many *different* patterns could he make altogether?

1



2



2. Number Patterns

All the next set of puzzles depends on curious and beautiful number patterns. From number crystals that grow before your very eyes to that classic of the IQ test, missing numbers in a series, you will find some, I hope, intriguing patterns to look at and play with.

Puzzle Boards

Jot down any four numbers on a board, like this:

1	4	
7	3	\

Write their differences. This means there will be no negative (minus) numbers at the end. For example, 4 and 1 gives 3, and so does 1 and 4.

1	4	4 − 1 = 3
7	3	7 − 3 = 4
7 − 1 = 6	4 − 3 = 1	\

Your table looks like this:

1	4	3
7	3	4
6	1	\

Take differences from the answers 3, 4 and 6, 1 and put them in the corner triangles:

4 − 3 = 1
6 − 1 = 5

1	4	3
7	3	4
6	1	5 \ 1

PUZZLE: Can you make up boards like this that give the *same corner numbers?* Can you find a rule for making them?

1	4	3
3	7	4
2	3	1 \ 1

Sum of the Whole Numbers

The sum of the numbers 1 through 10 is 55, as you can easily check. The sum of the numbers 1 through 100 is 5,050. Now can you write down from the pattern the sum for the numbers 1 through 1,000?

Sum 1 through 10 = 55
Sum 1 through 100 = 5,050
Sum 1 through 1,000 = ?

Number Crystals

A. Can you make this number pattern grow like a crystal by writing two more lines:

16 = 4 X 4
1,156 = 34 X 34
111,556 = 334 X 334

B. To grow this number crystal, keep adding 1 on the left and add 8 to the right half of the number, like this:

09 = 3 X 3
1,089 = 33 X 33
110,889 = 333 X 333

Can you write the next line of the crystal?

C. Grow this number crystal by writing one more line:

36 = 6 X 6
4,356 = 66 X 66

Number Carousel

Take the number 142,857 and multiply it by these numbers in turn: 1, 5, 6, 2, and 3. You should be able to check the lines from the growing pattern!

142,857 X 1 = 142,857
X 5 = 714,285 and so on

Now multiply by 8: 142,857 X 8 = 1,142,856

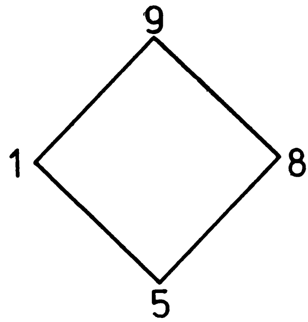
The pattern appears to be broken until you add the 1 on the left to the digit on the far right:

1,142,856 ⟶ 142,857

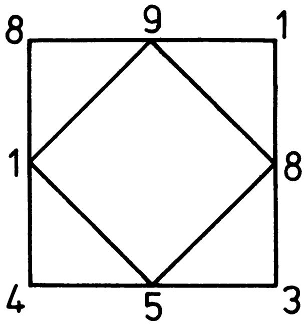
See if this pattern continues when you multiply by 9, 10, and 11.

Take-away Number Squares

This is just like the Take-away number triangle puzzle. Only this time you start with four numbers, one at each corner of a square. Here I have used 1, 5, 8, and 9.



Draw a square around this one to go through its corners. At each corner of this larger square put the difference of the numbers at the next-door corners of the small square: 4 (from 5 − 1), 3 (from 8 − 5), 1 from (9 − 8), and 8 (from 9 − 1):



Repeat the process until you come to a pattern of four numbers that does not change. What is this pattern? You should find the same pattern whatever four numbers you start with. Do you?

Take Any Three-Digit Number

Take any three-digit number with each digit different—say, 123. Now write the same three digits again to make a six-digit number; 123 becomes

123,123. Now divide by 7, then by 11, finally by 13, and I predict you will have the first three digits you began with. It works for *any* three digits. Can you say why it works?

Five 2s

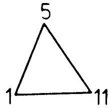
Can you write the numbers 0 through 10 using five figure 2s, no more and no less, and the usual signs: +, −, X, ÷, and parentheses? To start you off: $0 = 2 - 2/2 - 2/2$.

Four 4s

Using four 4s and the signs +, −, X, ÷, and parentheses, can you write the numbers 1 through 10? To start you off: $0 = 4 - 4 + 4 - 4$.

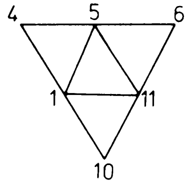
Take-away Number Triangles

This is a new number puzzle that has to do with finding a number pattern for taking away. Put any three numbers you like at the corners of a triangle, like this:

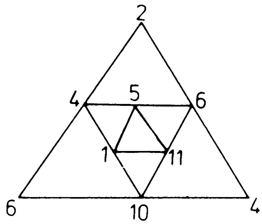


I have used 1, 5, and 11.

Now find the differences between next-door corners: $5 - 1 = 4$ and $11 - 1 = 10$ and $11 - 5 = 6$. Draw a larger triangle to go through these corners and put the differences 4, 10, and 6 at three corners of this new outside triangle, like this:



Repeat the process: draw another triangle outside the last one and put the new differences 6, 4, and 2 at the corners because $10 - 4 = 6$ and $10 - 6 = 4$ and $6 - 4 = 2$.



Now do this again two more times and you should finish up with a pattern of three numbers that does not change however many times you go on. What is this pattern? Begin with three other numbers and carry on taking away. Do you find you end up with the same pattern? The answer section will tell you what the pattern is.

Missing Numbers

Find the missing number in each pattern. These puzzles are very popular in IQ tests.

a. 2 5 ___ 11 14 17

b. 2 4 6 8 ___ 12

c. 2 7 12 17 22 ___ 32

d. 3 1 4 2 5 ___ 6

e. 1 4 9 16 25 ___ 49 64

f. 2 4 8 16 ___ 64 128

g. 0 2 6 14 30 62 ___ 254

h. 5 9 16 29 54 ___ 200

Sums in the Head

You should be able to do these sums in your head:

A. Multiply 999 by 3.

B. Group these three-digit numbers in pairs and add them quickly:

```
  645
  221
  304
  355
  779
+ 696
-----
```

C. Copy these numbers and beneath them put a third six-digit number. At once write (from left to right) the grand total:

```
604,253
283,012
```

We rule out 000,000, of course!

D. Which is easier to add, the sum on the left or the sum on the right?

```
  123,456,789         1
   12,345,678         12
    1,234,567         123
      123,456         1,234
       12,345         12,345
        1,234         123,456
          123         1,234,567
           12         12,345,678
+           1       + 123,456,789
-------------       -------------
```

9 in Ten Digits

The number 9 can be written as a fraction using all ten digits 1 through 9, like this:

$$9 = \frac{95{,}823}{10{,}647}$$

Can you find another way? There are actually six ways.

HINT: Try shuffling the same digits around.

Number Patterns

See if you can find the next two lines in this number pattern:

$$1 \times 2 \times 3 \times 4 + 1 = 5 \times 5$$
$$2 \times 3 \times 4 \times 5 + 1 = 11 \times 11$$
$$3 \times 4 \times 5 \times 6 + 1 = 19 \times 19$$

Check that both sides are equal.

Another Number Pattern

Finish the first three lines. Can you finish the last two lines?

$$37 \times (3 + 7) = 3^3 + 7^3 = ?$$
$$48 \times (4 + 8) = 4^3 + 8^3 = ?$$
$$111 \times (11 + 1) = 11^3 + 1^3 = ?$$
$$147 \times \underline{\qquad} \quad \underline{\qquad}$$
$$148 \times \underline{\qquad} \quad \underline{\qquad}$$

The little 3 above a number means you should multiply that number times itself three times. Thus $3^3 = 3 \times 3 \times 3$, and $7^3 = 7 \times 7 \times 7$.

Do the patterns multiply out correctly?

Reverse Sums

The sum 9 + 9 = 18 gives the reverse answer when the numbers are multiplied: 9 X 9 = 81. Find two more reversals, beginning 24 + . . . and 47 + ...

Palindromes in Numbers

A palindrome is a word that spells the same forward and backward, such as *pip, radar,* and *rotator.* An example of a number that is a palindrome is

121. Here is how to arrive at one. Add to any number its reverse; then add to the sum the sums reverse. Carry on until you reach a palindrome, like this:

$$\begin{array}{r} 38 \\ +\,83 \\ \hline 121 \end{array}$$

Now begin with the number 139.

The Next Palindromic Year

The last palindromic year was 1881—it reads the same forward or backward. What is the next palindromic year?

Corridors of Numbers

See how this number table is made up:

1	3	5	7	9	11
1	4	7	10	13	16
1	5	9	13	17	21
1	6	11	16	21	26
1	7	13	19	25	31
1	8	15	22	29	36

Each row starts with 1 on the left. The numbers in the first row go up by 2 each time, in the second row by 3, in the third row by 4, and so on. The table continues to the right and downward as far as you like. Now add the numbers in each ⌋-shaped corridor. The sum of the top left-hand corridor comes to 1 alone, which equals $1 \times 1 \times 1$ or 1^3. The sum of the second corridor is $1 + 4 + 3 = 8 = 2 \times 2 \times 2$ or 2^3. See if this pattern continues for the next few corridors shown.

What are the numbers along the diagonal from the top left to the bottom right? Take any section of this diagonal. The sum of all the numbers in a square built around that section is a square number. For example, take the section 9, 16, 25. The numbers in the square with that as diagonal are:

$$\begin{array}{l} 9 + 13 + 17 \\ + 11 + 16 + 21 = 144 = 12 \times 12 = 12^2 \\ + 13 + 19 + 25 \end{array}$$

Try this for the section 4, 9, 16.

Multiplying Equals Adding?!

Take two numbers and multiply them together. Now add the numbers. The two answers are the same. What are the two numbers? Well, they could both be 2, for 2 X 2 = 2 + 2. The puzzle is: How many other pairs of numbers work in this way?

CLUE: To start you off, try 1½ and a whole number less than 5.

Curious Centuries

Can you make 100, a century, out of the numbers 1 through 9, using the usual signs +, −, X, ÷, and parentheses? To start you off, here is how four centuries begin:

$$\begin{aligned} 100 &= 1 + 2 + 3 + \ldots \\ &= 123 - 45 \ldots \\ &= 123 - 4 - 5 \ldots \\ &= 1/2 + 6/4 + \ldots \end{aligned}$$

Times-Table Triangle

Here is an unusually simple number triangle. It's easy to build up. Yet it packs a mathematical punch! It's a multiplication ("times") table as well!

							1							
						2	3	4						
					5	6	7	8	9					
				10	11	12	13	14	15	16				
			17	18	19	20	21	22	23	24	25			
		26	27	28	29	30	31	32	33	34	35	36		
	37	38	39	40	41	42	43	44	45	46	47	48	49	
50	51	52	53	54	55	56	57	58	59	60	61	62	63	64

A. What do you see about the numbers on the right-hand sloping side? (Hint: They are all a special kind of number.)

B. Now for the mathematical punch. Pick any two numbers from one of the columns on adjacent rows. To start with, pick them near the top—say, 5 and 11. What is 5 X 11? The table will tell you. Look five rows down from the 5 and you find the answer, 55. Start with the smaller number and look down. Another example: What's 4 X 8? Find 4 and look four rows down to the answer, 32. We can write this briefly:

$$\begin{array}{r} 4 \\ \times\ 8 \\ \hline 32 \end{array}$$

(which is four rows down from the 4)

Copy the triangle, then add four or more rows. Then try these:

$$\begin{array}{r} 2 \\ \times\ 6 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ \times\ 7 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ \times\ 12 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ \times\ 13 \\ \hline \end{array}$$

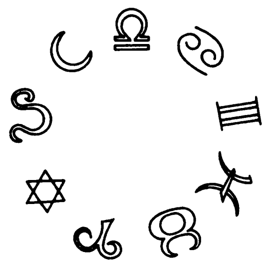
Of course, the triangle doesn't give *all* the multiplication tables.

3. Magic and Party Tricks with Numbers

I have described these tricks fully so that you can perform them for yourself in front of friends or a larger audience. I have tried to indicate the likely pitfalls and the sort of pattern to get you over the "tricky" parts. In fact, only one of the tricks requires any mental agility. The others either rely on the spectator's arithmetic skill or they work like a charm, purely from the (concealed) arithmetic underlying the trick. I hope you have fun trying them out.

Dial-a-Number Trick

I have performed this impressive but simple number trick with great effect on stage and for television. It depends upon the same idea as that used in "Fiddling by Numbers" on page 34. To present the trick, make a board of nine "Secret Signs" like the one shown here. You'll also need a blackboard



for your chosen spectator to write on. A sheet of paper clipped to a board will do as well. Invite the spectator to step up to the board. Now you turn your back and give the following instructions:

You say: "Write the last three digits of your telephone number on the board. Make sure the digits are all different, that's all. If they aren't, invent a three-digit number." Stress to your audience that you cannot possibly know what the digits are. (Indeed, you do not need to know.) It is as well to ask the audience to check that the subtracting and adding are being done correctly. "Write the digits in reverse order to form a new number," you say. "Then write the smaller of the two three-digit numbers beneath the larger." So if the digits were 376, he would write near it 673 and then put the 376 under it. "Take the smaller number from the larger," you say.

$$\begin{array}{r} 673 \\ -376 \\ \hline 297 \end{array}$$

"Then add the individual digits of the result." He does so and gets 18 (2 + 9 + 7), which he must not tell you.

You invite him over to the Chart of Secret Signs. Now you can face him. You ask him to count around clockwise on the chart, starting at the top as 1. Let him start counting to make sure he understands what to do and to build up tension. Then stop him, saying: "Now you agree that I cannot know your number. Yet I can predict you'll finish on the moon sign." Sure enough, if he did his sums correctly, he will count twice around the circle and land on the moon.

The trick depends on the fact that the sum of the digits will always be a multiple of 9; in this case it will come to 18. To heighten the effect, you can color the Secret Signs. Suppose the moon is green. Then select someone from your audience wearing something green. Then, at the Secret Sign Chart, predict *wrongly* that he will end on the red star, say. Then draw the audience's attention to the green in your "victim's" dress and appear to change your mind and say, "The green moon." With a little practice you can work up a most effective presentation of the trick.

The mathematics behind the trick is explained in the answer section.

Conjurer's Forces

Did you know conjurers can make you say a number? It's called "forcing" a number. You can try it on your friends. It goes like this:

You ask a friend for a number—any number—between 1 and 5. You'll find most people reply 3. Or ask for a number between 1 and 10 and most will say 7. You say to a friend: "Think of a number between one and fifty, but both figures have to be odd but not the same. For example, eleven won't do." Previously you have written 37 on a scrap of paper and handed it folded to the friend. The prompt of "eleven" is part of the act. When you say that, most people move on to the 30s. Now 33 will not do, and as 7 is the most popular number between 1 and 10, most answer 37. Your friend will be surprised to read 37 on the paper.

Here are several good tricks for guessing someone's age. They have an added advantage: They are very easy to perform in front of an audience; they do not depend on sleight of hand or great calculating skill.

How Old Are You?

"You won't tell me? All right, simply tell me the result of this little sum: Multiply your age by ten. From that take away nine times any single-digit number, such as one, two, three, up to nine itself. Done it? Tell me the result. . . . Now I know your age!"

Use this trick on your friends older than 9. For example, say your friend's age is 15, which times 10 is 150, and he chooses 4 for his digit. Product $4 \times 9 = 36$. He does this subtraction in his head: $150 - 36 = 114$. He calls out 114. This is what you do. Remove the last digit of the number he called out and add it to what remains. Here, 114 without the 4 is 11, and $11 + 4 = 15$. And this is his age. Perhaps, however, you had better check with the answer to see why the trick works before you use it.

For Someone Over 10 Years Old

Tell your friend to add 90 to his age, cross off the first digit of the result, and add it to the two digits left. Ask him for his answer. You then merely add 9 and tell him his age.

Suppose your friend is:	17
He adds 90:	+ 90
	107

He crosses off the first digit and adds it:

$$\begin{array}{r} \not{1}07 \\ +\ \ 1 \\ \hline 8 \end{array}$$

He tells you 8. You add on 9 and tell him he is 17.

Fiddling by Numbers

Here's an unusual mathematical joke. Did you hear of the boy whose violin playing was the despair of his parents? The squeaks and screeches he made on the instrument drove them mad. But his uncle was a kindly man and a mathematician. "The trouble is," the boy told him, "I can't get a note out of my violin. It never sings." At this his uncle's face lit up: "Ah, I think I know what's needed. Let's do it by numbers. As you say, it *never sings.*" So saying, he scribbled on a scrap of paper these words and numbers:

N	E	V	E	R	S	I	N	G	S
0	1	2	3	4	5	6	7	8	9

"Now choose any two numbers from *SINGS* and one from *NEVER.* Write them as a three-digit number," his uncle instructed. The boy chose, 8, 6, and 3. His uncle wrote the number 863. "Now reverse the figures and subtract the smaller number from the larger."

$$\begin{array}{r} 863 \\ -\,368 \\ \hline 495 \end{array}$$ (same digits in reverse order)

His uncle turned the answer back into letters: "*R-S-S* . . . Nope, that doesn't seem to have done the trick. Reverse them again. And you'd better *add* this time." So the boy wrote the reversed number underneath and added: $\underline{+\,594}$

The uncle went on: "If fiddling is to be your *forte,* as musicians say, you'd better multiply by 40, hadn't you?" This the boy did: $\underline{\times\ 40}$

"Now decode those numbers," the uncle said with a knowing look, "and I think you'll know what your violin needs to make it sing." The boy turned the numbers back into letters, and found that what he needed was _____. Can you work it out?

"The strange thing is, it works whatever pair of numbers you pick from *SINGS* and whatever single number you choose from *NEVER*. One thing, though. In the first number you write, the first digit must be at least 2 more than the last—564 won't work, for example, because 5 is only 1 more than 4. But if you try it on a friend, all you have to do is write 954 instead. You can write the first two numbers chosen in either order.

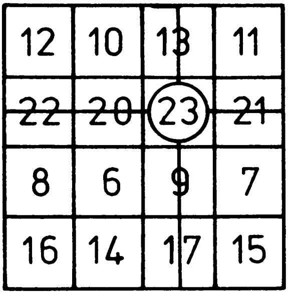
It also works for one number chosen from *SINGS* and two from *NEVER*. More surprisingly, it works for a pair of numbers chosen from *SINGS* and a pair from *NEVER* to make a four-digit number. Now see the answer to this puzzle.

Magic Matrix Force

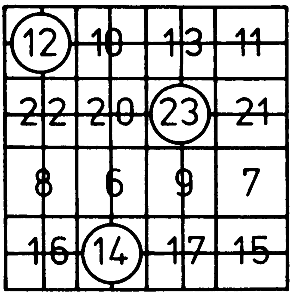
This breathtaking stunt depends on nothing more advanced th n a simple addition table. I have used it on stage and in clubs. A *matrix*, by the way, is simply a square array of numbers. And a "force" is what magicians use to make a chosen number reappear. The trick goes very like "The Calendar Trick."

You, the magician, ask for a number from your audience, between 30 and 100. We'll say it is 56. You now predict that you will give that number back by making up a magic square of numbers. You then proceed to fill in *very rapidly* and apparently in scrambled order a table with four numbers on a side, beginning with the numbers 6, 7, 8, and 9, as shown below left:

12	10	13	11
22	20	23	21
8	6	9	7
16	14	17	15



Next you ask for any number in the table to be selected. Suppose it is 23. You ask for a spectator to circle it and cross out the rest of the row and column it is in, as shown above right. If he continues in this fashion, the board might be completed as shown below leaving one number (7), which has to be circled: There's then no choice. Now you invite your audience to add up the four circled numbers—23, 12, 14, and finally 7, giving 56. This was the number your audience supplied and your matrix has "forced."



Filling in the table is the hardest part of the trick. With a little practice and no little nerve you will master it. You subtract 30 from the number supplied; then divide by 4 and note the remainder. For 56, you get 26 and then 6 with a remainder 2. Then 6 is the first number you put down in the table. You continue writing the numbers 7, 8, 9, and so on, keeping exactly to the same order of squares in each row. In our example we used middle left square, far right, far left, then middle right square. But in filling up the last line you do not put 18 above the 6; instead you add on the remainder of 2, making 20. Then you write the last three numbers in the same order. Write the numbers quickly, and your audience will believe you are writing them in scrambled order.

Using Someone's Age

Another age-telling trick depends on a neat sleight of mind. I'll explain how it works in the answer section. This time you predict a total using someone's age. Say to a friend: "Write down the year of your birth. Under that write the year of some great event in your life—for instance, when you saved someone from drowning or when you learned to ride a bicycle. Under that write down your age at the end of the current year, as of December thirty-first. Under that jot down the number of years ago that the great event took place. Now add up these four numbers."

At this point you surprise him by telling him the total.

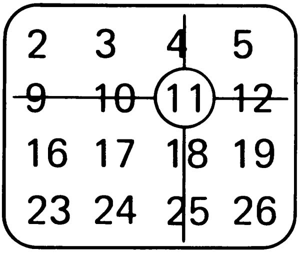
The Calendar Trick

This prediction trick was invented by the mathematician Mel Stover. Find an old calendar. On it a spectator blocks off any square of numbers, four numbers on a side, that he chooses, as shown:

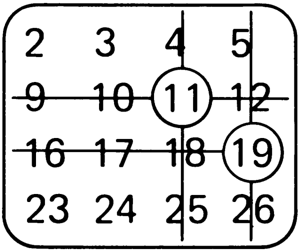
Su	M	Tu	W	Th	F	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

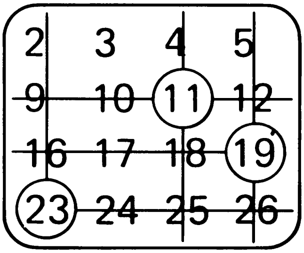
Turn away from your audience so that they think you need to hide your proceedings. At this point you hand a folded slip of paper to someone in the audience, with instructions not to open it until you say so.

With your back to the audience you say to your subject something along these lines: "Circle one of the dates in the blocked-off square." Unknown to you he circles, say, 11. "Cross out all the other dates in the row and column the circled date is in." The calendar now looks like this:



At your request he secretly circles another date (19) and crosses out the horizontal and vertical dates, as shown in the picture on the left, and then he repeats the process with yet another date (23), as shown in the picture on the right.





Only one date is left which is not circled or crossed out. Have him circle this date (3) and then add up all four dates circled. You ask the audience member to read aloud the number on the folded slip of paper. Everyone is amazed that the number, 56, is the same as the sum of the dates.

How did you manage to predict it? You add either two opposite corner dates (23 + 5 or 2 + 26 = 28) and then double the result. For an explanation, see "Magic Matrix Force."

Hundred Dollars for Five

I do a mathematical-magical stage routine in which I put this apparently attractive bet to my audience (I have changed the money to dollars here, although in England I work in pounds):

"I'll pay a hundred dollars to anybody who can give me five dollars in ten coins—half-dollars, quarters, and dimes only. You have to use at least one of each. One hundred for five. Any takers?" The audience usually says nothing. Some scribble on the backs of cigarette packs. But nobody has yet taken me up on this bet.

"Perhaps," I go on, "you haven't any small change? That's O.K. Just jot down on a scrap of paper how many coins of each kind–halves, quarters, and dimes–you must show me. And I'll pay you one hundred dollars." There can be no takers. Why?

No Questions Asked

This is a clever twist on the "think of a number" trick. In this trick you say what the "thought" number is without asking a single question. You can perform the trick with several friends all at once. Each friend picks a thought number from 51 through 100 (in this trick each can be different!). You are the magician, and you write a number from 1 through 50 and put it in an envelope.

In your mind subtract your "envelope" number from 99. Say the result aloud; then tell the friends that each must add it to his number, cross out the first digit of the sum, and add that same digit to the result. He must then take away the answer from his thought number to get his final answer. Your friends don't know it, but their final answers are all the same. Each in turn looks in the envelope to read his final answer. You could also write numbers in several separate envelopes–one for each friend.

How is the trick done? See the answers at the end of the book.

Lightning Sums

Here is a way to impress your friends as a lightning calculator. Ask a friend to pick any two numbers–2 and 5, say. Then he must write one under the other like this: 2
5

But he mustn't show you the list. Add the two numbers and write the sum (7) beneath them. Add the bottom two numbers, 5 and 7, and put their sum below (12). Repeat the process until there are ten numbers, as shown.

2
5
7
12
19
31
50
81
131
212

You ask to glance at the list; then quickly turn your back. You ask your friend to total the ten numbers. Long before he has finished, you announce the total: 550. How is it done?

When you quickly glanced at the list, you noted the fourth number up from the bottom—that is, 50. You multiply it by 11. This is easily done if you mentally see the number set out like this: 50

$$\begin{array}{r} 50 \\ +\ 50 \\ \hline \end{array}$$

Magic Year Number

Suppose the year is 1980. Say to a friend: "Write your shoe size. Ignore half sizes. Now multiply your shoe size by two, add five, and then multiply the result by fifty. Add the Magic Year Number—twelve hundred and thirty. Then take away the year of your birth. You have a four-digit number. The last two digits give your age." (The Magic Year Number changes from year to year: In 1979 it is 1,229; in 1980 it is 1,230; in 1981 it is 1,231; and so on.)

Actually, only the last two digits of the Magic Year Number matter. So you could use either 1,230 or simply 30, but using either might *j-u-s-t* give the trick away. See the answer section for the reason.

Suppose your friend's shoe size is 7½. She is 12 years old; so, since this is 1980, she was born in 1968. The shoe size, after the fraction is dropped, is 7. When she multiplies this by 2, she gets 14. Adding 5, she gets 19. After she multiplies this by 50 (or she could multiply by 100 and divide the result by 2), she gets 950. Adding the Magic Year Number for 1980 (1,230) yields 2,180. Subtracting the birth year (1968) yields 212. The last two digits reveal her age—12.

4. Magic Squares and Sliding-Block Puzzles

These puzzles date back to early China, where legend says Lo Shu found a 3-by-3 magic square scratched on the shell of a tortoise. We only show 3-by-3 and 4-by-4 magic squares. The magic of the squares is simply this: whether you add across, down, or along each diagonal, all the numbers in a row add up to the same magic number.

Magic Squares

See if you can recreate the Chinese magic in a large square made up of three squares on a side, using the numbers 1 through 9. When you have filled in the squares, or cells, the numbers across each row, up and down each column, and along each of its two main diagonals must all come to the same number—the magic number 15.

In this three-by-three "magic" the magic number is always 3 times the central cell. When you have filled in your "magic," you will find there are actually eight different large squares. You can turn it around in four different positions, and each position has a mirror image, giving a total of eight different "magics."

See the answers for an easy way to remember how.

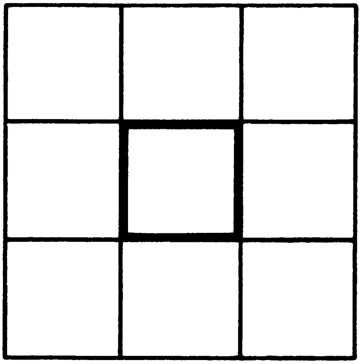
Cross Sums

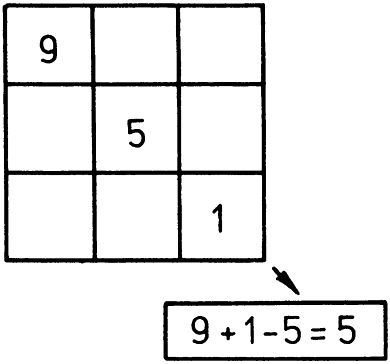
Arrange the consecutive numbers 1 through 9 in a cross like the one shown. But the across row must add up to the same sum as the down row. Can you do it?

		1		
		2		
6	7	3	8	9
		4		
		5		

A Number Square

Put the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 into the cells of a square like the one shown so that the sum of the two outer numbers less the number in the middle comes to 5. This must work for all four directions through the middle square—across, up and down, and along the two diagonals. Ignore the cases where all the squares are on the outside. For example, the diagonal of numbers shown in the second picture is correct.

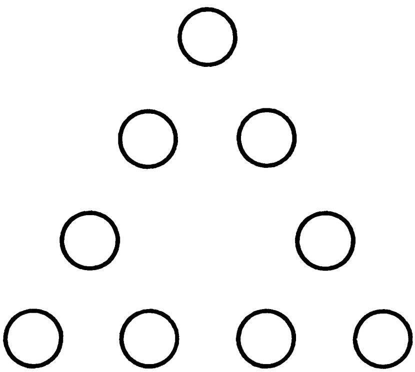




A Triangle of Numbers

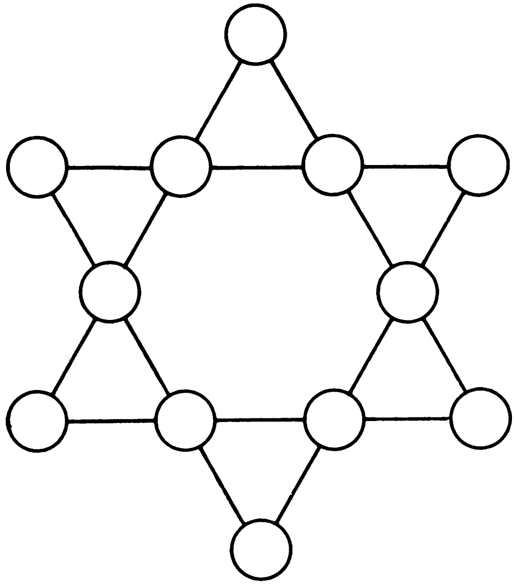
Can you put the numbers 1 through 9 into these discs so that each side of the triangle adds up to 20?

HINT: The corner discs add up to 15. So one of them must be 5.



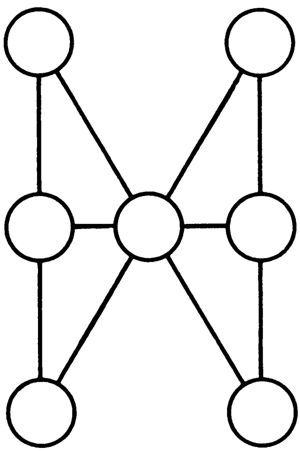
Star of David

Can you put the numbers from 1 through 12 in the discs on this star so that the sum of each of the six rows comes to 26?



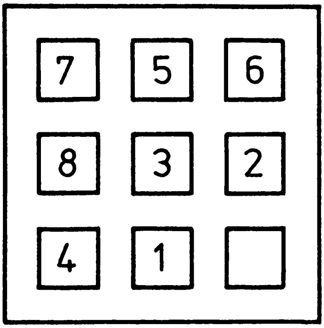
Number Rows

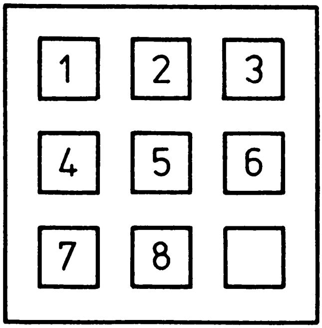
Use each number from 1 through 7 once only. Can you arrange them so that each row adds up to 12?



The Eight-Block Puzzle

Here is a reminder of the famous 15-block puzzle—a plastic puzzle you can buy in stores—where you have to slide numbered blocks about in a box. Eight blocks are numbered from 1 through 8 and put into a box, as shown in the picture on the left.





You can only move one block at a time into an empty space. No block may be lifted out of the box. The object is to shift the blocks around until you get them in the order shown in the picture on the right.

Well, it's not hard if you can have as many moves as you like. But see if you can do it in 22 moves. To record your moves, you only need note the numbers in the order they were shifted. Thus the first six moves are 2, 6, 5, 3, 1, and 2. This notation is quite clear. You can make your puzzle out of numbered slips of paper that you slide around on our picture. The same puzzle can be made out of lettered blocks, lettered *A* through *H*.

Four-Square Magic

The easiest way to make a four-by-four magic square is as follows: Draw a four-by-four grid with 16 squares. Lightly pencil in the numbers from 1 through 16. Now reverse the two main diagonals about the center. So 1 swaps places with 16, 6 with 11, 4 with 13, and 7 with 10:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

That done, add up the numbers along each row across, each column, and each of the two main diagonals. What's the magic number?

Surprisingly, you can swap rows or columns, and the four-by-four square is still magic. Swap the two middle columns so the square looks like this:

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Check that it is still magic. Take pairs of opposite sides in the dotted square and add them: 3 + 5 + 12 + 14 and 2 + 8 + 9 + 15. Do you see anything? Now take the squares of these numbers: $3^2 + 5^2 + 12^2 + 14^2 = 9 + 25 + \ldots$ and $2^2 + 8^2 + 9^2 + 15^2 = 4 + 64 + \ldots$. Carrying on, what do you see?

Some more surprising things about a four-by-four "magic" are:

The corners add up to 34

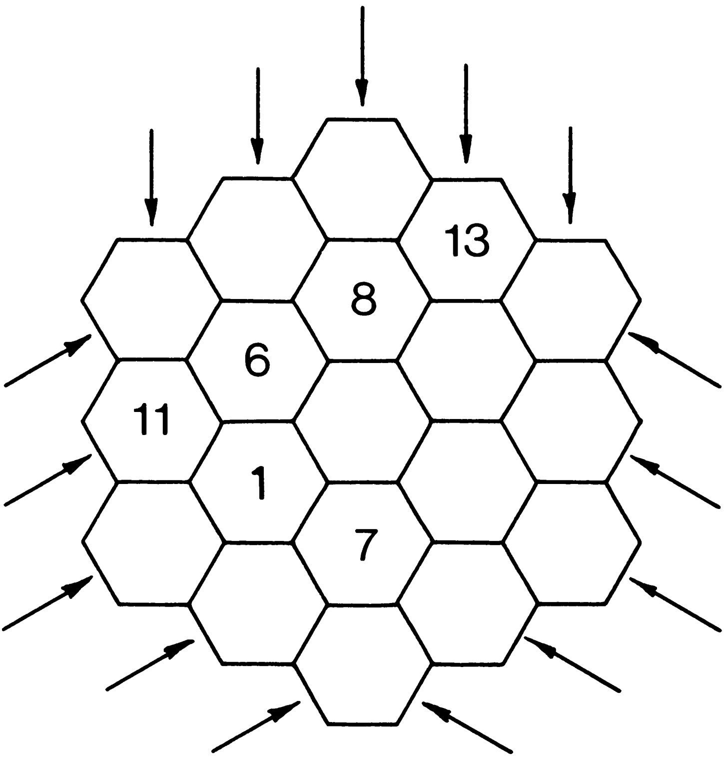
The five two-by-two squares, in the corners and the center, all add up to 34

In each row one pair of adjacent numbers adds up to 15, and the other to 19

The painter Albrecht Dürer made an engraving called *Melancholia.* In it he shows a four-by-four magic square—just the second one shown here. If you look at the bottom of the middle columns you can tell the date he made the engraving. What was the date?

A Magic Honeycomb

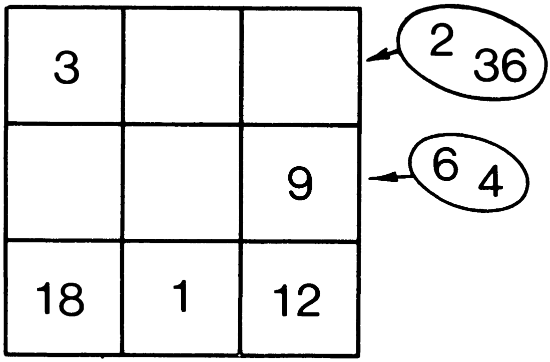
Use the numbers from 1 through 19. Can you put them in the bee cells so that each arrowed row adds up to 38?



Multi-Magic Square

A multi-magic square is like an ordinary magic square except that you multiply numbers in each row instead of adding them. Each row, column, and diagonal multiply to the same number, the multi-magic number. Can you see what it is?

I have started you off on the smallest multi-magic square with the lowest numbers possible. You have to copy the square and fit in the numbers shown to the right of the square in their correct cells. There are four ways of doing this, but only one of them is right.

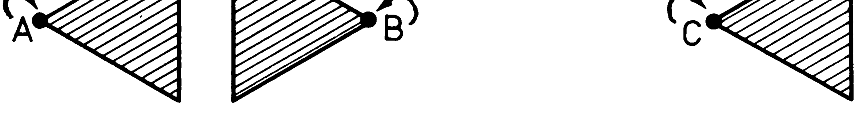


5. Illusions

Optical illusions are drawings that fool the eye. They are not only fascinating in themselves, but they provide the material for excellent puzzles. Seeing is believing, they say. But can you always believe what you see? The next batch of puzzles will test your belief. But I must let you into a secret. It is not only the eye that is fooled by the pictures that follow; the brain too takes part in the deception. We see what we have learned to see. So strong is this habit that we refuse to see the evidence before our eyes. Instead we see what fits in with our experience. I should also mention that you may not see all the illusions; not every illusion may work for you. It depends by and large on your seeing "habits." Most of the pictures should puzzle you. Don't look at the answers until you have checked the illusions with a ruler.

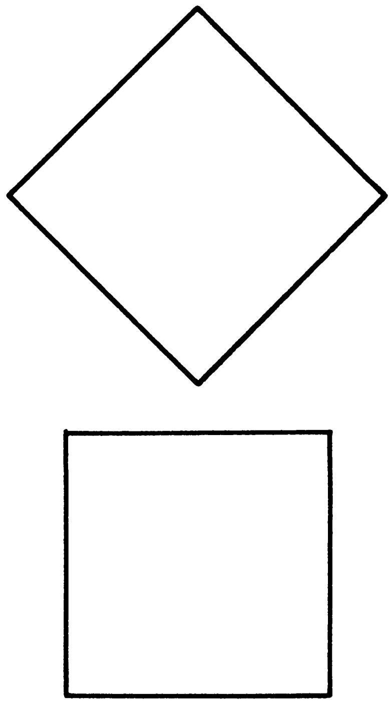
Illusive Lengths

Which is bigger, the length AB or BC?



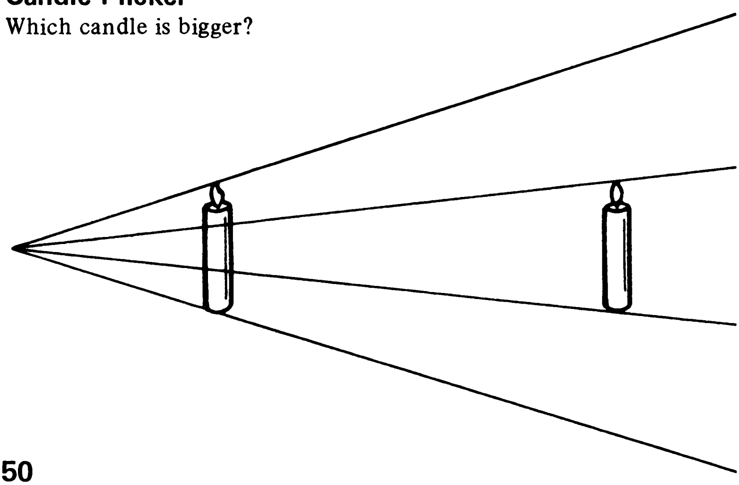
Diamond-Square Puzzle

Which is a diamond and which a square? And which is the larger?



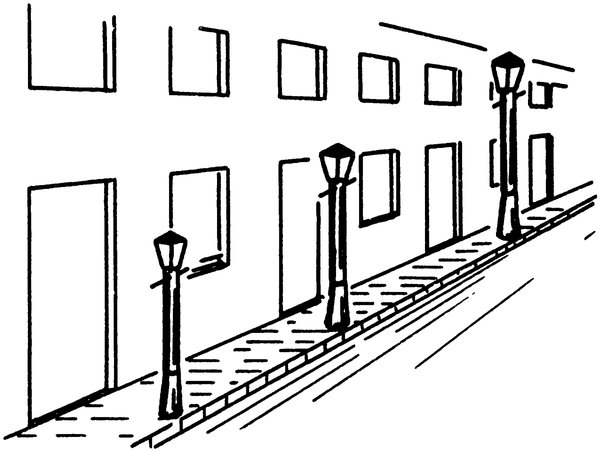
Candle Flicker

Which candle is bigger?



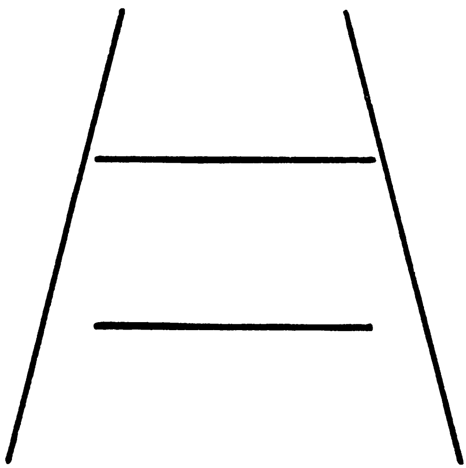
The Three Streetlamps

Cities used to have attractive streetlamps to light the sidewalks at night. Which of the lamps shown here is the tallest?



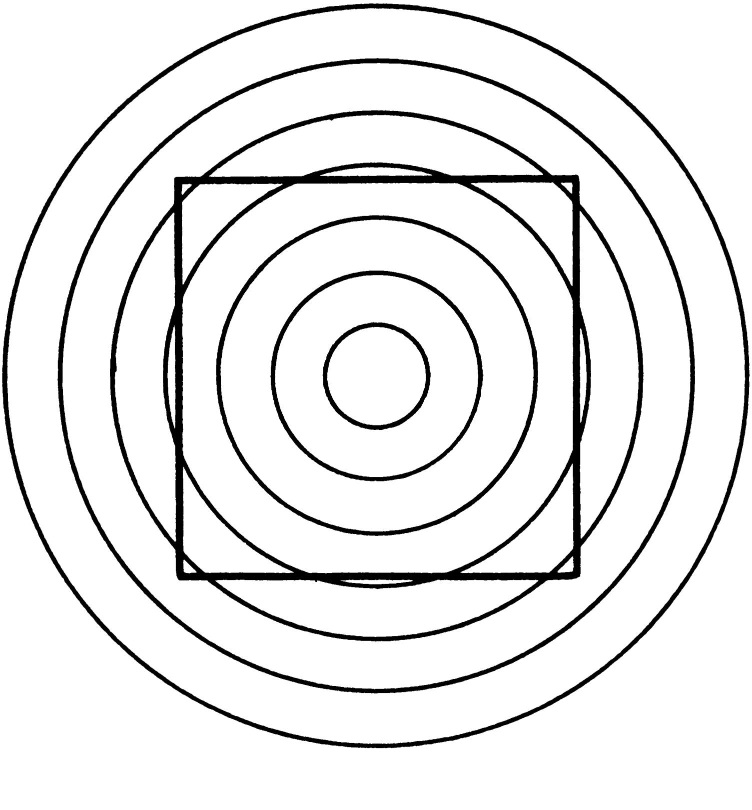
Railroad Track

This is a very famous illusion. Which of the two horizontal lines—that is, the ties between the tracks—is the longer?



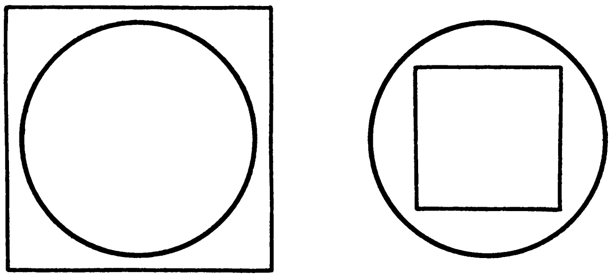
Target Square

The target circles look true enough. But what about the square?



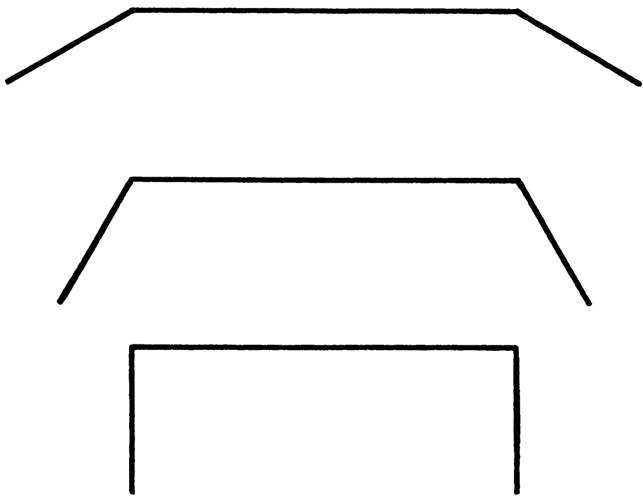
Squaring the Circles

Which of the two circles is bigger?



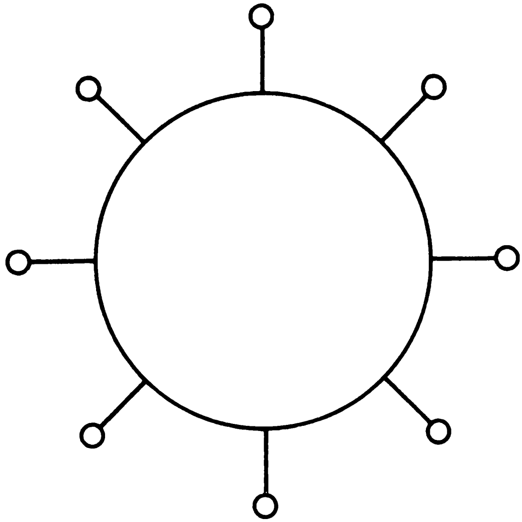
The Flap Tabletop

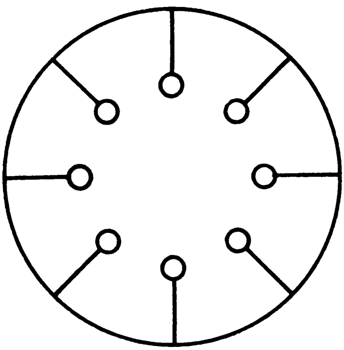
You know those tables with flaps on two sides. The picture shows three flap tables. Which tabletop is the longest?



Run Rings Around You?

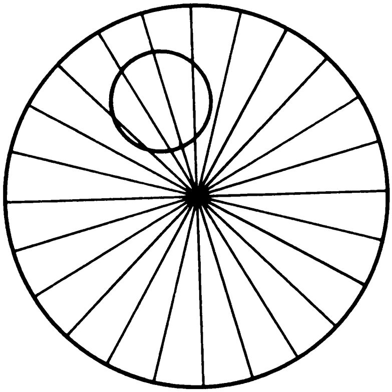
Which of the large circles is bigger?





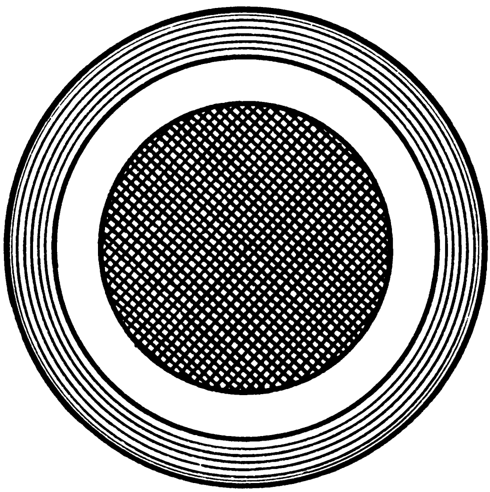
Circle Counting

How many circles are there here?



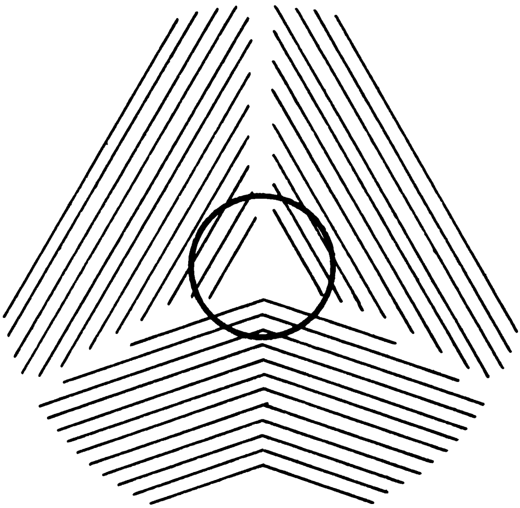
Bull's-eye and Ring

Which has the greater area, the bull's-eye or the outer ring?



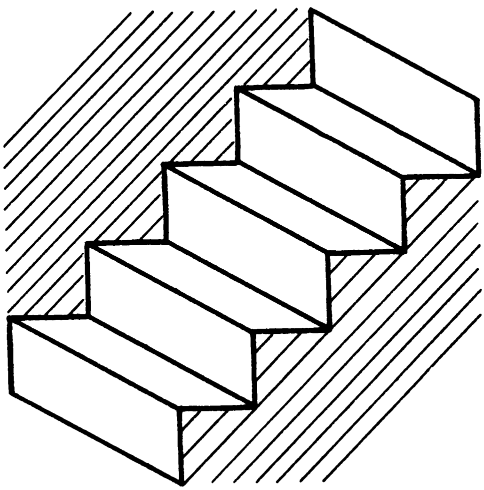
Circle Sorcery

Something has happened to the circle here. How much is bent?



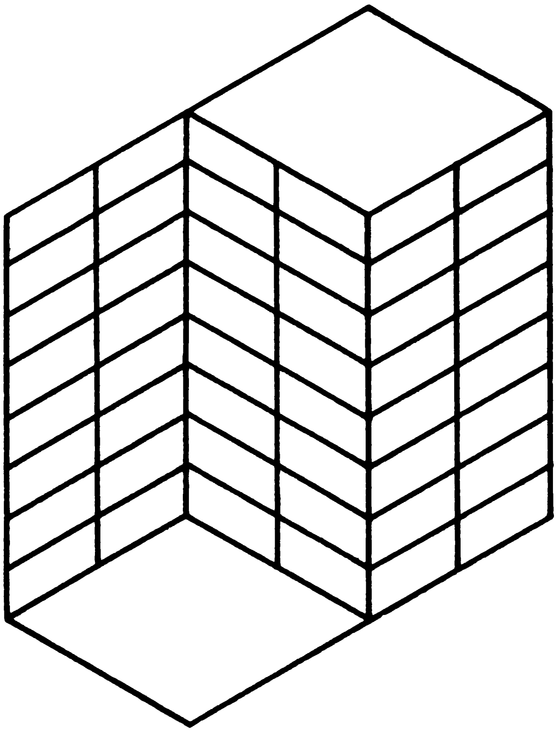
Upstairs, Downstairs

One moment you seem to be looking upstairs, the next downstairs. Can you believe your eyes? There is no answer to this puzzle.



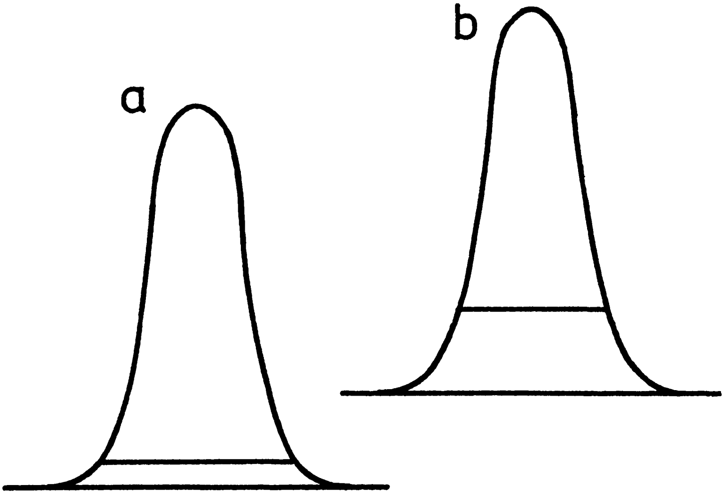
The Tricky Tower

Try to fix this figure before your eyes. You cannot: It shifts as if one moment you are looking down on it, the next under it. There is no answer.



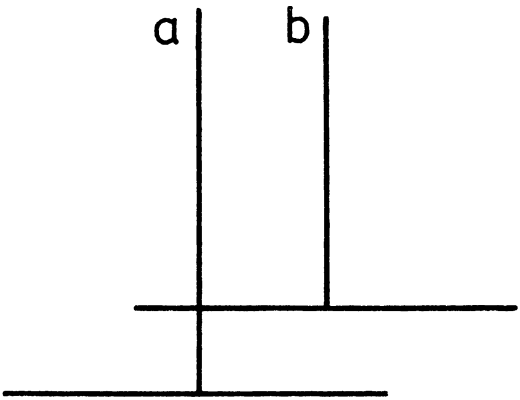
Two Tall Hats

Look at the top lines of the hatbands on the two hats here. Which line is half the width of its brim?



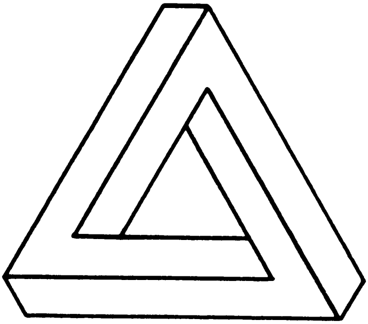
The Two T's

Which T shape has equal height and width, *a* or *b*?



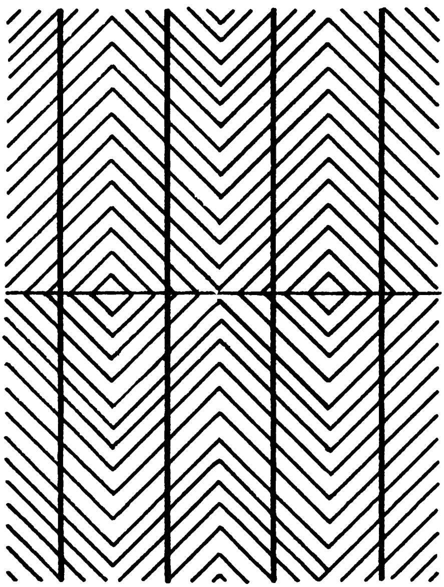
Impossible Triangle

This has to be the most famous illusion invented this century. Two psychologists, both called Penrose, dreamed it up. There is no answer. It is used in many works of modern art to fool the eye of the beholder.



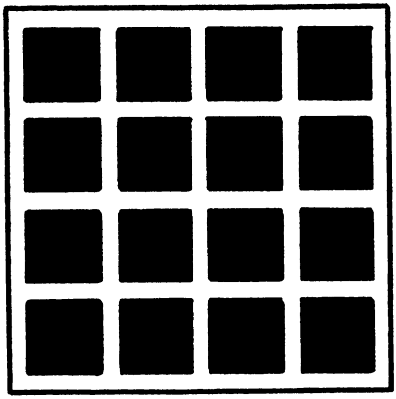
The Herringbone Pattern

The four heavy lines are meant to be straight. Are they?



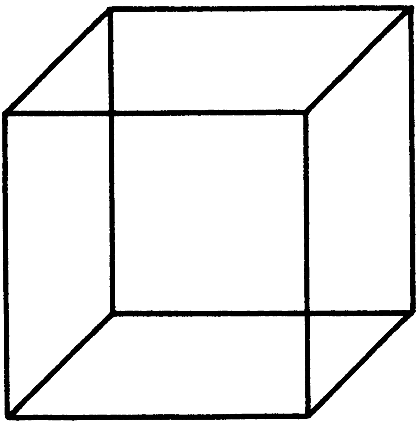
The Blinking Squares

Stare at this grid of squares, and I bet you will see gray patches come and go in the white crossings. And yet, as you can see, there is no gray printed on the paper. There is no answer.



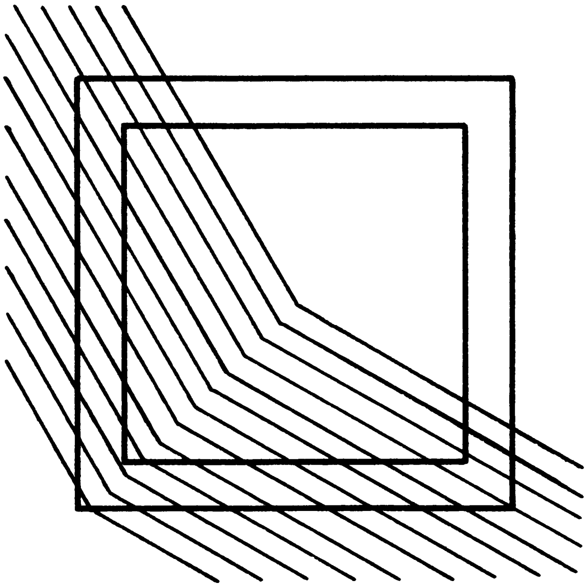
The Bamboozle Box

Actually, this is called the Necker cube after the scientist who noticed the effect. The box seems to flip in and out before your very eyes. There is no answer.



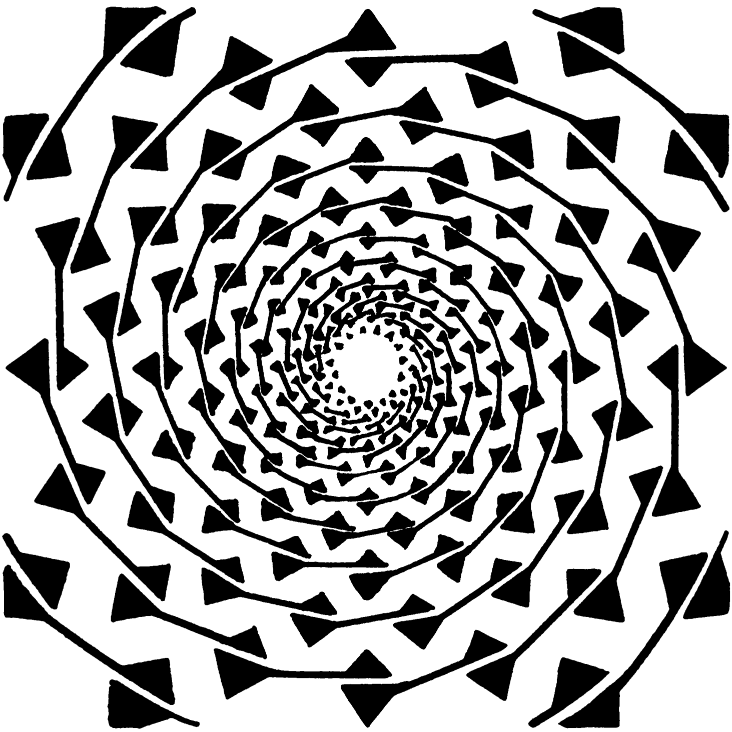
The Picture Frame

You'd better check the frame. It doesn't look square.



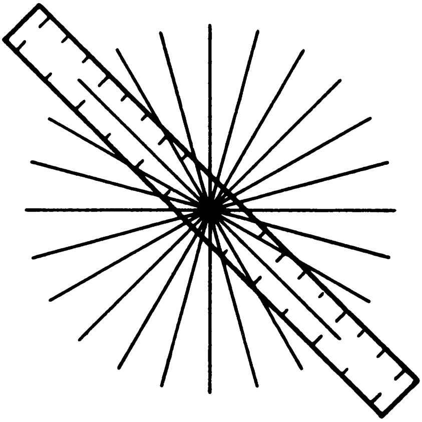
The Fraser Spiral

Here's another very famous illusion. Just follow around the whorls of the spiral and what do you see?



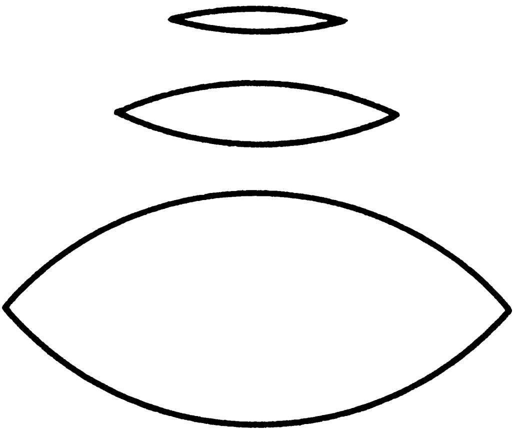
The Swollen Ruler

What's happened to the ruler? Has it swollen?



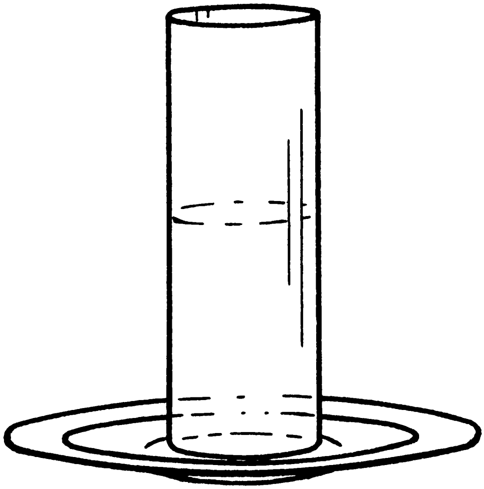
Zooming Lenses

Look at the surfaces of these three lenses. Which do you think is most curved, the little lens or the big lens?



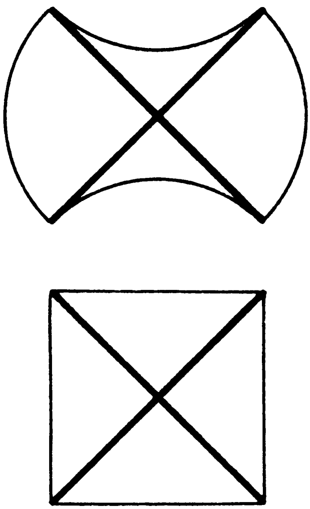
The Long Glass

Well, the long glass is rather taller than the plate it rests on is wide, isn't it? Guess by how much, then check with a ruler.



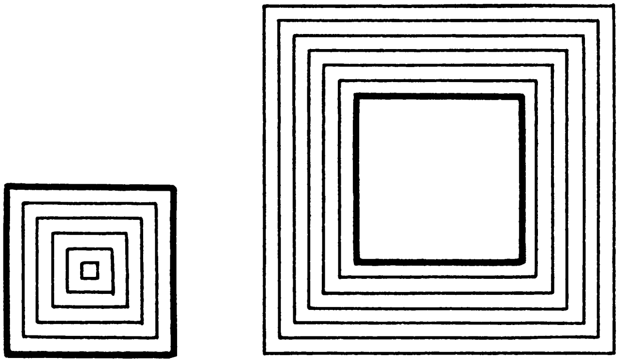
The Black Crosses

How much smaller is the black cross in the square than the black cross in the top figure?



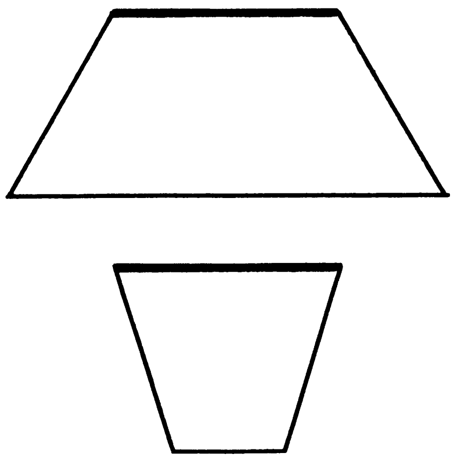
The Blacker Squares

Which of the two blacker squares shown here is the larger?



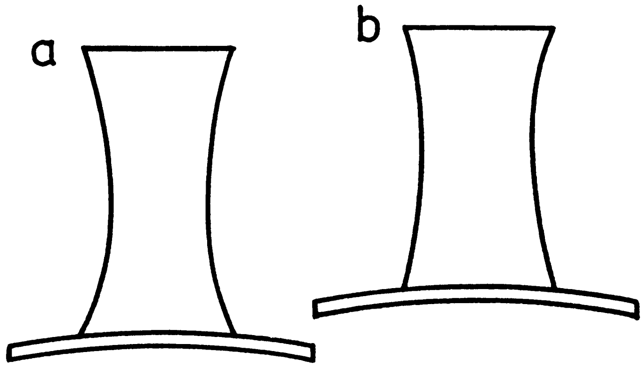
Lampshade and Flowerpot

Look at the heavy lines in the two drawings. Which do you think is longer?



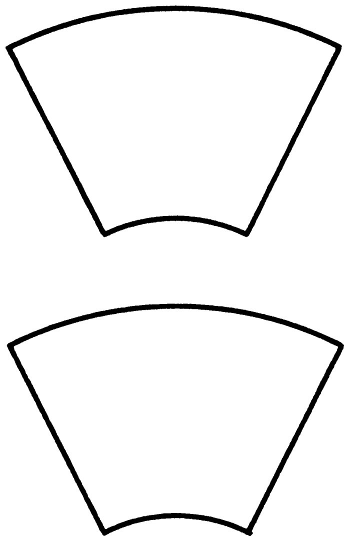
The Top Hats

I can tell you that the height and width of one of the hats shown here are equal. The question is, which hat, *a* or *b*?



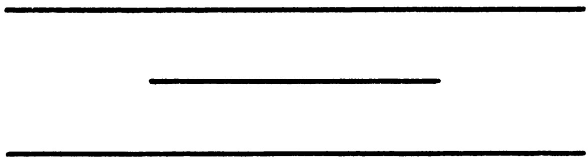
The Fans

Which is the wider fan?



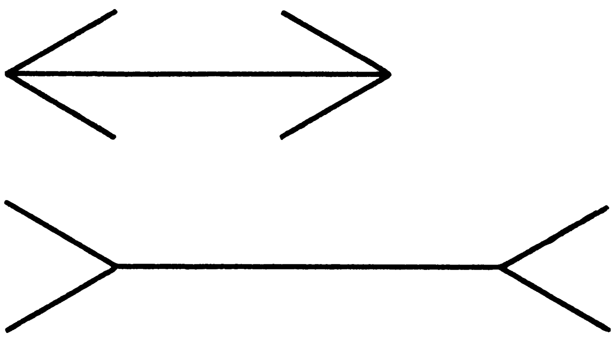
The Long and Short of It

Is the middle line half as long as the longer ones?



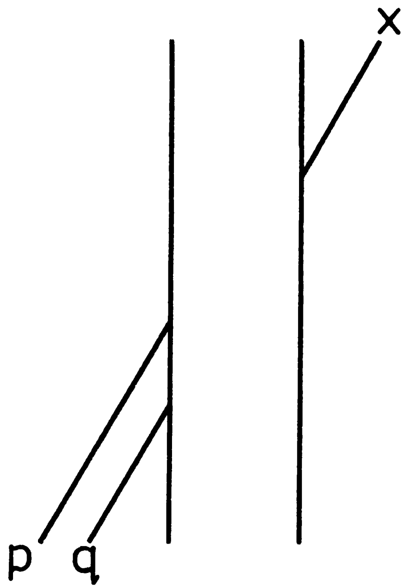
The Artful Arrows

A very well-known illusion. Obviously, the double-headed arrow is shorter than the other figure. But by how much?



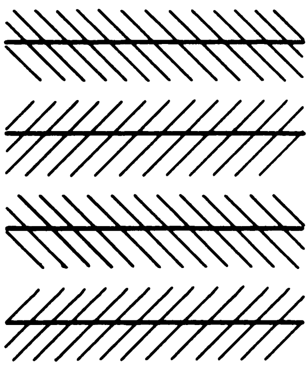
Straight As an Arrow

Run your eye along line *x*. Think of it as an arrow passing through a door, the two vertical lines. Which line, *p* or *q*, is the continuation of the arrow?



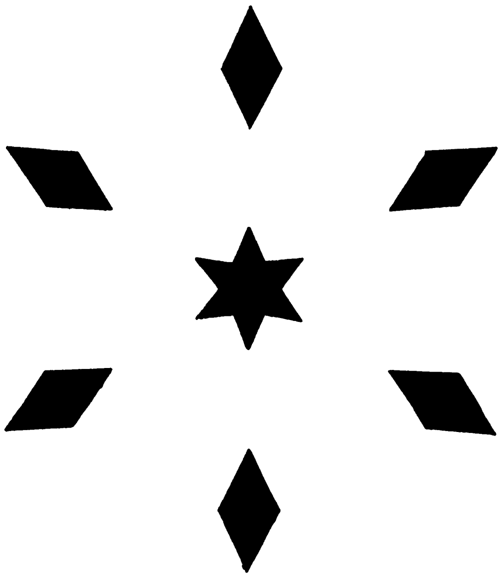
Skew Lines?

When lines are not parallel, they are called skew. Well, are these horizontal lines skew or parallel?



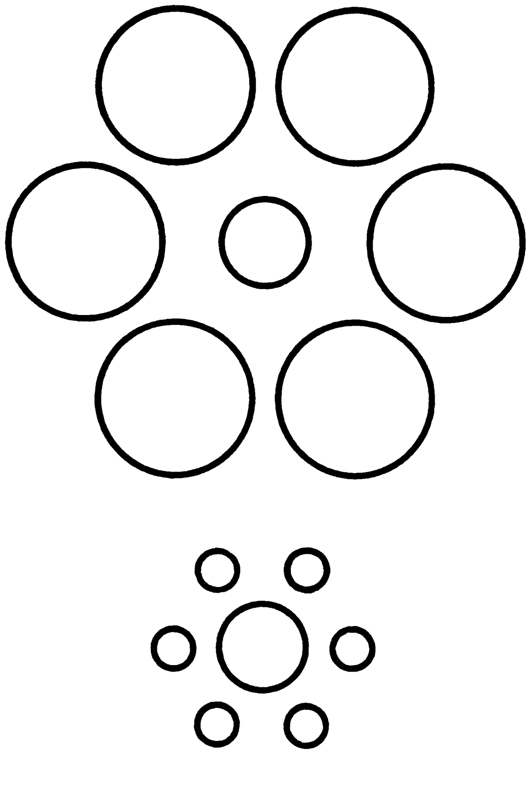
Stars and Diamonds

Look at the distance between a tip of the star and the tip of the nearest diamond. Is the length of a diamond the same as this distance?



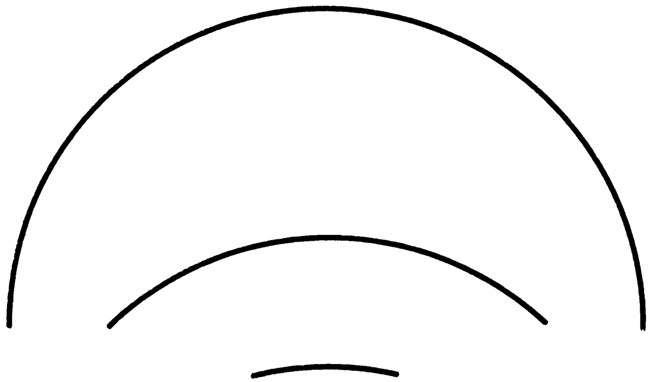
Circles Before Your Eyes

Look at the center circles in each group. Which one is larger?



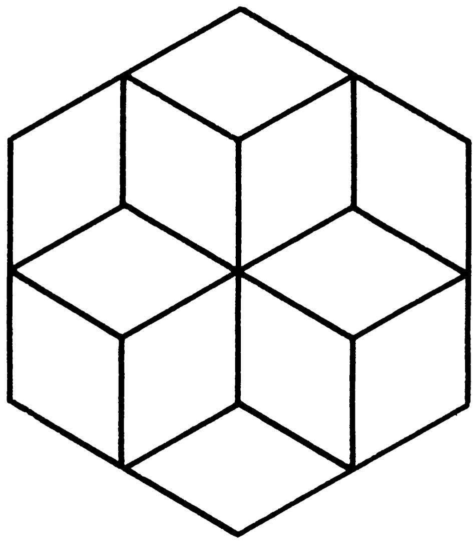
Arcs More or Less Curved

Which of the arcs shown is the most curved?



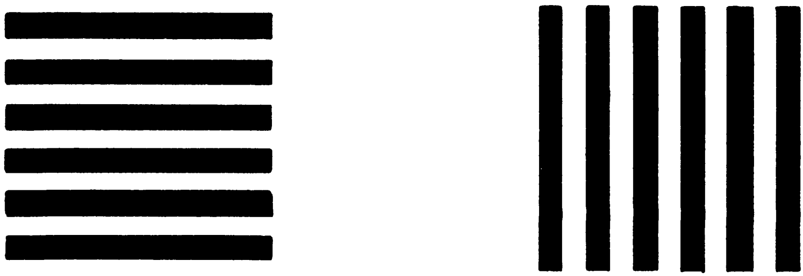
Funny Figure

Stare at this figure and you'll be able to see it in eight different ways.



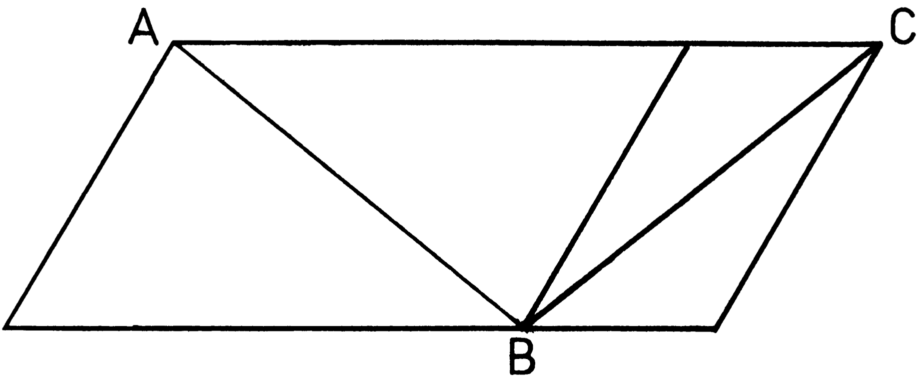
Hickman's Squares

This illusion was created by the artist who drew all the pictures in this book, Pat Hickman. Can you tell which of these figures is not a square? Or are they both squares?



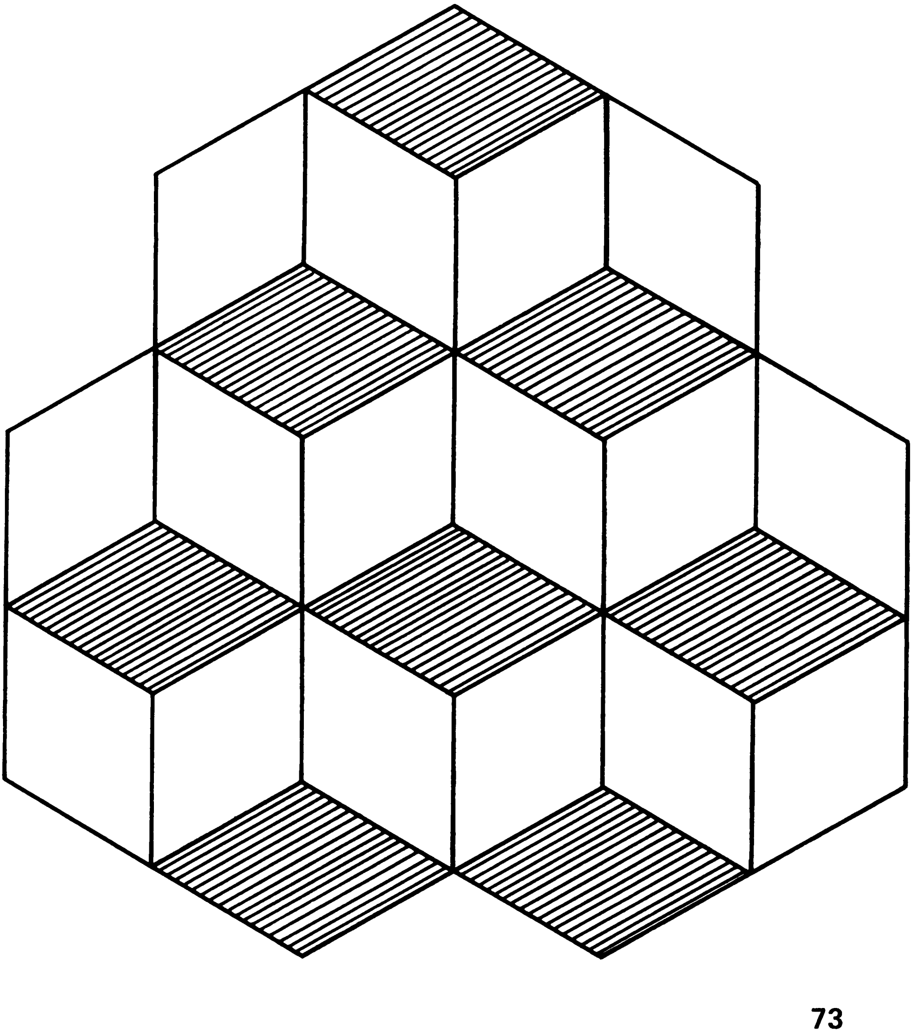
Devilish Diagonals

Which is longer, *AB* or *BC*? Guess first, then measure and see for yourself.



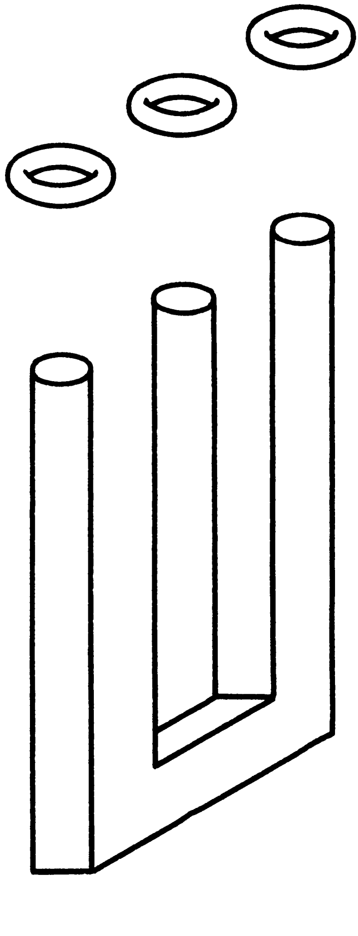
The Extra Cube

Here is a well-known optical illusion. Look hard at these six cubes. Can you see by any chance an extra cube?



Rods and Rings

This illusion depends upon the same principle as the impossible triangle. There are three rings about to slip onto three rods. First you see it, then you don't! There is no answer.



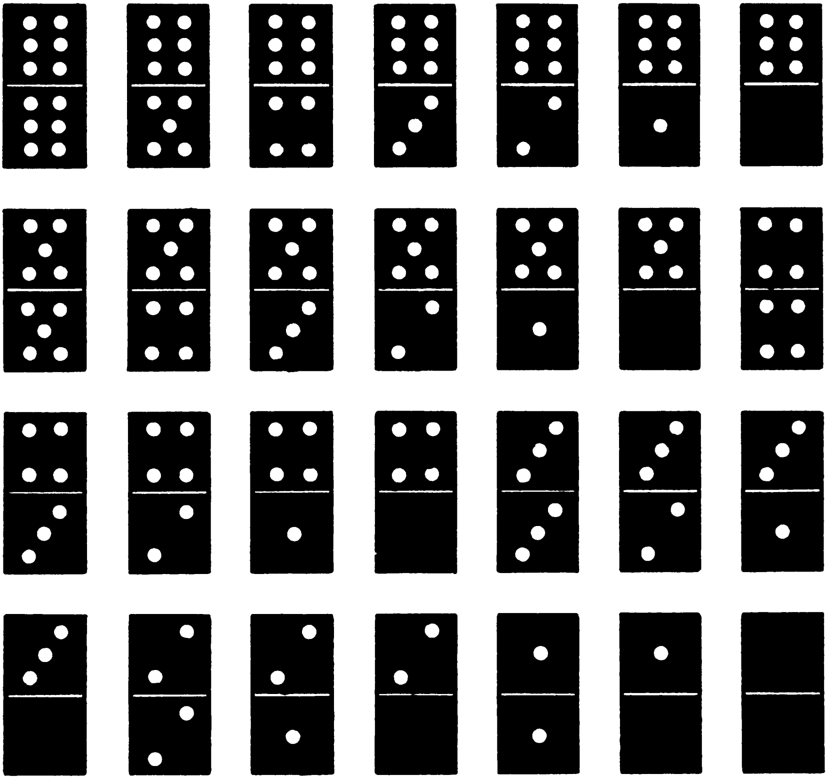
6. Dominoes and Dice

The next batch of puzzles is about dominoes and dice. Dice, as you probably know, are small cubes with six sides that are numbered by dots from one to six, so placed that the sums of the dots on a side and the opposite side equal seven. Dice is probably one of the oldest of all games. They have been found in ancient Egyptian tombs and the ruins of Babylon.

Dominoes are derived, in remote times, from dice. They are oblong tiles, usually known as bones, marked in two squares, with from zero up to six dots on each square. It is the only parlor game to use the number 0. Dominoes are marked with all possible combinations of numbers that can be rolled with two dice. There are 21 combinations. The number 0 is always added in the form of a blank, making 28 bones in all, as shown in the picture below.

The basic rule of play is: You add to a chain of dominoes by playing a bone with its end matching an end of the chain. So you play a 6 against a 6, a 5 against a 5, a blank against a blank, and so on.

A domino on which both squares have the same number of dots is called a doublet. The bone at the top left of the picture on the next page is a doublet.



Dominoes—with a Difference

Try playing dominoes where you put down a tile whose numbers do *not* match at either end of the chain. A good rule is: The difference between numbers on adjacent tiles must be 1. Can you form closed chains this way?

Or play to this rule: The sum of the numbers on adjacent tiles must be 7. Thus, next to 3 you play 4, next to 6 a 1, and so on. This way you can make closed chains.

The Dot's Trick

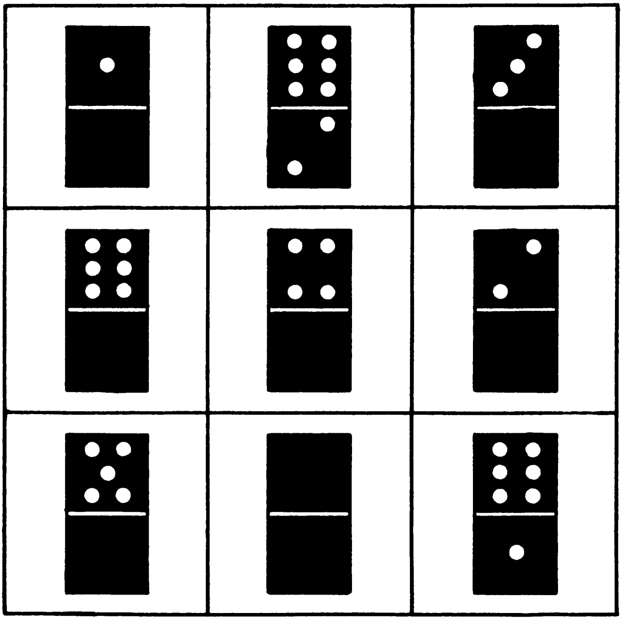
Empty a box of dominoes on the table and spread them out. Put them in a chain according to the usual domino rule: Ends next to each other must match. Say 3 is at one end of the chain. How many dots will be at the other end? Do it in your head. Then check with real dominoes. How is the trick done?

Domino Trick

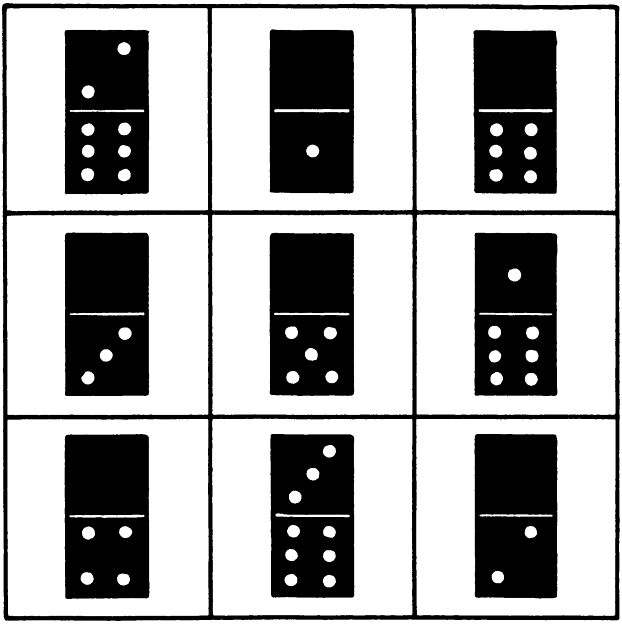
Secretly pocket a domino. Make sure it is not a doublet, since this may make the trick too obvious. Ask a friend to make a chain of the remaining 27 dominoes, which he thinks are a full set. You predict what the numbers at the ends of the chain will be. They will be the very numbers on the bone you pocketed. How does it work? "The Dot's Trick" should tell you.

Magic Domino Squares

The first picture here shows a three-by-three magic square made out of dominoes instead of the usual nine numbers. You count the total value of both squares of a domino as the cell's number. For example, the top row reads 1 + 8 + 3; its sum is 12. What is the magic number for the square? Add up all the rows, columns, and both diagonals.

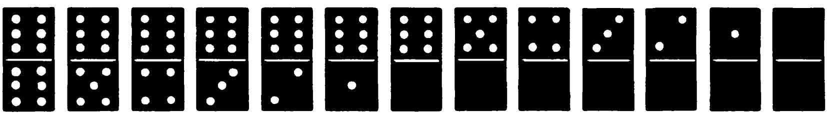


Look at the domino square on the next page. What is its magic number?



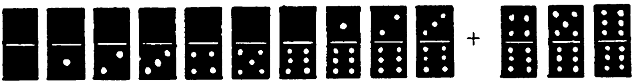
Party Trick

Put a box of dominoes face down in a row. You turn your back and ask a friend to move any number of bones, up to 12, from the *right* end of the row to the *left.* After he does this, you turn around and turn over a bone. The number of its dots is the number of bones moved! The trick is, you previously picked out all 13 bones with a 6 or a blank on them and then placed them face down as in our picture. Then you put the rest of the bones to their right. When you turn around, you turn over the thirteenth bone from the left. Why does it work?



Domino Fractions

From a box of dominoes take out all the doublets (double 6s, double 5s, and so forth) and any bones with a blank on them. That leaves fifteen bones to play with. You can consider them as fractions. For example:

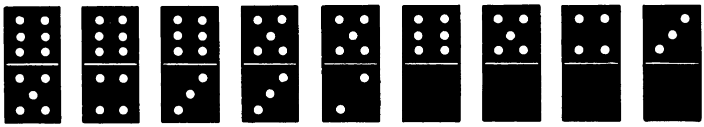


$$= \frac{3}{6} + \frac{4}{6} = \frac{7}{6} = 1\frac{1}{6}$$

Put down the fifteen bones in three rows of five. Now, can you make the sum of each row come to 10?

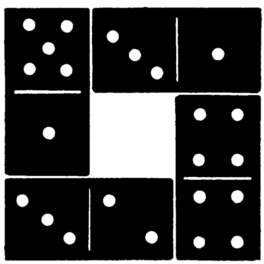
Another Magic Square

Can you form a magic square from these nine bones with a magic number of 21?



Domino Window

The picture shows a window formed by four dominoes. Count the dots along each side. What do the four sums each come to? Make up three more domino windows like this. The sums for the different windows need not be the same as this one, but in each window the sum of each side should be the same.



Reading the Bones

This trick is the same as giving back to somebody the two numbers he first thought of. Here the two numbers are read from the two squares of dots on a domino bone.

Ask a friend to secretly pick a domino. Tell him to multiply one of its numbers by 2, add 4 to the result, multiply the sum by 5, add in the other number on the bone, and tell you the answer. You then tell him the numbers on his domino.

How is it done? What you do is subtract 20 from the two-digit answer he gives you. So if he said 35, you subtract 20, leaving 15, which means he picked the bone 1-5. If he said 23, you take away 20, leaving just 3, which means the bone 0-3.

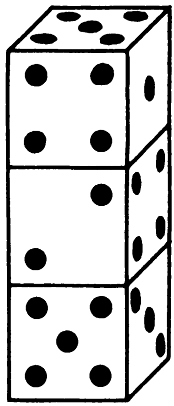
Suppose he chooses the bone 0-2. He multiplies 0 by 2, which gives 0; then he adds 4 to get 4. He multiplies 4 by 5, getting 20, and adds the other number 2 on the bone to get 22, which he tells you. You take away 20, leaving 2. You then tell him he chose the bone 0-2.

Do you know why it works?

Hidden Faces of Three Dice

Here is a very effective trick with dice, well known to most conjurers, which always seems to baffle the audience. I have used it with complete success—even on an audience of math teachers!

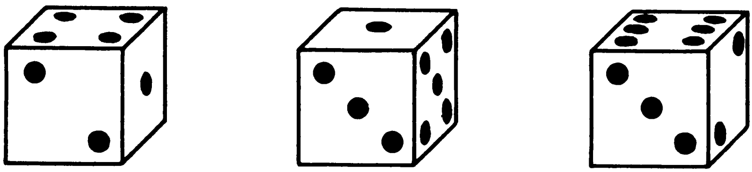
Take three dice and stack them in a pile. Explain to your subject that he is to stack the dice himself any way he likes and then add up the spots on the hidden faces—that is, the five hidden faces. (This means "not the top face," but on no account say this or you might give the trick away!) You turn your back, and he stacks the dice. You turn around and pretend to read the total in his mind. You can make play of certain numbers to heighten the audience's amazement. For example: If you work it out as 20, you ask if he has perfect vision; if it is 18, you might ask when he or she went to college; and so forth. The sum in the picture is 16. How do you work it out?



Three Dice in a Row

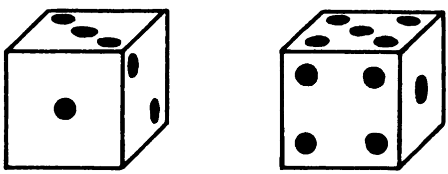
Another amazing trick you can do with three dice is this one: With your back turned, ask your subject to set the dice in a row and add up the spots on the three tops of the dice. Then he is to choose any one of the dice, pick it up, and add in the spots on the bottom face to the sum he has already got. Without putting that die down he is to toss it and note the spots on the top face and add them into the sum he has so far, and to remember the grand total (of the five faces). Instruct him to arrange the dice in a line. Then you say that you cannot possibly know which die he picked up and tossed, yet you will read his mind and tell him the grand total. How is it done?

CLUE: When you turn around, you quickly read off the sum of the spots on the top faces of the dice. Here are the dice at the end of this trick. The magician says the subject's grand total is 18. How does he know?



What Did the Dice Show?

Yet another dice trick. Give a friend a pencil and paper and two dice. (You can do this trick with three dice, but the working is lengthier.) Turn your back and ask him to arrange the dice in order so that he can read the spots on the top faces from left to right and make a two-digit number. For example, he would read 35 on the dice shown. Ask him to peek at the bottom faces and write what they show in the same left-to-right order and attach it to the first number to make up a four-digit number. He would write 42 with the dice in our picture (since opposite sides add up to 7) to make 3,542. Tell him to divide this number by 11 and give you the result. You then tell him what the top faces show.



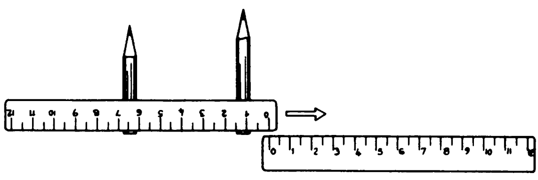
The method is: Take away 7 from the result he gives you, then divide by 9. In our example, 3,542 ÷ 11 = 322, which is the number he gives you. You take away 7: 322 − 7 = 315. Then you divide 9: 315 ÷ 35: the number on the top faces! Can you explain it?

7. Physics Puzzles

The next batch are all puzzles in physics—that is, the laws of nature. You can either solve them by scientific flair or, if you can rig up the experiments, by trial and error. My aim has been to astound or amuse you rather than to test your powers of reasoning. In other words, do not be put off if at first you can see no way to solve one of these puzzles. They have eluded the grasp of some of the greatest scientists who have ever lived, particularly the problems on chance and coin tossing. The puzzles on measuring water out of cans of different capacities can, however, be cracked by simply taking thought.

Ruler Rolling

Rest a ruler on two round pencils as shown here:



Slide the ruler forward, keeping it firmly pressed down on the pencils so it does not slip over them. When the pencils have rolled forward 2 inches, how far will the ruler have moved?

Dollar Bill for Free?

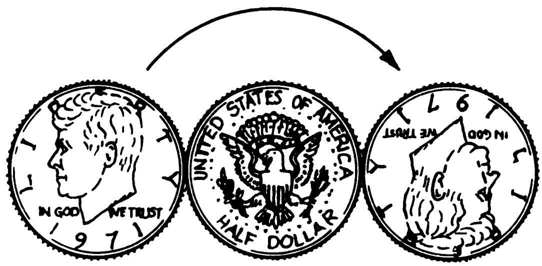
Balance a pile of about six or more coins on top of a dollar bill resting on the rim of a glass. Now bet a friend that he can't pull the bill free without toppling the coins or touching the glass.

The Coin-rolling Bet

Take two coins. Half-dollars will do. Place one coin tails up, the other next to it heads up and with the picture of the President's head as shown. Begin to roll the heads-up coin around the first, making sure there's no slipping.

Now bet a friend that the heads-up coin will end up with the President's head upside down, as shown. The friend is pretty sure to take your bet . . . and lose.

Why?



Monkey Puzzle

Lewis Carroll wrote, as you know, the famous Alice books. He was also, as you may not know, an Oxford professor of mathematics. One day he set himself this problem:

A rope passes over a free-running pulley—no friction at all! A bunch of bananas hanging from one end of the rope exactly balances a monkey clinging to the other end. The monkey begins to climb the rope. What happens to the bananas?

Don't be put off if you find this monkey business too much for you! As Carroll himself wrote: "It is very curious the different views taken by good mathematicians." One said the bunch of bananas goes up with increasing speed; another that it goes up at the same speed as the monkey does; while a third said the bananas went down.

Not only is the pulley frictionless, it is also weightless, and the rope, too, has no weight. (This is quite common in scientific problems.)

A Weighty Problem

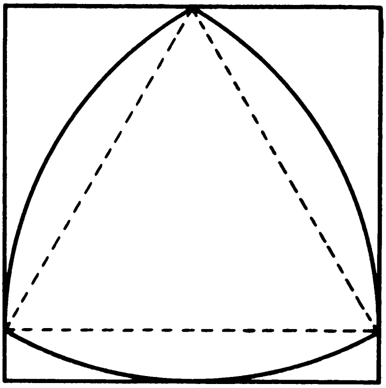
A balance scale has only two weights, 1 ounce and 4 ounces. In only three weighings split 180 ounces of seed into two bags of 40 and 140 ounces.

Tug of War

Sam and Sally take separate ends of a rope in their hands. Sam pulls on his end with a force of 100 pounds. Sally tugs with a pull of 100 pounds. What is the tension of the rope?

Rolling Rouleau

Rouleau is a French word, pronounced "roo-loh." It means a roller, like a common, or garden, roller. But it is rather more unusual than that! It is really the strangest yet simplest invention of math you've ever seen. The curved figure shown here inside the square and outside the dotted triangle is a *rouleau.* It's easy enough to draw. Place the point of a compass on each of the corners of the triangle and draw an arc through the other two corners.



Cut this *rouleau* out of thick card or, better, plywood. Rest a ruler on the edge of it and push the ruler forward with no slipping. You'll be surprised to discover that the ruler moves smoothly along, without bobbing up and down.

If you put an axle in the middle of the *rouleau,* you'll find it works perfectly well as a wheel. How do you find its middle? It will be the center of the dotted triangle (an *equilateral* triangle). Join each corner to the middle of the opposite side with a line; these lines all cross at the triangle's center.

You can design *rouleaux* (plural of *rouleau*) with five or seven curved sides or more. The British 50-pence coin is a seven-sided *rouleau.* Rest a ruler on its edge, and the ruler will slide smoothly over it, just as it would over a round roller.

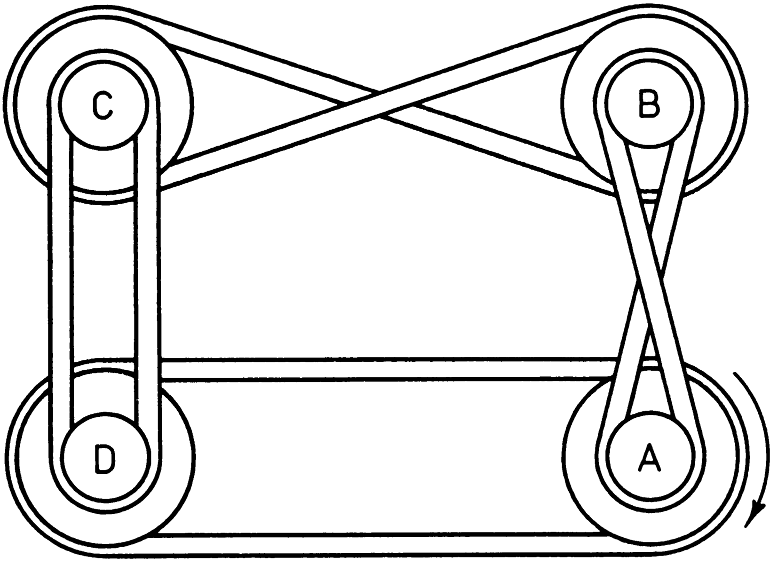
There is one more surprise. Look at the *rouleau* in our picture.. It fits so snugly in the square box. Yet, believe it or not, it can turn inside the square. This is more than a mathematical curiosity. The *rouleau* shape is used in the revolutionary (pun intended!) Wankel engine named after its inventor. Where the everyday automobile has pistons that move up and down in a cylinder, the Wankel engine has a *rouleau*-shaped piston that turns in a square-shaped box.

(There is no answer.)

Belts

Wheels *A*, *B*, *C*, and *D* are connected by belts as shown. Wheel *A* turns clockwise, as the arrow shows. Can all four wheels turn without the belts slipping? If so, which way does each wheel turn, clockwise or counter-clockwise?

Now imagine all four belts are crossed. Can the wheels all turn now? What if one or three belts are crossed?



Magnetic Puzzle

You have two identical-looking bars of iron. One is a magnet, with north and south poles; the other is not. You don't know which is which. The puzzle is to find out. How can you tell just by pushing the bars around and seeing how each pulls (or pushes) the other? Try it out and see for yourself.

HINT: Remember, a magnet pulls iron things toward itself. An iron bar is magnetic (which means a magnet can pull it), but it is not necessarily a magnet (which would do the pulling).

Leaky Can

Imagine taking an empty can and punching three holes in its side—one near the top, one halfway up, and one near the bottom. Place the can in the sink. Fill the can with water from a faucet and keep it just topped up to the brim all the time. Water streams from the three holes in gentle curves known as parabolas and hits the sink floor a little way from the can.

PROBLEM: Does the water spurt out fastest from the can from the bottom, middle, or top hole?

Measuring Water by Cans

Jack wanted a quart of water. He only had two cans for measuring the water. One held five quarts and the other three. He found he could measure out a quart with the two cans. He filled the three-quart can and then poured the water into the five-quart can. What did he do next to measure out just one quart?

More Measuring

Jack wanted to measure out two quarts of water. He only had two cans, a five-quart and a three-quart can. How did he do it?

Yet More Measuring

Jack wanted to measure six pints using only a nine-pint can and a four-pint can. How did he to it?

Bottle in the Lake

A youngster plans to row at a steady speed in a straight line across a big lake from the boathouse to an island one-half mile away. As he sets off, the boathouse clock strikes noon. At that moment his bottle of soda, perched on the stern of the rowboat, falls into the lake. Thinking the bottle empty, he rows on blithely until he reaches the island, at 12:30. Then he remembers the bottle still had some ginger ale in it. (He's also conservation-minded.) So he turns the boat around instantly and rows back to the bottle at the same steady speed. As he fishes the bottle out of the water, the boathouse clock strikes 1. Well, how fast did the youngster row?

Birthday Match

Next time you are at a party ask if any two people, including yourself, have the same birthday. They don't have to be the same age. That is, the year of birth does not have to match. For instance, two partygoers might both have their birthday on May 23. What do you think the chances of a match-up would be among 23 people? Among 60 people? Remember, there are 365 days in a year, ignoring leap year. The answer will surprise you.

Tossing Two Coins

Toss two coins. What are the odds that two heads will turn up? This problem has puzzled some of the greatest minds. So don't worry if you don't get it right; if you get the wrong answer, it may be the same one as the great minds gave! Then you can say: "Great minds think alike."

Heads or Tails

Take a dime and toss it. What are the odds that it will fall heads up? Perhaps you know it is 50-50, or ½. There's a 50-50 chance, as they say, it will fall heads up and the same chance it will fall tails up. Say you tossed the dime and it fell heads up ten times in a row. What is the chance of it falling heads on the next throw? Do you think it is still ½? Or do you think it's more likely to fall tails to keep up the law of averages because it has had a long run of heads? Another "great minds think alike" problem!

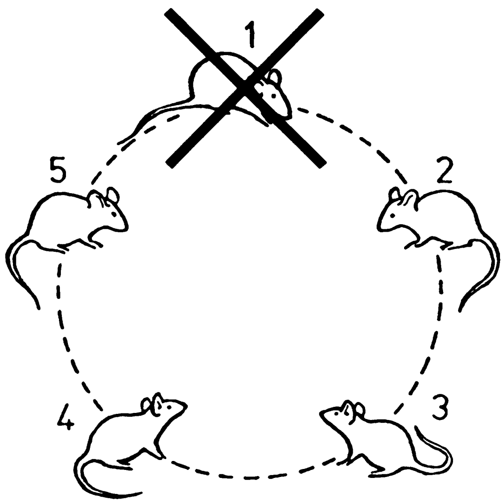
ANSWERS

1. Number Problems

All the Fun of the Fair
19 + 6 + 25 = 50; teapot, doll, coconut.

Chessboard Problem
The answer is (1 X 1) + (2 X 2) + (3 X 3) + and so on up to 8 X 8, or 1 + 4 + 9 + . . . + 64 = 204 squares in all. (Algebra books give a formula for writing out what is called the sum of the squares.) For a six-by-six board it is 1 + 4 + 9 + . . . + 36 = 91.

Cat and Mice
Suppose Puss starts at mouse number 1, marked with a cross in the diagram. It will help if you draw one like this. Go around clockwise through positions 2, 3, 4, and 5, which you cross out. Crossing out each fifth dot goes in this order: 1, 2, 4, 5, 3. So the white mouse must be at position 3, if Puss starts at position 1 and moves clockwise. Or he could go counterclockwise from position 1; then the white mouse must be at position 4.



A Question of Ages
Sam 6 years, May 2 years.

Another Question of Ages
Uncle 60 years, girl 20 years.

Teen-Age Problem
Jo 13 years, Sam 9 years.

15 Shuffle
Move the 9 to the first pile: 9 + 1 + 2 + 3 = 15.

Birthday Paradox
He was born on February 29; 1792 was a leap year. So he only had a birthday every fourth year. (1800 was not a leap year.)

Word Sums

$$\begin{array}{r} 764 \\ +\ 764 \\ \hline 1{,}528 \end{array} \qquad \begin{array}{r} 534 \\ +\ 534 \\ \hline 1{,}068 \end{array}$$

Easy as ABC?

$$\begin{array}{r} 888 \\ 777 \\ +\ 444 \\ \hline 2{,}109 \end{array} \qquad \begin{array}{r} 888 \\ 666 \\ +\ 555 \\ \hline 2{,}109 \end{array}$$

The Missing Dollar
There *is* no problem! It is simply the words that hoodwink you. The following table makes clear where the money is all the time.

	Girls' Pockets	*Store's Till*	*Saleslady's Purse*
1. To start with	\$30		
2. Girls pay for radio		\$30	
3. Saleslady takes \$5		\$25	\$ 5
4. Saleslady gives \$1 each to girls	\$ 3	\$25	\$ 2

As you see, there is always \$30 kicking about. In the end the girls have paid \$27, which equals \$25 to the shop plus \$2 to the saleslady. You do not add \$27 to \$2, as the problem suggests. It is meaningless! You would be adding what the girls have paid to what the saleslady has gained. But you could say: The girls' losses ($27) equals the saleslady's gain (an ill-gotten $2) plus the store's gain ($25). Or, to put it another way, the girls' losses plus the saleslady's loss equals the store's gain—that is, $27 + (− $2) = $25. But that − $2 may not appeal to you, in which case use the first way of putting it.

You Can't Take It (All) with You
To count $1 million would take 11 days 13 hours 46 minutes 39.9 seconds, or just over 11½ days. With no sleep—the counting was nonstop—this would be more than flesh and blood could stand! You *might* manage to count for 2½ days, bringing you $216,000.

Tear 'n' Stack
Believe it or calculate it, the answer is as high as the moon. The calculation goes like this: 1 tear makes a stack 2 sheets, or 2 thou, thick; 2 tears make 4 thou; 3 tears make 8 thou, or 2 X 2 X 2 thou. So 47 tears make a stack 2 X 2 X 2 X . . . (47 times) thou thick, or 140,737,488,355,328 thousandths of an inch high. Divide this by 1,000 to bring it to inches, then by 12 to give feet, then by 5,280 to give miles. The result is about 221 million miles, which is about the distance to the moon (actually 250 million miles).

Grains of Wheat
A sack of wheat held about a million grains of wheat. To finish covering the first 20 squares a total of 1,048,575 grains was needed. The first three answers in the second box are all about right! The last answer *might* be!! But nobody has counted the grains of sand, so far as I know.

A Sweet Problem
Five lumps in one cup, two lumps in the second cup, and three lumps in the third cup. Then sit the third cup in the second cup so the second cup now has in it five lumps!

Stock Taking
15 sheep.

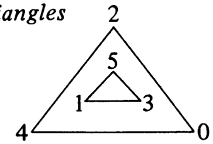
Slobodian Coin Puzzle
There are four ways. Break it down like this. First way: 1 + 1 + 1 + 1 + 1 + 1; second way: 1 + 1 + 1 + 1 + 2; third way: 1 + 1 + 2 + 2; fourth way: 2 + 2 + 2 + 2.

Tug of War
In the last event you can replace the dog by two girls and a boy because, as the second tug of war showed, they are equally matched. The last tug of

war then becomes a contest between five girls and a boy on the left against four boys on the right. But the first event showed that five girls are as strong as four boys—that is, a boy is stronger than a girl. The tug of war comes down to a contest between five boys on the left and five girls on the right. So the left-hand team will win. This all supposes that each girl pulls as hard as the next, and similarly for the boys. If you are clever you could also solve the puzzle by algebra.

Check-out Check
Take away the 20 cents for the two sticks of gum. Then the three bars must have cost $2.00, or 200 cents. But 200 cannot be divided by 3 to leave a whole number of cents.

Puzzle Triangles



Nice Work If You Can Get It!
The boss hired her because she was smart enough to see the second rate of pay was much, much better. So much better he couldn't pay her the second rate for long. For at the end of the tenth day she would have earned by the first rate $100 in all. But by the second rate she would have earned 2 X 2 X 2 X 2 X 2 X 2 X 2 X 2 X 2 cents which is 512 cents on the tenth day alone; that is 2 times itself 9 times (the first day she only earned 1 cent). Her total earnings would be this 2 times itself 10 times less 1 (you can take this on trust), which comes to $10.23.

By the end of the twenty-seventh day she would have earned $270 by the first rate; by the second rate, she would be a dollar millionaire! She would have earned a total of 2 times itself 27 times less 1 cents, or $1,342,177.27. Very nice work!

12 Days' Gifts
78 gifts.

Dividing-the-Line Code
He counts the number of letters in the message and then notes the next higher number in the series—16, 32, 64, 128, and so on. That will be the key number.

Holiday Message
GOOD HUNTING.

Bottle and Cork
Bottle four cents, cork 1 cent. Were you stuck? Then try guessing: cork

two cents, then bottle five cents (three cents more): total seven cents, no good. And so on.

Juggling and Balancing

You can solve it by trial and error, since the answer is only going to be a *few* mugs, or you can use letters (that is, algebra). Let m stand for a mug, b for a bottle, p for a plate, and j for a jug. Then picture A shows $j = b$. Picture B shows $j = m + p$. So we know $b = m + p$. That is, a bottle weighs the same as a mug and a plate together.

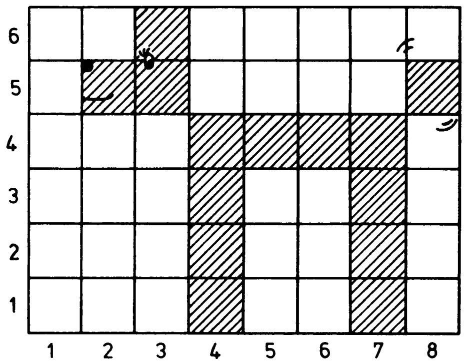
Now for the trick: $3b = 3m + 3p$ simply by tripling up. But picture C shows $2b = 3p$. So $3b = 3m + 2b$. Take two bottles from both sides: $1b = 3m$. That is, a bottle balances three mugs. But a bottle balances a jug. So a jug balances three mugs.

Hidden Animal

LEOPARD.

Pictures by Numbers

A dog.



Number Tracks

A: 6, 12, 18—numbers divisible by 6.
B: 2, 4, 8, 10, 14, 16, 20.
C: 3, 9, 15.
D: 1, 5, 7, 11, 13, 17, 19.

Solve It in Your Head

Adding the equations gives: $10x + 10y = 50$, or $x + y = 5$. Taking the lower equation from the top equation gives $4x - 4y = 4$, or $x - y = 1$. You want two numbers, x and y, that add up to 5 with a difference of 1; $x = 3, y = 2$ is the answer.

Movie Times
6:30; 10:30.

Time, Please
One o'clock.

Picture and Frame
Most people think (wrongly) that the picture costs $1 and the frame costs half a dollar more, or $1.50. But then the total cost would be $2.50, not $2 as stated. Algebra, if you know it, makes the problem easy! If not, guess. Put your guesses in a table:

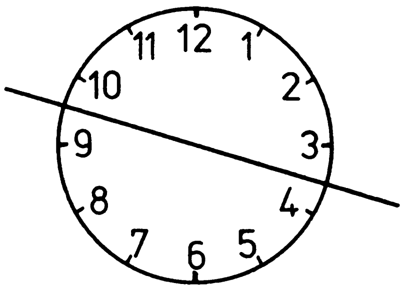
Frame	$0.25	$0.50	$0.75
Picture (costs 50¢ more)	0.75	1.00	1.25
Total	$1.00	$1.50	$2.00

Answer: $0.75 for the frame and $1.25 for the picture.

Idle Ivan and the Devil
We'll solve this puzzle backward in words. Before the third crossing Ivan must have had 4 rubles. But he had given the devil 8 rubles just before that, so he had had 12 rubles after the second crossing. Before the second crossing, then, he had 6 rubles, and before paying the devil he must have had 14 rubles. Before the first crossing he had 7 rubles, the sum he started with.

Happy Landing
Noon.

Cracked-Clock Problem
Split the clockface as shown. Each half comes to 39.



Stamp-Strip Puzzle
The other ways are *a d c b* (did you notice the first stamp is shown printed the wrong way?), *b a d c*, and *a c d b*.

The Smallest Flock

We solve the problem in the same way as we did the hint. The obvious but wrong answer is $(2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10) + 1$. We have counted an extra 2 in the 4 (simply multiplying the original 2 by one more 2 would have covered 4); 6 is already covered by 2×3; 8 is covered by another 2 on top of the two 2s we already have; for 9 we only need another 3; and the 10 is covered by the 2×5. So the smallest flock that obeys the rules is $(2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3) + 1 = 2{,}521$. You notice that the answer is composed of the product of primes (a prime is a number only divisible by itself or 1) plus one. And so the answer must be a prime.

The answer suggests another way of cracking the problem. One less than the answer (2,520) must be cleanly divisible by 2, 3, 4, . . . up to 10. So we know $2 \times a = 2{,}520$, where a is a whole number, and $3 \times b$, another number, $= 2{,}520$. So we can write $2 \times a = 3 \times b = 4 \times c = 5 \times d = 6 \times e = 7 \times f = 8 \times g = 9 \times h = 10 \times i$, where the letters are whole numbers. But $2 \times a$ and $3 \times b$ and $4 \times c$ and $10 \times i$ are "covered" by the others. So the answer is $(5 \times 7 \times 8 \times 9) + 1 = 2{,}521$.

Letter Frame-up

Begin at bottom left corner: *A rolling stone gathers much speed.*

The Farmer's Will

17 horses won't divide cleanly. So they borrowed a horse, making 18 horses, which *will* divide. Ann got 9 horses, Bob 6, and Charlie 2, making 17 in all. They could return the borrowed horse.

The Checkers Match

10 games; each of the five children has to play four others. This suggests 20 games. But this would be counting each pair of players twice. Best way to solve it is to draw a network, as mathematicians call it. Put five dots roughly in a ring on paper for the five players. Then join all the pairs of dots with lines to stand for the games. You'll find there are ten such lines.

The Spelling Bee

8 ways to spell out *rath* and 16 ways to spell out *raths;* you double the number of ways at each row. "And the grave land turtles squeaked out."

Number Oddity

Because $12{,}345{,}679 \times 9 = 111{,}111{,}111$.

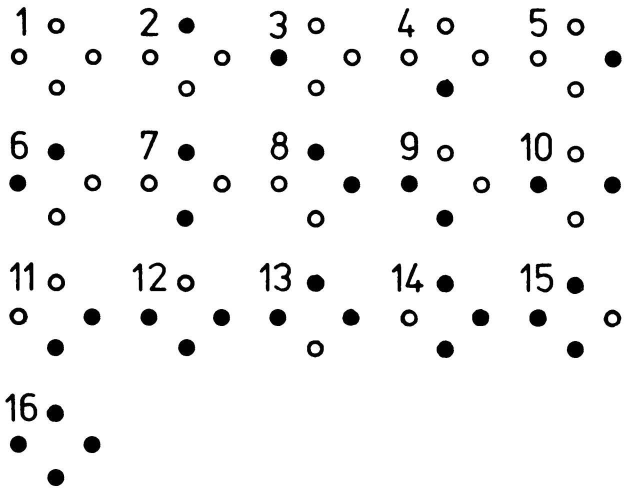
Six 1s are 24?

$1 + 1 + 11 + 11 = 24$.

Pairing Puzzle

$1 + 8 = 2 + 7 = 3 + 6 = 4 + 5 = 9$.

Diamonds of Marbles
16 different patterns.



2. Number Patterns

Puzzle Boards
There is no known rule for making up these boards. If you can find one, you are a mathematician of the first order!

Sum of the Whole Numbers
500,500.

Number Crystals

A. 11,115,556 = 3,334 X 3,334
1,111,155,556 = 33,334 X 33,334

B. 11,108,889 = 3,333 X 3,333

C. 443,556 = 666 X 666

Number Carousel

142,857 X 1 = 142,857
X 5 = 714,285
X 4 = 571,428
X 6 = 857,142
X 2 = 285,714
X 3 = 428, 571

The pattern of numbers reading across and down goes in a cycle like this:

$\times 7 = 999{,}999$
$\times 8 = 1{,}142{,}856$—which is 1 in front of the initial number.

The pattern continues:

$142{,}857 \times 9 = 1{,}285{,}713 \quad (285{,}713 + 1 = 285{,}714)$

and so on.

Take-away Number Squares

You should end up with four 0s: 0, 0, 0, 0. In fact any four numbers you pick will always end up in four 0s. The explanation is a little too involved to go into, but is like that of the previous puzzle.

Take Any Three-Digit Number

The way to see how it works is to reverse the process. You end up with your three-digit number; then multiply it in turn by 7, 11, and 13, which is the same as multiplying it by 1,001. So 123 becomes $123 \times 1{,}001 = 123{,}123$, which is what you began with in the trick. This is why it works, whatever three-digits you begin with.

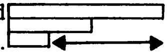
Five 2s

$1 = 2 + 2 - 2 - 2/2$
$2 = 2 + 2 + 2 - 2 - 2$
$3 = 2 + 2 - 2 + 2/2$
$4 = 2 \times 2 \times 2 - 2 - 2$
$5 = 2 + 2 + 2 - 2/2$
$6 = 2 + 2 + 2 + 2 - 2$
$7 = (22 \div 2) - 2 - 2$
$8 = 2 \times 2 \times 2 + 2 - 2$
$9 = 2 \times 2 \times 2 + 2/2$
$10 = 2 + 2 + 2 + 2 + 2$

Four 4s

$1 = 44/44 = 4 - 4 + 4/4 = (4 + 4)/(4 + 4)$
$2 = 4/4 + 4/4$
$3 = (4 + 4 + 4)/4$
$4 = (4 - 4)/4 + 4$
$5 = [(4 \times 4) + 4]/4$
$6 = (4 + 4)/4 + 4$
$7 = 4 + 4 - 4/4$
$8 = 4 + 4 + 4 - 4$
$9 = 4 + 4 + 4/4$
$10 = (44 - 4)/4$

Take-away Number Triangles

You should end up with the trio of numbers 2, 2, and 0. The strange thing is, whatever three numbers you begin with, you will end up with two identical numbers and a zero, such as 1, 1, and 0; 3, 3, and 0; and so on. To simplify things, note that after the first subtraction you always get one number to be the sum of the other two: 11, 5, and 1 became 6, 4, and 10, and we see immediately that 6 + 4 = 10. You can see this with rods, where the difference between the longest and the shortest rod equals the sum of differences between the other two pairs.  This means the problem is not really about "any three numbers" but about any two numbers and their sums. Also, further subtractions always replaces the biggest number by the differences between the other two. With these rules it is possible to arrive at an explanation, but it is too involved for such a book as this!

Missing Numbers

a. 8 (numbers go up in 3s).
b. 10 (numbers go up in 2s).
c. 27 (numbers go up in 5s).
d. 3 (there are two patterns: 1, 2, 3, and 3, 4, 5, 6).
e. 36 (they are all squares).
f. 32 (each number is double the previous one).
g. 126 (differences are 2, 4, 8, 16, . . .).
h. 103 (two patterns: 2, 3, 4, 5, . . . and 3, 6, 12, 24, . . . ; the numbers are 2 + 3, 3 + 6, 4 + 12, 5 + 24, . . .).

Sums in the Head

A. Do it this way: 999 is 1,000 less 1. When we multiply this quantity by 3, we get 3,000 less 3, or 2,997.

B. Group the numbers as follows: (645 + 355), (221 + 779), and (304 + 696). The answer to each of the three sums is 1,000. So the grand total is 3,000.

C. You write 112,734, and the total is 999,999. All you do is write the number that makes the figures in each column add up to 9.

D. The answer is the same for both—137,174,205—though it is easier to do the sum on the left because it is set out correctly for adding. But it makes no difference if you use a calculator.

9 in Ten Digits

$$9 = \frac{97{,}524}{10{,}836} = \frac{57{,}429}{06{,}381} = \frac{95{,}742}{10{,}638} = \frac{58{,}239}{06{,}471} = \frac{75{,}249}{08{,}361}$$

Number Patterns

$$4 \times 5 \times 6 \times 7 + 1 = 29 \times 29$$
$$5 \times 6 \times 7 \times 8 + 1 = 41 \times 41$$

To get the right-hand side, you take the second and third number in the multiplication on the left, multiply them together, and take away 1. So for the first line here, $(5 \times 6) - 1 = 30 - 1 = 29$. For the second $(6 \times 7) - 1 = 41$. Yes, the pattern goes on and on.

Another Number Pattern

$$3^3 + 7^3 = 370$$
$$4^3 + 8^3 = 576$$
$$11^3 + 1^3 = 1{,}332$$
$$147 \times (14 + 7) = 14^3 + 7^3 = 3{,}087$$
$$148 \times (14 + 8) = 14^3 + 8^3 = 3{,}256$$

Reverse Sums

$$24 + 3 = 27, \text{ and } 24 \times 3 = 72$$
$$47 + 2 = 49, \text{ and } 47 \times 2 = 94$$

Palindromes in Numbers

$$\begin{array}{r} 139 \\ +\ 931 \\ \hline 1070 \\ +\ 0701 \\ \hline 1771 \end{array}$$

The Next Palindromic Year

1991

Corridors of Numbers

The next three corridors are $3^3 = 27$, $4^3 = 64$, $5^3 = 125$, and $6^3 = 216$.

Multiplying Equals Adding?!

When one number is 3, the other is $\frac{3}{2}$, because $3 + \frac{3}{2} = 3 \times \frac{3}{2}$. There are, actually, an infinite number of answers. Other number pairs that work are: 4 and $\frac{4}{3}$, 5 and $\frac{5}{4}$, 6 and $\frac{6}{5}$, 7 and $\frac{7}{6}$, and so on.

Curious Centuries

$$\begin{aligned} 100 &= 1 + 2 + 3 + 4 + 5 + 6 + 7 + (8 \times 9) \\ &= 123 - 45 - 67 + 89 \\ &= 123 - 4 - 5 - 6 - 7 + 8 - 9 \\ &= \frac{1}{2} + \frac{6}{4} + \frac{5 + 3}{8} + 97 \end{aligned}$$

Times-Table Triangle
A. All are squares. **B.** 12 (those two rows down from 2), 21 (three rows down from 3), 72 (six rows down from 6), and 91 (seven rows down from 7).

3. Magic and Party Tricks with Numbers

Dial-a-Number Trick
Look at the numbers in the puzzle:

$$\begin{array}{r} 673 \\ -376 \\ \hline \end{array}$$

When you take away, you "borrow" 10 first in the ones column and then in the tens column. Then you must "pay back" 1 in the tens column and the hundreds column.

6 − 3	7 − 7	3 − 6
6 − 3 − 1	10 + 7 −7 − 1	10 + 3 − 6

Don't *do* the take-away as you normally would. Then you can see the 6s cancel (first and third columns), then the 7s (second column), and the 3s (first and third columns), leaving just two 10s less two 1s, or 18.

"How Old Are You?"
The trick relies on a place-value shuffle by playing about with hundreds, tens, and ones in a disguised way. It also depends on the number 9 being 1 less than 10—the number system we count in. It is usually explained by algebra. However, we will do so using *typical* numbers.

Suppose your friend is 37 years old. Let's see what happens.

Original number: 37 = 3 tens + 7.
Multiply by 10: 370 = 3 hundreds + 7 tens.
Take away 9 × a number (5, say):*

− 9 fives = − 10 fives + 1 five
= − 5 tens + 5.

So we have then: 370 − (9 × 5)
= (3 hundreds + 7 tens) − 5 tens + 5.

The final step in the trick is to add the far right figure (5) to the other two, which become tens and ones instead of hundreds and tens:

3 tens + 7 − 5 + 5,

which is 3 tens + 7, or 37, your friend's age.

It works, and you can test it, with any numbers that fit the trick.

* You note we don't call 9 fives 45.

For Someone Over 10 Years Old
The trick here is that the subject adds 90, which is the same as adding 1 hundred and taking away 1 ten. Another place-value shuffle. He also crosses off the first digit (the hundreds place), which must be 1, and *adds* it. After you yourself add on another 9, 10 will have been added in all. But 1 ten was subtracted before, so we are back where we started.

Fiddling by Numbers
43560 which becomes RESIN.

Using Someone's Age
The year of anybody's birth plus his age must always equal the present year. The year of the great event plus the number of years ago that it happened also must always equal the present year. So the total is twice the present year.

Example: Suppose it is June 1980. Jim is 13. His birthday is in September, when he will be 14.

He was born in:	1966	
Big event took place in (say):	1977	
His age as of December 31:	14	
The great event happened:	+ 3	years ago
	3960	

The total you predict is twice the present year, or 2 × 1980, or 3,960.

Hundred Dollars for Five
You cannot win the bet. Here's why. (If you don't like algebra, skip it and take it on trust.) Suppose you *did* find a way with x half-dollars ($x \times 50$ cents), y quarters ($y \times 25$ cents), and z dimes ($z \times 10$ cents). The total must add up to $5, or 500 cents. So we can write:

$$50x + 25y + 10z = 500.$$

Dividing through by 5, we get:

$$10x + 5y + 2z = 100. \qquad \text{(Equation 1)}$$

But there must be 10 coins. That is,

$$x + y + z = 10.$$

An algebra trick follows. Double everything in this last equation:

$$2x + 2y + 2z = 20. \qquad \text{(Equation 2)}$$

And subtract Equation 2 from Equation 1:

$$8x + 3y = 80.$$

The idea was to get rid of the z, the number of dimes.

Divide by 8: $x + 3/8y = 10.$

Now you can only have a whole number of each coin, so y must be 8. (It cannot be any other multiple because 10 is the limit.) If $y = 8$, then $x =$

7, and the money subtotal (without *any* dimes, since we haven't yet figured in z) is then $(50 \times 7) + (25 \times 8)$, which is 550, or 50 cents more than the desired grand total.

So there is no combination that fits the terms of the bet.

No Questions Asked

The only way to explain the trick is by algebra. You, the magician, write a number x and put it in the envelope. A friend thinks of a "thought" number, y. You take x from 99 and announce the result, $99 - x$. The friend adds this to his thought number to get $99 - x + y$. This is the same as $100 - 1 - x + y$. He crosses off the first digit and adds it to the number that remains. He ends up with $y - x$, which he takes from his thought number y:

$$\begin{aligned} & y - (y - x) \\ &= y - y + x \\ &= x \end{aligned}$$

All that is left is x, the number in the envelope.

Magic Year Number

This trick uses the same idea as "Using Someone's Age." It is simply: The current year minus somebody's birth year always gives their age. The rest of the calculation is pure hokum to fool the spectator. The shoe size doesn't matter a bit. Multiplying it by 2 then by 50—that is, by 100—puts the shoe into the hundreds place. As you are only likely to meet people below 100 years of age, you are only interested in the last two digits. It is important, however, that the last two digits are 50, as I will show in the next paragraph. When the shoe size was doubled, the result was an even number. It was necessary to add 5 (or *any* other odd number) to this to make an odd number; when this new number is multiplied by 50, the last two digits will be 50.

As I said, only the last two digits of the Magic Year Number matter: the first two digits are a blind. The reason: The first three steps give you 100 times the shoe size, plus 250. So shoe size 6½ (or 6) gives 850, 7 gives 950, 8 gives 1,050, and so on. Forget everything but the last two digits, 50. Add the Magic Year Number (1,230) to it, giving 1,280. The last two digits correspond to—surprise, surprise!—the current year. All you are doing in the last two digit places is adding 50 and then 30 (which comes from the Magic Year Number), which gives the last two digits of the current year.

4. Magic Squares and Sliding-Block Puzzles

Magic Squares

There is an easy way to make up a three-by-three magic square. Some puzzle books give a complicated way of memorizing writing numbers or diagonals. But you may not recall which cell (little square) to start on. So write the numbers to be used in a row:

1 2 3 4 ⑤ 6 7 8 9

Put the middle number, 5, in the middle of the magic square. Now pair off extreme numbers 1, 9; then the next pair 2, 8; and so on. Add each pair to 5 to get the constant magic number, 15. Remember 4, 5, 6, is a diagonal (picture *1*) and the rest follows with very little trial and error.

(*1*)

		6
	5	
4		

(*2*)

		1		
	8		6	
3		5		7
	4		2	
		9		

Clearly, the top left cell cannot have 9 because 9 + 6 already equals 15. So try 8 in the top left cell; then 2, its "mate" (because 8 + 2 = 10) must go in the bottom right cell (see picture *2*). The remaining "mates"—1, 9 and 3, 7—are shown outside the square ready to be put into the empty cells, which you have chosen with very little trial and error.

Here is one of the eight 3-by-3 magics:

2	7	6
9	5	1
4	3	8

Cross Sums

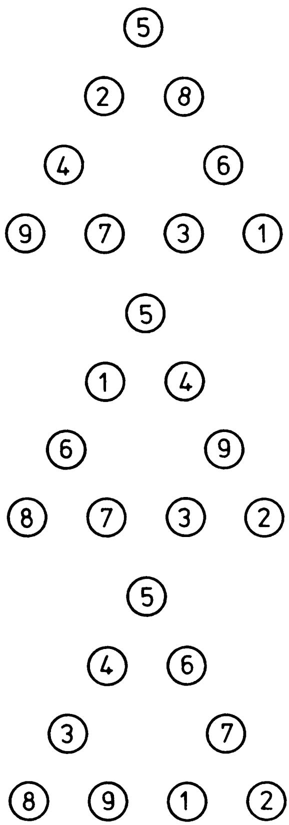
		2		
		5		
3	4	1	7	8
		6		
		9		

A Number Square

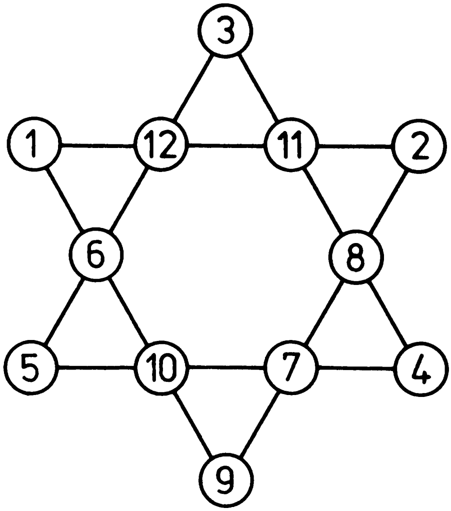
Any of the solutions to "Magic Squares" will do. It is stipulated that the two outside numbers must add up to 10, but since the rows and columns along the outside don't have to obey any rule, it is an easier problem than "Magic Squares."

A Triangle of Numbers

Here is the method behind the first solution diagrammed here. Put 5 at one corner; then 9 and 1 can go in the other two corners. Now take away the corner pairs of numbers from 20. There are three possible number triangles. In the first one you can see that 9 + 5 from 20 leaves 6, and that means the two middle discs on that side can only be filled by numbers adding to 6: 5, 1 or 4, 2. The 5 and the 1 have been used in the corners. So they must be 4 and 2.

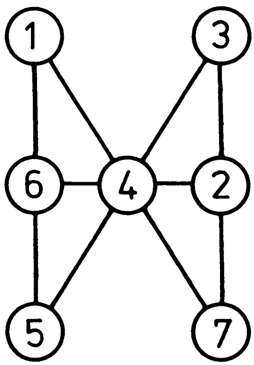


Star of David



Number Rows

The end circles of each row must add to 8 because 4 has to go in the middle circle.



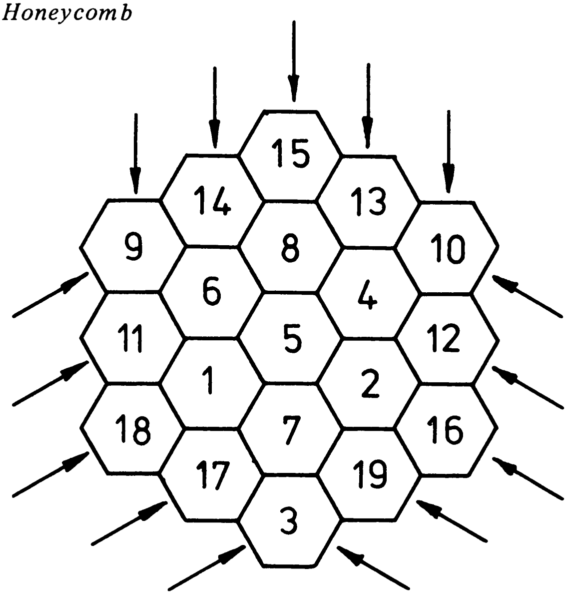
The Eight-Block Puzzle

Move the blocks in this order: 2, 6, 5, 3, 1, 2, 6, 5, 3, 1, 2, 4, 8, 7, 1, 2, 4, 8, 7, 4, 5, 6.

Four-Square Magic

3 + 5 + 12 + 14 is equal to 2 + 8 + 9 + 15. Also the sums of the squares are equal. Date of Dürer's engraving was 1514.

A Magic Honeycomb



Multi-Magic Square

3	36	2
4	6	9
18	1	12

5. Illusions

Illusive Lengths
AB is same length as *BC.*

Diamond-Square Puzzle
Both are squares, the same size.

Candle Flicker
Both are the same size.

The Three Streetlamps
All three are the same height.

Railroad Track
Both ties are the same length.

Target Square
The square *is* square.

Squaring the Circles
Both circles are the same size.

The Flap Tabletop
All three tabletops are the same length.

Run Rings Around You?
Both circles are the same size.

Circle Counting
Two perfect circles.

Bull's-eye and Ring
Surprisingly, both have the same area.

Circle Sorcery
It is not bent: It is a perfect circle.

Two Tall Hats
The top of band of hat *a* is half the brim's width. This is not true of hat *b*.

The Two T's
a has same height and width, *b* hasn't.

The Herringbone Pattern
The four heavy lines *are* straight. The background pattern fools the eye.

The Picture Frame
The frame *is* square.

The Fraser Spiral
They are not whorls of a spiral but, believe it or not, circles.

The Swollen Ruler
The ruler has not swollen.

Zooming Lenses
The little lens.

The Long Glass
The plate is wider than the glass is tall.

The Black Crosses
They are equal.

The Blacker Squares
They are equal.

Lampshade and Flowerpot
They are equal.

The Top Hats
Hat *a*.

The Fans
They are equal.

The Long and Short of It
The middle line is just half the length of the other lines, although it looks longer.

The Artful Arrows
Arrow shafts are the same length.

Straight As an Arrow
Line *q* is a continuation of the arrow *x*.

Skew Lines?
The horizontal lines are parallel.

Stars and Diamonds
The length of each diamond *is* the same as the distance between tip of the star and diamond tip.

Circles Before Your Eyes
Incredibly, both inner circles are the same size.

Arcs More or Less Curved
All these arcs are equally curved. That is, all are part of the same size circle.

Hickman's Squares
Both figures are squares.

Devilish Diagonals
Both diagonals AB and BC are the same length.

The Extra Cube
Turn the book upside down and a cube magically seems to appear.

6. Dominoes and Dice

Dominoes—with a Difference
Playing "1 difference" you can make a closed chain.

The Dot's Trick
Each value appears in pairs throughout the set of dominoes. So an unpaired 3 at one end of the line must have its "mate" at the other end.

Domino Trick
The trick works for the same reason as "The Dot's Trick": each value has a pair in the set. So the two values on the domino you pocket must appear at the ends, since you left only one of each value and they can only appear in pairs on the inside of the chain.

Magic Domino Squares
In the magic square all rows and columns and both diagonals add up to the magic number 12. In the second magic square the magic number is 15.

Party Trick
The trick works for this reason: With no tiles moved, the thirteenth tile is 0-0, which means that no tiles have been moved. Move one tile to the left end, and the thirteenth tile now shows 0-1, indicating one tile has been moved . . . and so on.

Domino Fractions

$$\frac{4}{1}+\frac{2}{3}+\frac{5}{2}+\frac{4}{2}+\frac{5}{6}=10$$

$$\frac{6}{1}+\frac{1}{3}+\frac{3}{4}+\frac{5}{3}+\frac{5}{4}=10$$

$$\frac{2}{1}+\frac{5}{1}+\frac{4}{6}+\frac{6}{3}+\frac{2}{6}=10$$

Another Magic Square

To solve, write the total values in order:

3 4 5 6 ⑦ 8 9 10 11

and circle the middle number, 7. Now pair the outer numbers 3 and 11 and so on inward. Put 7 in the middle of your magic square and then juggle the pairs in lines that pass through 7.

Answer:

10	3	8
5	7	9
6	11	4

in numbers

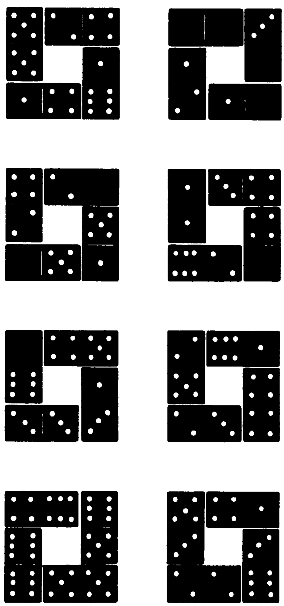
or

$\frac{4}{6}$	$\frac{0}{3}$	$\frac{3}{5}$
$\frac{0}{5}$	$\frac{2}{5}$	$\frac{3}{6}$
$\frac{0}{6}$	$\frac{5}{6}$	$\frac{0}{4}$

in dominoes

Domino Window

Dots along each side come to 9. Here are other possibilities:



Reading the Bones

The adding and multiplying you ask of your friend amounts to this: He multiplies one number on the domino bone by 10 and adds in the other number *plus* 20, which you then mentally subtract. The multiplying by 10 is concealed in two stages—first he multiplies by 2, then, after adding in 4, by 5. He now has one number times 10 plus 4 X 5, which is 20. After you discard the 20, you are left with the first number times 10 (and therefore in the tens place) plus the second number (which is in the ones place).

Hidden Faces of Three Dice

All you need do is glance at the top face and take its number from 21. The top face in the picture was 5, so the sum was 16. The secret is simple: Opposite faces of dice add up to 7. So the three pairs of opposite faces add up to three 7s, or 21. Take away the number on the top face to get the sum for the hidden faces.

Three Dice in a Row

All you have to do is add 7 to the sum of the spots on the top faces, which was 4 + 1 + 6 = 11. So the subject's total was 18. This is how it works: After the subject sets up the three dice in a row originally, he only handles one of the dice; so when you add up the spots on the top faces, two faces are the same for you and for your subject. You could not know the original top or bottom of the third die, which he has added in individually, but you know that the *total* of opposite sides *must* be 7. After the subject has tossed this third die, he adds in the spots on the top face. This is the face you see and also add in. All you have to do is account for the top and bottom face before that die was tossed—that is, you add in 7.

What Did the Dice Show?

We'll use our sample to explain how the trick works. The number 3,542 could be written as 35 hundreds plus (7 − 3) tens plus 7 − 5. That is, $35h + (7 - 3)\,t + 7 - 5$, or $35h + 70 - 30 + 7 - 5$, which is $35h - 35 + 77$. Now 35 hundreds less 35 is 35 ninety-nines. So we now have (35 X 99) + 77. The subject had to divide by 11, which gave (35 X 9) + 7. What you did was to take away 7, leaving 35 X 9; then divide by 9, giving 35. Of course it works for any pair of numbers on the dice. Try it and see.

7. Physics Puzzles

Ruler Rolling
The ruler travels twice as far as the pencils—that is, 4 inches.

Dollar Bill for Free?
The secret is to pull the bill out so quickly the coins don't move with it. And the way to do this is to hold the free end of the bill firmly between the forefinger and thumb of one hand and with the outstretched forefinger of the other hand strike the stretched dollar bill. This brings the bill free of the coins without dislodging them—provided you move quickly and without hesitation.

The Coin-rolling Bet
The rolled coin finishes right side up again if rolled all the way around the stationary coin—just as if it had revolved once completely about its own rim.

Monkey Puzzle
The bananas are pulled up at the same speed as the monkey hauls himself up the rope.

A Weighty Problem
Split the 180 ounces evenly into the two pans, making two lots of 90 ounces. Split one panload again between the two pans to make two lots of 45 ounces. Weigh that against the two weights, 5 ounces in total, and 40 ounces of seed. You now have 40 ounces of seed in one pan, which you tip into one bag; the rest, 140 ounces, goes in the other bag.

Tug of War
100 pounds.

Belts
Yes. *B* turns counterclockwise; *C* and *D* turn clockwise. The wheels can also turn if all four belts are crossed but not if one or three belts are.

Magnetic Puzzle
Move each bar in turn up to the middle of the other bar and at right angles to it. The magnet will attract the other when it is brought up. The non-magnet will not do so.

Leaky Can
Physics shows that the water spurts out fastest from the middle hole. But the math is a little too advanced to go into here.

Measuring Water by Cans
Jack refilled the three-quart can and poured two quarts of the water into the five-quart can till it was full, leaving one quart left in the three-quart can.

More Measuring
Jack measures out one quart as in the previous puzzle. Then he empties the five-quart can and pours this one quart into it. Then he fills the three-quart can again and pours the water into the five-quart can, which is now holding four quarts. He fills the three-quart can again and pours off one quart into the five-quart can, which is now full. The three-quart can now holds two quarts of water.

Yet More Measuring
Jack filled the nine-pint can and then poured off four pints to fill the four-pint can, which he then emptied. This left five pints in the nine-pint can. He then poured four of these pints into the four-pint can, leaving one pint in the nine-pint can. He emptied the four-pint can again and then poured the one pint from the bigger can into it. He filled the nine-pint can again and poured three pints from it to fill the four-pint can. This left six pints in the bigger can.

Bottle in the Lake
He rows one mile in one hour. So his speed was one mile per hour.

Birthday Match
Surprisingly, you only need 60 people at the party to be almost certain to find two people with the same birthday. Of course, this does not mean their birthday is the same as *yours.* With 23 people you have a 50-50 chance of finding two people with birthdays that match.

Tossing Two Coins
These are the possibilities: head head, head tail, tail head, and tail tail. That is, four ways, only one of which gives the two heads. Many great minds thought there were only *three* ways, because they forgot that head tail is not the same as tail head.

Heads or Tails

The odds are still ½ that it will fall heads. The odds of any coin falling heads or tails is *always* 50-50. One way to look at it is this: The coin has no memory: It cannot remember how many times it has fallen one way or the other. Another way is to say: You can toss ten different dimes or one dime ten times. Well, obviously each dime cannot tell how the others are falling—heads or tails—can it? The fact remains, however, that in the long run the number of heads and tails *does* even out. In one experiment 30 pennies were shaken in a box and tossed; this was done 100 times, making 3,000 tosses in all. And 1,492 heads were gotten. That's only 8 tosses short of half (1,500).