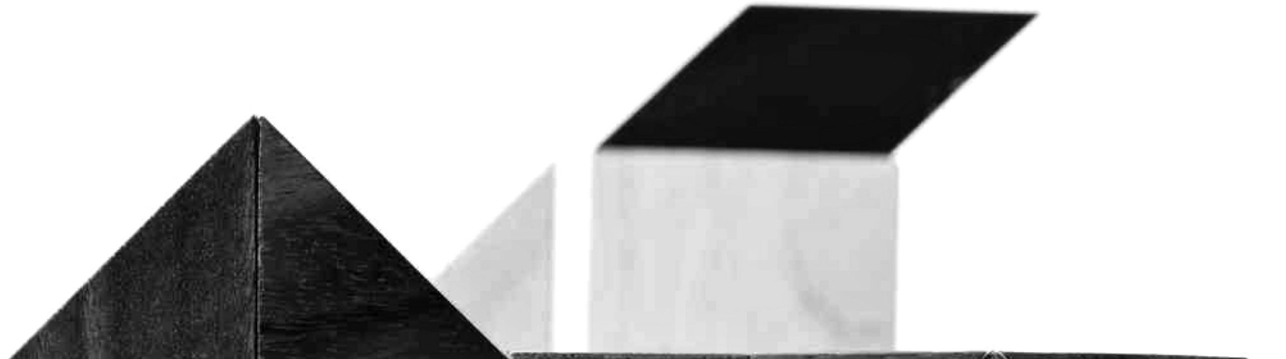
OSVALDO DOLCE
JOSÉ NICOLAU POMPEO

FUNDAMENTOS DE MATEMÁTICA ELEMENTAR

Geometria plana

9



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RESOLUÇÕES

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JOSÉ NICOLAU POMPEO

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COMPLEMENTO PARA O PROFESSOR

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Atual Editora

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Apresentação

Este livro é o *Complemento para o professor* do volume 9, Geometria plana, da coleção *Fundamentos de Matemática Elementar*.

Cada volume desta coleção tem um complemento para o professor, com o objetivo de apresentar a solução dos exercícios mais complicados do livro e sugerir sua passagem aos alunos.

É nossa intenção aperfeiçoar continuamente os *Complementos*. Estamos abertos às sugestões e críticas, que nos devem ser encaminhadas através da Editora.

Agradecemos aos professores Manoel Benedito Rodrigues e Carlos Nely Clementino de Oliveira a colaboração na redação de soluções que são apresentadas neste *Complemento*.

Os Autores.

Sumário

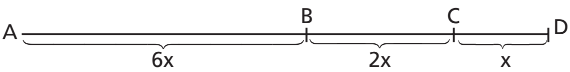
CAPÍTULO II — Segmento de reta

17. $AD = 36 \Rightarrow 9x = 36 \Rightarrow x = 4$

$AB = 6x = 24$ cm

$BC = 2x = 8$ cm

$CD = x = 4$ cm



18. Hipótese Tese

$\overline{PA} \equiv \overline{QB} \Rightarrow \overline{PQ} \equiv \overline{AB}$

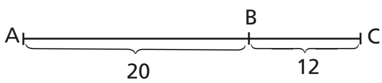
Demonstração

Observando o segmento $\overline{AQ}$ comum a $\overline{PQ}$ e $\overline{AB}$, temos:

$\overline{PA} \equiv \overline{QB} \Rightarrow \overline{PA} + \overline{AQ} = \overline{AQ} + \overline{QB} \Rightarrow \overline{PQ} \equiv \overline{AB}$

19. Temos duas possibilidades:

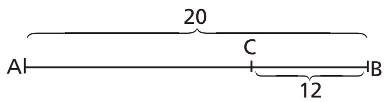
1ª) B está entre A e C



$AC = AB + BC \Rightarrow$

$\Rightarrow AC = 20 + 12 \Rightarrow AC = 32$ cm

2ª) C está entre A e B



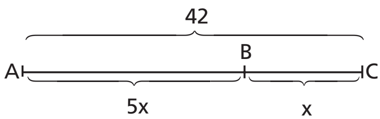
$AC + BC = AB \Rightarrow$

$\Rightarrow AC + 12 = 20 \Rightarrow AC = 8$ cm

20. $5x + x = 42 \Rightarrow x = 7$ cm

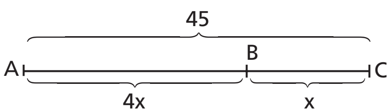
$AB = 5x \Rightarrow AB = 35$ cm

$BC = x \Rightarrow BC = 7$ cm



21. Temos duas possibilidades:

1ª) B está entre A e C

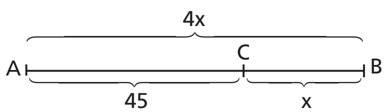


$4x + x = 45 \Rightarrow x = 9$ cm

$AB = 4x \Rightarrow AB = 36$ cm

$BC = x \Rightarrow BC = 9$ cm

2ª) C está entre A e B



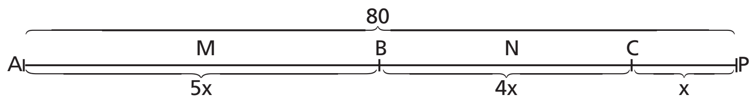
$45 + x = 4x \Rightarrow x = 15$ cm

$AB = 4x \Rightarrow AB = 60$ cm

$BC = x \Rightarrow BC = 15$ cm

22. Temos três possibilidades:

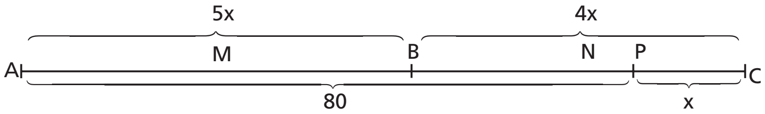
1ª)



$5x + 4x + x = 80 \Rightarrow x = 8$ cm

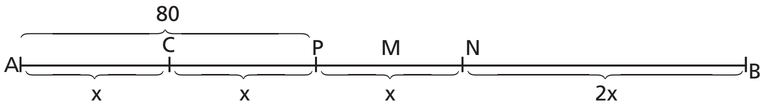
$MN = MB + BN \Rightarrow MN = 2{,}5x + 2x \Rightarrow MN = 36$ cm

2ª)



1) $BP + PC = BC \Rightarrow BP + x = 4x \Rightarrow BP = 3x$
2) $AB + BP = 80 \Rightarrow 5x + 3x = 80 \Rightarrow x = 10$ cm
3) $MN = MB + BN \Rightarrow MN = 2{,}5x + 2x \Rightarrow MN = 45$ cm

3ª)

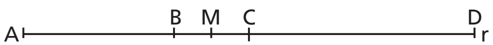


1) $BP + PC = BC \Rightarrow BP + x = 4x \Rightarrow BP = 3x$
2) $BN + NP = BP \Rightarrow 2x + NP = 3x \Rightarrow NP = x$
3) $AC + BC = AB \Rightarrow AC + 4x = 5x \Rightarrow AC = x$
4) $AP = 80 \Rightarrow 2x = 80 \Rightarrow x = 40$ cm
5) Se o ponto M dista 2,5x do ponto A, então M é ponto médio de $\overline{PN}$.

Logo, $MN = \dfrac{x}{2}$ e então $MN = 20$ cm.

23. Hipótese Tese

$\overline{AB} \equiv \overline{CD} \Rightarrow \overline{AD}$ e $\overline{BC}$ têm o mesmo ponto médio



Demonstração

Seja M o ponto médio de BC. Temos:

$$\overline{AM} \equiv \overline{AB} + \overline{BM} \equiv \overline{CD} + \overline{MC} \equiv \overline{MD}$$

Como $\overline{AM} \equiv \overline{MD}$, M também é ponto médio de $\overline{AD}$.

24. Hipótese Tese

$$\overline{AC} \equiv \overline{BD} \Rightarrow \begin{cases} 1)\ \overline{AB} \equiv \overline{CD} \\ 2)\ \overline{BC} \text{ e } \overline{AD} \text{ têm o mesmo ponto médio} \end{cases}$$

Demonstração

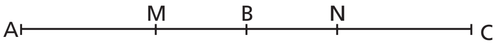
1) Observando o segmento BC, temos:

$$\overline{AC} \equiv \overline{BD} \Rightarrow \overline{AC} - \overline{BC} \equiv \overline{BD} - \overline{BC} \Rightarrow \overline{AB} \equiv \overline{CD}$$

2) Análogo ao exercício 23.

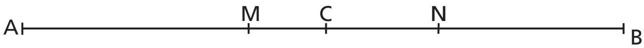
26. Temos duas possibilidades:

1ª)



$$MN = MB + BN \Rightarrow MN = \frac{AB}{2} + \frac{BC}{2} \Rightarrow MN = \frac{AB + BC}{2}$$

2ª)



$$MN = MC + CN \Rightarrow MN = (BM - BC) + CN \Rightarrow$$

$$\Rightarrow MN = (BM - BC) + \frac{BC}{2} \Rightarrow MN = BM - BC + \frac{BC}{2} \Rightarrow$$

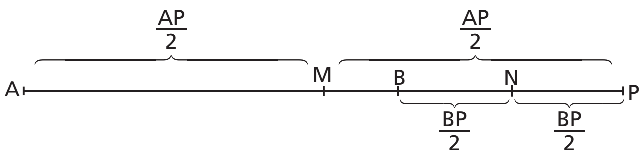
$$\Rightarrow MN = BM - \frac{BC}{2} \Rightarrow MN = \frac{AB - BC}{2}$$

28. O segmento $\overline{MN}$ terá medida constante e igual à metade do segmento $\overline{AB}$.

Justificação

Temos três casos a analisar:

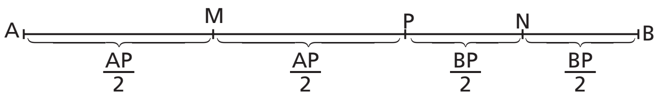
1º)



Neste caso temos:

$$MN = MP - NP \Rightarrow MN = \frac{AP}{2} - \frac{BP}{2} \Rightarrow MN = \frac{AP - BP}{2} \Rightarrow MN = \frac{AB}{2}$$

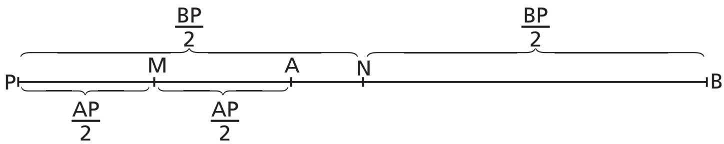
2º)



Neste caso temos:

$$MN = \frac{AP + BP}{2} \Rightarrow MN = \frac{AB}{2}$$

3º)



Neste caso temos:

$$MN = PN - PM \Rightarrow MN = \frac{BP}{2} - \frac{AP}{2} \Rightarrow MN = \frac{BP - AP}{2} \Rightarrow MN = \frac{AB}{2}$$

CAPÍTULO III — Ângulos

55. ângulo $\rightarrow x$ complemento $\rightarrow (90° - x)$

"O ângulo mais triplo do complemento é igual a 210°."

$x + 3 \cdot (90° - x) = 210° \Rightarrow 2x = 60° \Rightarrow x = 30°$

59. ângulo $\rightarrow x$

complemento do ângulo: $(90° - x)$

complemento da metade: $\left(90° - \frac{x}{2}\right)$

triplo do complemento da metade: $3 \cdot \left(90° - \frac{x}{2}\right)$

suplemento do triplo do complemento da metade: $180° - 3\left(90° - \frac{x}{2}\right)$

$180° - 3\left(90° - \frac{x}{2}\right) = 3 \cdot (90° - x) \Rightarrow \frac{9x}{2} = 360° \Rightarrow x = 80°$

60. ângulo $\rightarrow x$

complemento do dobro do ângulo $\rightarrow (90° - 2x)$

suplemento do complemento do ângulo $\Rightarrow 180° - (90° - x)$

$180° - (90° - x) - \frac{90° - 2x}{3} = 85° \Rightarrow x = 15°$

65. Sejam *x* e *y* os ângulos.

$\left.\begin{array}{l} \frac{x}{y} = \frac{2}{7} \\ x + y = 180° \end{array}\right\} \Rightarrow (x = 40°, y = 140°)$

O complemento do menor é igual a $90° - x = 90° - 40° = 50°$.

68. ângulo $\rightarrow x$

complemento do ângulo $\rightarrow (90° - x)$

suplemento do ângulo $\rightarrow (180° - x)$

"O triplo do complemento mais 50° é igual ao suplemento."

$3 \cdot (90° - x) + 50° = 180° - x \Rightarrow 2x = 140° \Rightarrow x = 70°$

72. *x* e *z* são opostos pelo vértice $\Rightarrow x = z$

x e *y* são suplementares $\Rightarrow y = 180° - x$

"*x* mede a sexta parte de *y*, mais metade de *z*."

$x = \frac{180° - x}{6} + \frac{x}{2} \Rightarrow 6x = 180° - x + 3x \Rightarrow x = 45°$

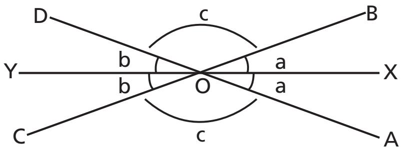
$y = 180° - 45° \Rightarrow y = 135°$

74. Os ângulos são da forma 2k, 3k, 4k, 5k e 6k e somam 360°.

$2k + 3k + 4k + 5k + 6k = 360° \Rightarrow 20k = 360° \Rightarrow k = 18°$

O maior ângulo é de $6k = 6 \cdot 18° = 108°$.

75. Hipótese: $\begin{cases} A\hat{O}B \equiv C\hat{O}D \\ \overrightarrow{OX}, \overrightarrow{OY} \text{ são bissetrizes} \end{cases}$



Tese: $\overrightarrow{OX}$ e $\overrightarrow{OY}$ são semirretas opostas

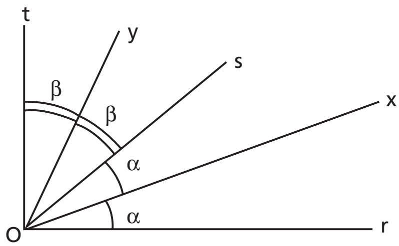
Demonstração

O ângulo entre $\overrightarrow{OX}$ e $\overrightarrow{OY}$ é dado por $(a + b + c)$

$2a + 2b + 2c = 360° \Rightarrow a + b + c =$
$= 180°$

Portanto, $\overrightarrow{OX}$ e $\overrightarrow{OY}$ são semirretas opostas.

77.



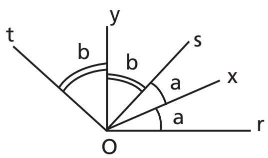
Hipótese		Tese
$r\hat{O}s$ e $s\hat{O}t$ adjacentes e complementares Ox e Oy, respectivas bissetrizes	$\Leftrightarrow$	$x\hat{O}y = 45°$

Demonstração

Sejam a medida de $r\hat{O}x = x\hat{O}s = \alpha$ e a medida de $s\hat{O}y = y\hat{O}t = \beta$:

$\alpha + \alpha + \beta + \beta = 90° \Rightarrow$
$\Rightarrow 2\alpha + 2\beta = 90° \Rightarrow$
$\Rightarrow \alpha + \beta = 45° \Rightarrow x\hat{O}y = 45°$

78.



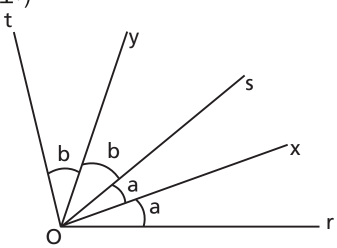
$2a + 2b = 136°$

$a + b = 68°$

Resposta: o ângulo formado pelas bissetrizes é igual a 68°.

79. Temos duas possibilidades:

1ª)

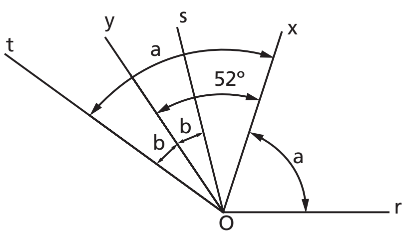


Ox e Oy são bissetrizes

$\left.\begin{array}{l} a + b = 52° \\ 2a = 40° \end{array}\right\} \Rightarrow$

$\Rightarrow 2a + 2b = 104° \Rightarrow$
$\Rightarrow 40° + 2b = 104° \Rightarrow$
$\Rightarrow 2b = 64°$

2ª)



Ox e Oy são bissetrizes

$\left.\begin{array}{l} a - b = 52° \\ 2b = 40° \end{array}\right\} \Rightarrow$

$\Rightarrow a - 20° = 52° \Rightarrow$
$\Rightarrow a = 72° \Rightarrow$
$\Rightarrow 2a = 144°$

CAPÍTULO IV — Triângulos

91. a) $\begin{cases} AB = AC \\ AB = BC \end{cases} \Rightarrow \begin{cases} x + 2y = 2x - y \\ x + 2y = x + y + 3 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x - 3y = 0 \\ y = 3 \end{cases} \Rightarrow (x = 9,\ y = 3)$

$AB = x + 2y \Rightarrow AB = 15$

O perímetro do triângulo ABC é igual a $3 \cdot 15 = 45$.

b) $AB = AC \Rightarrow 2x + 3 = 3x - 3 \Rightarrow x = 6$

$AB = 2x + 3 \Rightarrow AB = 15$; $AC = AB \Rightarrow AC = 15$; $BC = x + 3 \Rightarrow$
$\Rightarrow BC = 9$

O perímetro do triângulo ABC é igual a $AB + AC + BC = 39$.

92. Sejam ℓ a medida dos lados congruentes, *b* a medida da base e *p* o semiperímetro. Temos:

$$\begin{cases} p = 7{,}5 \\ 2\ell = 4b \end{cases} \Rightarrow \begin{cases} \dfrac{2\ell + b}{2} = 7{,}5 \\ \ell = 2b \end{cases} \Rightarrow (\ell = 6\text{ m},\ b = 3\text{ m})$$

Resposta: Os lados do triângulo medem 3 m, 6 m e 6 m.

98. $\triangle ABC \equiv \triangle DEC \Rightarrow \begin{cases} \hat{A} = \hat{D} \\ \hat{B} = \hat{E} \end{cases} \Rightarrow \begin{cases} 3\alpha = 2\alpha + 10° \\ \beta + 48° = 5\beta \end{cases} \Rightarrow (\alpha = 10°,\ \beta = 12°)$

100. $\triangle CBA \equiv \triangle CDE \Rightarrow \begin{cases} AC = CE \\ AB = DE \end{cases} \Rightarrow \begin{cases} 2x - 6 = 22 \\ 35 = 3y + 5 \end{cases} \Rightarrow (x = 14,\ y = 10)$

Os perímetros são iguais; portanto, a razão entre eles é 1.

101. 1) $\triangle PCD \equiv \triangle PBA \Rightarrow \begin{cases} PD = PA \\ CD = AB \end{cases} \Rightarrow \begin{cases} 3y - 2 = 2y + 17 \\ x + 5 = 15 \end{cases} \Rightarrow (x = 10,\ y = 19)$

2) $\triangle PCD \equiv \triangle PBA \Rightarrow AB = CD$ (1)

$$\left.\begin{array}{l} PC = PB \\ P\hat{B}C = P\hat{C}B\ (\triangle PBC \text{ é isósceles}) \\ CA = BD \text{ (usando (1) e o fato} \\ \qquad\qquad \text{de BC ser comum)} \end{array}\right\} \xRightarrow{LAL} \triangle PCA \equiv \triangle PBD$$

Logo, a razão entre os perímetros destes triângulos é igual a 1.

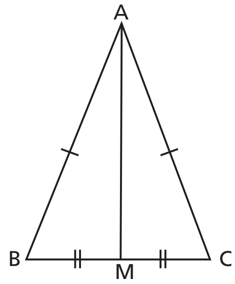
108. Hipótese — Tese

$$\left.\begin{array}{l} \triangle ABC \text{ é isósceles} \\ AM \text{ é mediana} \\ \text{relativa à base} \end{array}\right\} \Rightarrow M\hat{A}B = M\hat{A}C$$

Demonstração

$\left.\begin{array}{l} AB = AC \text{ (hipótese)} \\ BM = MC \text{ (hipótese)} \\ AM \text{ comum} \end{array}\right\} \xRightarrow{\text{LLL}} \triangle ABM \equiv \triangle ACM$

Logo, $M\hat{A}B \equiv M\hat{A}C$ e concluímos que AM é bissetriz do ângulo $\hat{A}$.



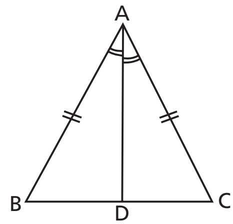
109. Hipótese Tese

$\left.\begin{array}{l} \triangle ABC \text{ é isósceles} \\ \overline{AD} \text{ é bissetriz} \\ \text{relativa à base} \end{array}\right\} \Rightarrow \begin{array}{l} \overline{AD} \text{ é mediana} \\ (\text{isto é, } \overline{BD} \equiv \overline{DC}) \end{array}$

Demonstração

$(AB = AC;\ B\hat{A}D = C\hat{A}D;\ AD \text{ comum}) \Rightarrow$

$\xRightarrow{\text{LAL}} \triangle ABD \equiv \triangle ACD \Rightarrow \overline{BD} \equiv \overline{DC}$



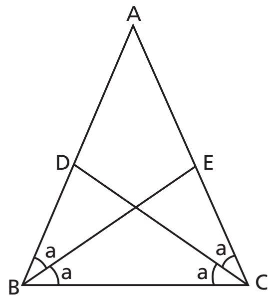
111. Hipótese Tese

$\left.\begin{array}{l} \triangle ABC \text{ é isósceles} \\ \text{de base } \overline{BC} \\ \overline{CD} \text{ é bissetriz de } \hat{C} \\ \overline{BE} \text{ é bissetriz de } \hat{B} \end{array}\right\} \Rightarrow \overline{CD} \equiv \overline{BE}$

Demonstração

$(E\hat{B}C = D\hat{C}B;\ BC \text{ comum};\ E\hat{C}B = D\hat{B}C) \Rightarrow$

$\xRightarrow{\text{ALA}} \triangle CBD \equiv \triangle BCE \Rightarrow \overline{CD} \equiv \overline{BE}$



112. Hipótese Tese

$\left.\begin{array}{l} AM \text{ é bissetriz} \\ AM \text{ é mediana} \end{array}\right\} \Rightarrow \triangle ABC \text{ é isósceles}$

Demonstração

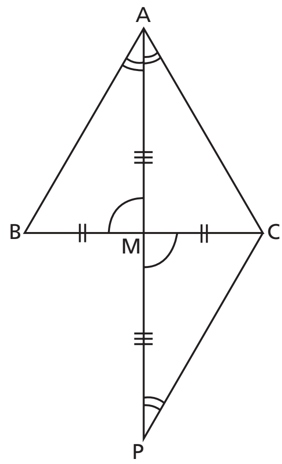
1) Tomemos P sobre a semirreta $\overrightarrow{AM}$ com M entre A e P e $MP = AM$.

2) $(\triangle AMB \equiv \triangle PMC$ pelo LAL$) \Rightarrow$
$\Rightarrow (B\hat{A}M \equiv C\hat{P}M$ e $\overline{AB} \equiv \overline{PC})$

3) $(B\hat{A}M \equiv C\hat{P}M;\ AM \text{ (bissetriz)}) \Rightarrow$
$\Rightarrow C\hat{P}M \equiv C\hat{A}M$

Donde sai que $\triangle ACP$ é isósceles de base $\overline{AP}$. Então: $\overline{AC} \equiv \overline{PC}$.

4) De $\overline{AB} \equiv \overline{PC}$ e $\overline{PC} \equiv \overline{AC}$ obtemos $\overline{AB} \equiv \overline{AC}$. Então, o $\triangle ABC$ é isósceles.



Desigualdades nos triângulos

114. Seja *x* o terceiro lado. Temos:

$|8 - 21| < x < 8 + 21 \Rightarrow 13 < x < 29$

Se *x* é múltiplo de 6 entre 13 e 29 (exclusive), então $x = 18$ cm ou $x = 24$ cm.

115. $|20 - 2x - (2x + 4)| < x + 10 < 20 - \cancel{2x} + \cancel{2x} + 4 \Rightarrow$

$$\Rightarrow \begin{cases} x + 10 < 24 \\ |16 - 4x| < x + 10 \end{cases} \Rightarrow \begin{cases} x < 14 \\ -x - 10 < 16 - 4x < x + 10 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x < 14 \\ -x - 10 < 16 - 4x \\ 16 - 4x < x + 10 \end{cases} \Rightarrow \begin{cases} x < 14 \\ x < \dfrac{26}{3} \\ x > \dfrac{6}{5} \end{cases} \Rightarrow \frac{6}{5} < x < \frac{26}{3}$$

116. Aparentemente temos duas possibilidades: 38 cm ou 14 cm.

Mas um triângulo de lados 14 cm, 14 cm, 38 cm não existe, pois não satisfaz a desigualdade triangular.

O triângulo de lados 38 cm, 38 cm, 14 cm satisfaz a desigualdade triangular.

Resposta: 38 cm.

117. $AC = b = 27$, $BC = a = 16$, $AB = c$ é inteiro

$\hat{C} < \hat{A} < \hat{B} \Rightarrow c < 16 < 27 \Rightarrow c < 16$

O valor máximo de AB é 15.

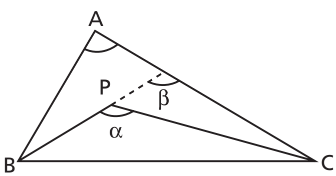
122. Sejam: *a*: hipotenusa; *b*, *c*: catetos. Temos:

$$\begin{cases} a > b \\ a > c \end{cases} \Rightarrow 2a > b + c \Rightarrow a > \frac{b + c}{2}$$

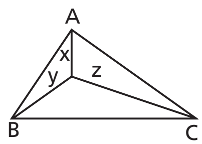
123. $a < b + c \Rightarrow 2a < a + b + c \Rightarrow a < \dfrac{a + b + c}{2}$

124. De acordo com o teorema do ângulo externo, temos: $\alpha > \beta$.

$(\alpha > \beta, \beta > A) \Rightarrow \alpha > A$



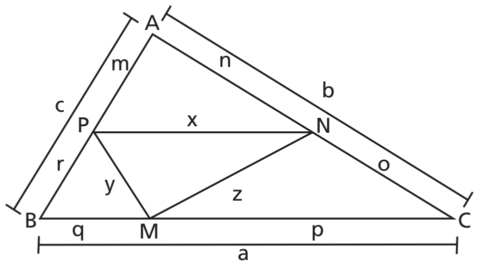
126.
$$\begin{cases} c < x + y < a + b \\ b < x + z < a + c \\ a < y + z < b + c \end{cases} \Rightarrow$$



$\Rightarrow a + b + c < 2(x + y + z) < 2(a + b + c) \Rightarrow$

$\Rightarrow \dfrac{a + b + c}{2} < x + y + z < a + b + c$

127. $\begin{cases} x < m + n \\ y < r + q \\ z < o + p \end{cases} \Rightarrow \begin{cases} x + y + z < \underbrace{(p + q)} + \underbrace{(n + o)} + \underbrace{(m + r)} \\ x + y + z < a + b + c \end{cases}$

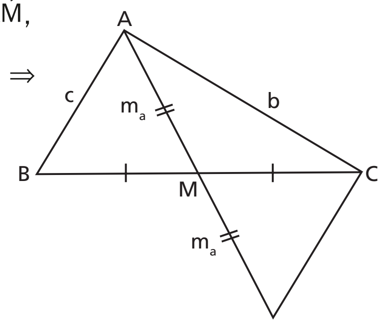


128. 1) Tomemos A' sobre a semirreta $\overrightarrow{AM}$, com M entre A e A' e $MA' = m_a$.

2) ($\triangle AMB \equiv \triangle A'MC$ pelo caso LAL) $\Rightarrow$ $\Rightarrow A'C = c$

3) No $\triangle AA'C$ temos:
$|b - c| < 2m_a < b + c \Rightarrow$

$\Rightarrow \dfrac{|b - c|}{2} < m_a < \dfrac{b + c}{2}$

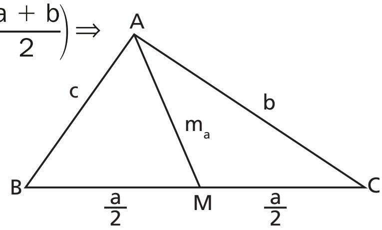


129. 1) De acordo com o exercício 128, temos:

$\left(m_a < \dfrac{b + c}{2}; m_b < \dfrac{a + c}{2}; m_c < \dfrac{a + b}{2}\right) \Rightarrow$

$\Rightarrow m_a + m_b + m_c < a + b + c$

2) $\triangle ABM$: $c < m_a + \dfrac{a}{2}$. Analogamente, $b < m_c + \dfrac{c}{2}$, $a < m_b + \dfrac{b}{2}$.



Somando membro a membro as desigualdades, temos $\dfrac{a + b + c}{2} <$ $< m_a + m_b + m_c$.

CAPÍTULO V — Paralelismo

147. a) Os ângulos internos são dados por $(180° - \alpha)$, $(180° - \beta)$ e $(180° - \gamma)$. Como a soma destes deve ser igual a dois retos, temos: $(180° - \alpha) + (180° - \beta) + (180° - \gamma) = 180° \Rightarrow \alpha + \beta + \gamma = 360°$.

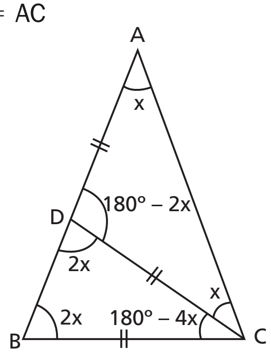
b) De modo análogo:
$(360° - \alpha) + (360° - \beta) + (360° - \gamma) = 180° \Rightarrow \alpha + \beta + \gamma = 900°$

148. a) $\hat{C} = x + 15° \Rightarrow (x + 15°) + (x + 15°) + x = 180° \Rightarrow x = 50°$

b) $A\hat{C}B = 180° - 4x$. $\triangle ABC$ é isósceles de base $\overline{BC} \Rightarrow 180° - 4x = x \Rightarrow$
$\Rightarrow x = 36°$

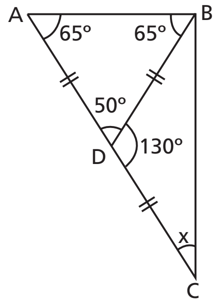
c) $$\left.\begin{array}{l} \hat{A} = 180° - (x + 70°) \\ \hat{C} = \dfrac{180° - \hat{A}}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} \hat{A} = 110° - x \\ x = \dfrac{180° - (110° - x)}{2} \end{array}\right\} \Rightarrow x = 70°$$

149. d) $AB = AC$



1) $\triangle ACD$ é isósceles $\Rightarrow$
$\Rightarrow A\hat{C}D = x$
2) $A\hat{D}C = 180° - 2x$
3) $C\hat{D}B = 2x$
4) $\triangle CBD$ é isósceles $\Rightarrow$
$\Rightarrow C\hat{B}D = 2x$
5) $B\hat{C}D = 180° - 4x$
6) $\triangle ABC$ é isósceles $\Rightarrow$
$\Rightarrow 180° - 4x + x = 2x \Rightarrow$
$\Rightarrow x = 36°$

f) 1) $\triangle ABD$ é isósceles $\Rightarrow$
$\Rightarrow A\hat{B}D = 65°$
2) $A\hat{D}B = 50°$
3) $B\hat{D}C = 130°$
4) $\triangle D\hat{B}C$ é isósceles $\Rightarrow$
$\Rightarrow x = 25°$



g) $$\begin{cases} x + y = 2x + 10° \\ x + y + 2x + 10° + y = 180° \end{cases} \Rightarrow \begin{cases} -x + y = 10° \\ 3x + 2y = 170° \end{cases} \Rightarrow x = 30°, y = 40°$$

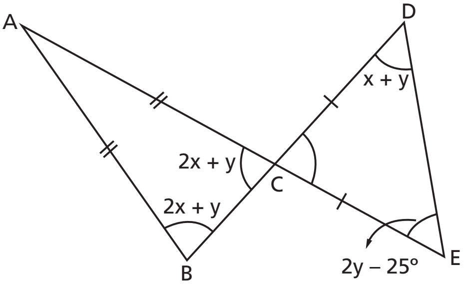
h) 1) $\triangle ABC$ é isósceles $\Rightarrow A\hat{C}B = 2x + y$

2) $E\hat{C}D = 180° - (x + y) - (2y - 25°)$

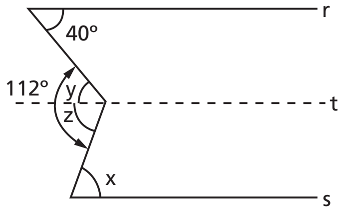
3) $E\hat{C}D = A\hat{C}B$ (o.p.v.)

4) $\triangle CDE$ é isósceles $\Rightarrow x + y = 2y - 25°$

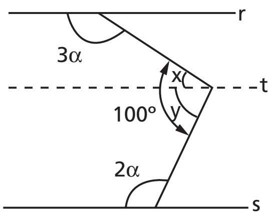
3) e 4) $\Rightarrow \begin{cases} 180° - (x + y) - (2y - 25°) = 2x + y \\ x + y = 2y - 25° \end{cases} \Rightarrow x = 15°, y = 40°$



152. Construímos a reta *t*, t // r, t // s.
t divide o ângulo de 112° em dois outros: *y* e *z*.
y = 40° (alternos internos)
y + z = 112° ⇒ z = 72°
z = x (alternos internos) ⇒ x = 72°

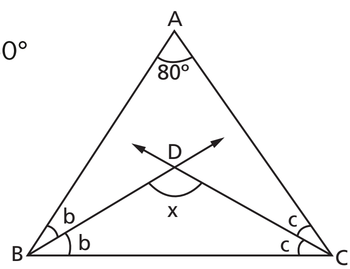


154. Construímos a reta *t*, t // r, t // s.
t divide o ângulo de 100° em *x* e *y*.
$x = 180° - 3\alpha$ (colaterais internos)
$y = 180° - 2\alpha$ (colaterais internos)
$x + y = 100° \Rightarrow 360° - 5\alpha = 100° \Rightarrow$
$\Rightarrow \alpha = 52°$



165. Do $\triangle ABC$ temos:
$2b + 2c + 80° = 180° \Rightarrow b + c = 50°$

Do $\triangle BCD$ temos:
$b + c + x = 180° \Rightarrow$
$\Rightarrow 50° + x = 180° \Rightarrow x = 130°$

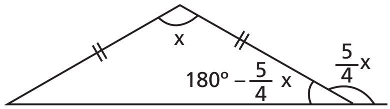


167. ângulo do vértice: x

ângulo da base: $\left(180° - \frac{5}{4}x\right)$

$x + 2\left(180° - \frac{5}{4}x\right) = 180° \Rightarrow$

$\Rightarrow x = 120°$



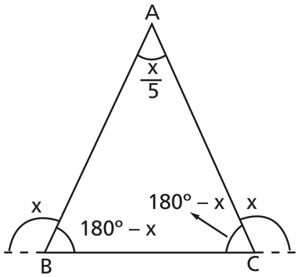
Resposta: os ângulos medem 120°, 30° e 30°.

168. Seja x o valor dos ângulos externos em B e C. Temos:

$\hat{A} = \frac{2x}{10} \Rightarrow \hat{A} = \frac{x}{5}$

$\frac{x}{5} + 2 \cdot (180° - x) = 180° \Rightarrow x = 100°$

$\hat{A} = \frac{x}{5} \Rightarrow \hat{A} = 20°$



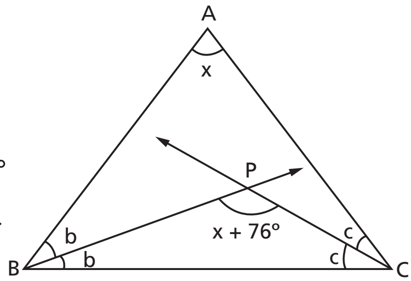
169. $\triangle ABC$: $x + 2b + 2c = 180° \Rightarrow$

$\Rightarrow b + c = \frac{180° - x}{2}$

$\triangle PBC$: $x + 76° + b + c = 180°$

$x + 76° + \frac{180° - x}{2} = 180° \Rightarrow$

$\Rightarrow x = 28°$



170. $\left.\begin{array}{l} \alpha \text{ é ângulo externo do } \triangle ABD \Rightarrow \alpha = \frac{\hat{A}}{2} + \hat{B} \\ \beta \text{ é ângulo externo do } \triangle ACD \Rightarrow \beta = \frac{\hat{A}}{2} + \hat{C} \end{array}\right\} \Rightarrow \alpha - \beta = \hat{B} - \hat{C}$

174. $\triangle CDE$: $C\hat{D}E = 180° - 6\beta$;

$E\hat{C}D = 100°$

$x + 100° + (180° - 6\beta) = 180° \Rightarrow$

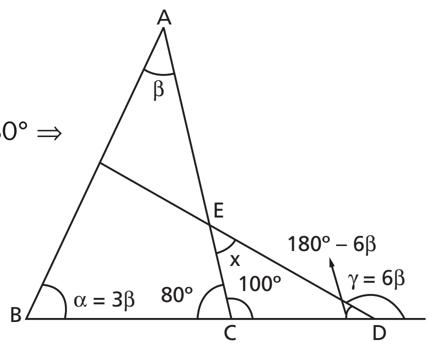
$\Rightarrow x = 6\beta - 100°$

$\triangle ABC$: $3\beta + \beta = 100° \Rightarrow$

$\Rightarrow \beta = 25°$

$x = 6\beta - 100° \Rightarrow$

$\Rightarrow x = 50°$



176. Considere as figuras:

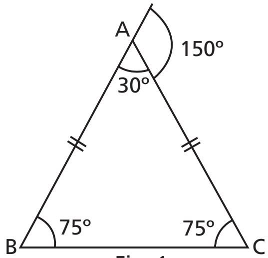


Fig. 1

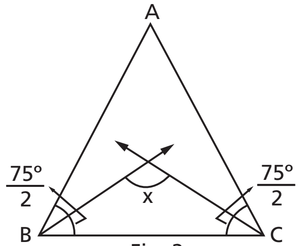


Fig. 2

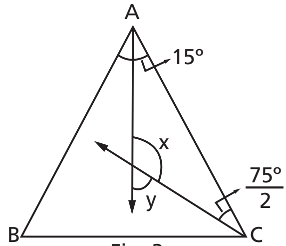


Fig. 3

a) É fácil deduzir (figura 1) que os ângulos medem 30°, 75° e 75°.

b) De acordo com a figura 2, temos $x + \dfrac{75°}{2} + \dfrac{75°}{2} = 180°$. Donde vem: $x = 105°$.

c) De acordo com a figura 3, temos:

$x + 15° + 37°\ 30' = 180° \Rightarrow x = 127°\ 30'$

$y = 15° + 37°\ 30' \Rightarrow y = 52°\ 30'$

179.
$$\left.\begin{array}{l}\text{primeiro ângulo: } x\\ \text{segundo ângulo: } x - 28°\\ \text{terceiro ângulo: } x + 10°\end{array}\right\} \Rightarrow \begin{array}{l} x + (x - 28°) + (x + 10°) = \\ = 180° \Rightarrow x = 66°\end{array}$$

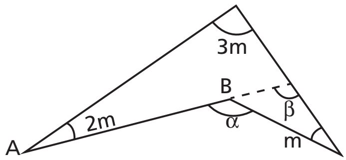
Resposta: os ângulos medem 66°, 38° e 76°.

182. Prolonguemos a reta $\overleftrightarrow{AB}$.

Na figura temos:

$\beta = 2m + 3m \Rightarrow \beta = 5m$

$\alpha = \beta + m \Rightarrow \alpha = 6m$



183. 1) $\hat{A} + \hat{B} + \hat{C} = 180° \Rightarrow$

$\Rightarrow \hat{B} + \hat{C} = 180° - \hat{A}$

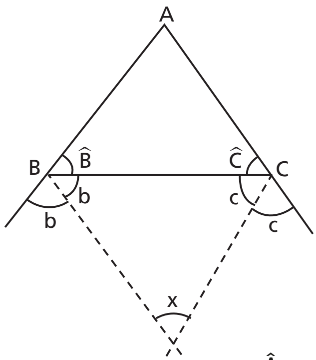
2) $\left.\begin{array}{l}2b + \hat{B} = 180°\\ 2c + \hat{C} = 180°\end{array}\right\} \Rightarrow$

$\Rightarrow 2(b + c) = 360° - (\hat{B} + \hat{C})$

3) $2(b + c) = 360° - (\hat{B} + \hat{C}) \Rightarrow$

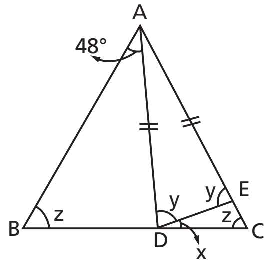
$\Rightarrow 2(b + c) = 360° - (180° - \hat{A}) =$

$= 180° + \hat{A}$



4) $x + (b + c) = 180° \Rightarrow x + \dfrac{180° + \hat{A}}{2} = 180° \Rightarrow x = 90° - \dfrac{\hat{A}}{2}$

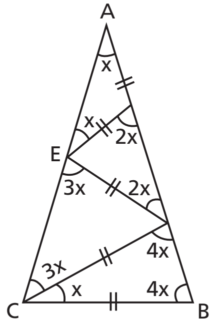
184. Na figura marcamos os ângulos de mesma medida.



$\triangle ABD$: $x + y = z + 48°$
$\triangle ACD$: $y = z + x$

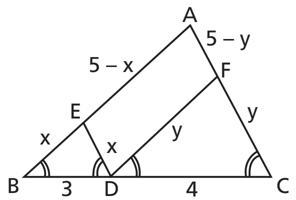
Subtraindo membro a membro:
$x = 48° - x \Rightarrow x = 24°$

185. Seja *x* a medida do ângulo $\hat{A}$. Temos:



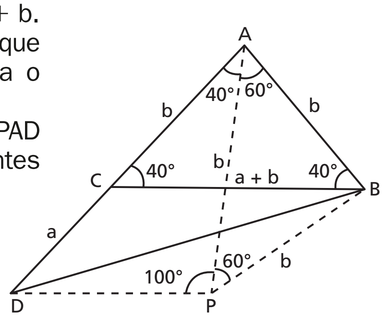
1) $\triangle AEF$ é isósceles $\Rightarrow F\hat{E}A = x$
2) $D\hat{F}E$ é externo ao $\triangle AEF \Rightarrow D\hat{F}E = 2x$
3) $D\hat{E}C$ é externo ao $\triangle AED \Rightarrow D\hat{E}C = 3x$
4) $\triangle CDE$ é isósceles $\Rightarrow D\hat{C}E = 3x$
5) $B\hat{D}C$ é externo ao $\triangle ACD \Rightarrow B\hat{D}C = 4x$
6) $\triangle BCD$ é isósceles $= C\hat{B}D = 4x$
7) $AC = AB \Rightarrow B\hat{C}D = x$
8) $\hat{A} + \hat{B} + \hat{C} = 180° \Rightarrow$
$\Rightarrow x + 4x + 4x = 180° \Rightarrow x = 20°$

187. Na figura temos:



1) $\triangle ABC$ é isósceles $\Rightarrow A\hat{B}D = A\hat{C}D$
2) $E\hat{D}B$, $A\hat{C}D$ correspondentes $\Rightarrow$
$\Rightarrow E\hat{D}B = A\hat{C}D$
3) $F\hat{D}C$, $A\hat{B}D$ correspondentes $\Rightarrow$
$\Rightarrow F\hat{D}C = A\hat{B}D$
4) $\triangle EBD$ e $\triangle FDC$ são isósceles
Indiquemos por *x* e *y* os lados de mesma medida desses triângulos. Temos: $AE + ED + DF + AF = (AE + x) + (y + AF) =$
$= 5 + 5 = 10$ cm.

188. 1) Indiquemos as medidas $AB = AC = b$ e $CD = a$, donde obtemos $BC = a + b$.
2) Tracemos $\overline{AP}$ com $AP = b$, de modo que $B\hat{A}P = 60°$. Obtemos dessa forma o triângulo equilátero APB de lado *b*.

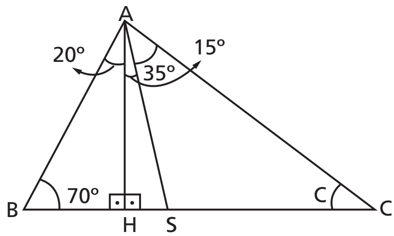


3) Consideremos agora os triângulos PAD e ABC. Note que eles são congruentes pelo caso LAL.
Logo: $PD = AC = b$ e $A\hat{P}D = 100°$.
4) De $PD = b$ concluímos que o $\triangle PBD$ é isósceles. Neste triângulo PBD, como $\hat{P} = 160°$, concluímos que $\hat{B} = \hat{D} = 10°$.
5) Finalmente, de $A\hat{B}P = 60°$, $D\hat{B}P = 10°$ e $C\hat{B}A = 40°$, concluímos que $C\hat{B}D = 10°$.

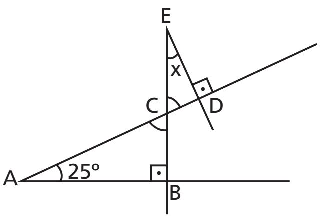
CAPÍTULO VI — Perpendicularidade

192.

1) $(A\hat{H}B = 90°, \hat{B} = 70°) \Rightarrow$
$\Rightarrow H\hat{A}B = 20°$
2) $\overline{AS}$ é bissetriz $\Rightarrow S\hat{A}C = 35°$
3) $\triangle ABC$: $(\hat{A} = 70°, \hat{B} = 70°) \Rightarrow$
$\Rightarrow \hat{C} = 40°$



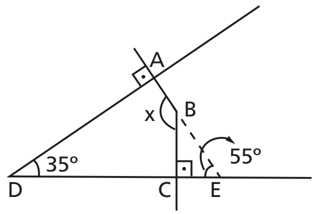
193. a)



1) $A\hat{C}B = 65°$

2) $A\hat{C}B$ e $D\hat{C}E$ são o.p.v. $\Rightarrow$
$\Rightarrow D\hat{C}E = 65°$
3) $x = 90° - D\hat{C}E \Rightarrow x = 25°$

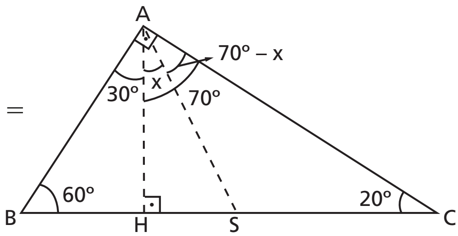
b)



1) Prolongamos $\overline{AB}$ até cortar $\overrightarrow{CD}$ em E.
2) $\triangle AED$: $\hat{E} = 55°$
3) $\triangle BCE$: $x = 90° + 55° \Rightarrow$
$\Rightarrow x = 145°$

198.

1) $\triangle ABH \Rightarrow H\hat{A}B = 30°$
2) $\triangle ACH \Rightarrow H\hat{A}C = 70°$
3) $S\hat{A}C = 70° - x$
4) $\overline{AS}$ é bissetriz $\Rightarrow x + 30° =$
$= 70° - x \Rightarrow x = 20°$



199.

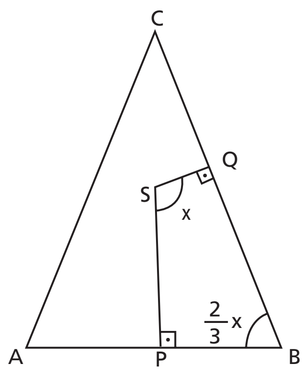
1) Usando o resultado do exercício 194:

$$x + \frac{2}{3}x = 180° \Rightarrow x = 108°$$

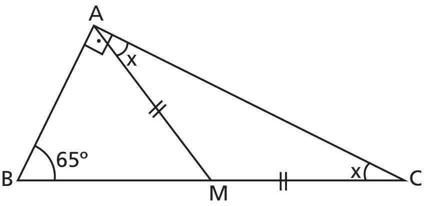
2) $\hat{B} = \frac{2}{3}x \Rightarrow \hat{B} = 72° = \hat{A}$

3) $\hat{A} + \hat{B} + \hat{C} = 180° \Rightarrow \hat{C} = 36°$

Resposta: os ângulos medem 36°, 72° e 72°.



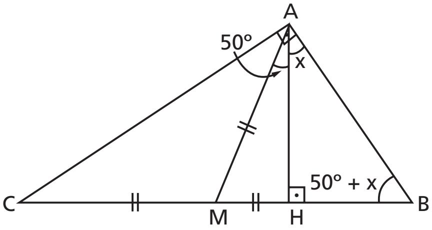
201. a)



1) $AM = MC \Rightarrow \hat{C} = x$

2) $\triangle ABC$: $x + 90° + 65° = 180° \Rightarrow$
$\Rightarrow x = 25°$

b)



1) $AM = MB \Rightarrow \hat{B} = 50° + x$

2) $\triangle ABH$: $x + 50° + x = 90° \Rightarrow$
$\Rightarrow x = 20°$

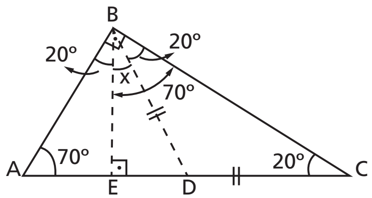
203. 1) $\triangle AEB$: $A\hat{B}E = 20°$

2) $A\hat{B}E = 20° \Rightarrow E\hat{B}C = 70°$

3) $\triangle ABC$: $\hat{C} = 20°$

4) $\overline{BD}$ é mediana $\Rightarrow DB = DC \Rightarrow$
$\Rightarrow D\hat{B}C = \hat{C} = 20°$

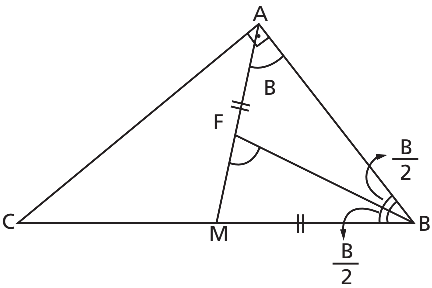
5) $E\hat{B}D + D\hat{B}C = 70° \Rightarrow$
$\Rightarrow E\hat{B}D + 20° = 70° \Rightarrow E\hat{B}D = 50°$



204. 1) AM é mediana $\Rightarrow$
$\Rightarrow AM = MB \Rightarrow B\hat{A}M = \hat{B}$

2) BF é bissetriz $\Rightarrow A\hat{B}F = \dfrac{\hat{B}}{2}$

3) x é externo ao $\triangle ABF \Rightarrow$
$\Rightarrow x = \hat{B} + \dfrac{\hat{B}}{2} = \dfrac{3\hat{B}}{2}$

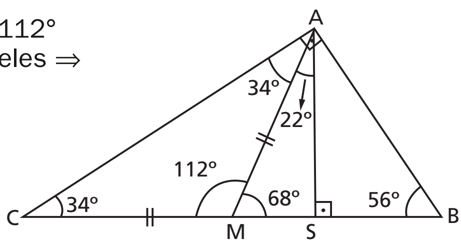


205. 1) $\triangle AMS$: $\hat{M} = 68°$, $A\hat{M}C = 112°$

2) $AM = MC \Rightarrow \triangle AMC$ isósceles $\Rightarrow$
$\Rightarrow \hat{C} = M\hat{A}C = 34°$

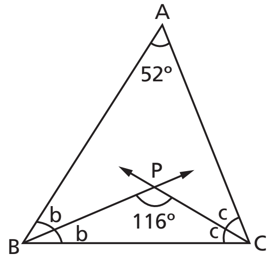
3) $\triangle ABC$: $\hat{B} = 56°$

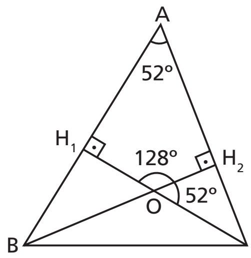
Resposta: $\hat{B} = 56$; $\hat{C} = 34°$.



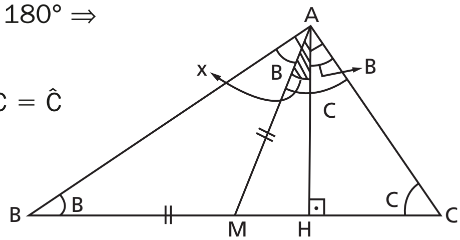
207. 1) $\triangle PBC$: $b + c + 116° =$
$= 180° \Rightarrow b + c = 64°$
2) $\triangle ABC$: $2b + 2c + \hat{A} =$
$= 180° \Rightarrow \hat{A} = 52°$

3) Usando o resultado do exercício 194, temos:
$H_1\hat{O}H_2 = 128° \Rightarrow H_2\hat{O}C = 52°$

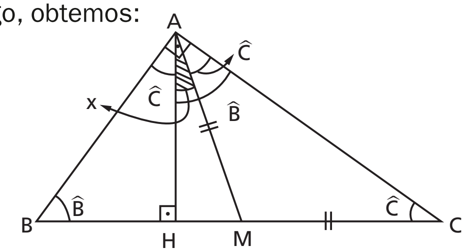




208. 1) $\triangle ABC$: $= 90° + \hat{B} + \hat{C} = 180° \Rightarrow$
$\Rightarrow \hat{B} + \hat{C} = 90°$
2) $\triangle ACH$: $H\hat{A}C = \hat{B}$
3) $\triangle AMC$ é isósceles $\Rightarrow M\hat{A}C = \hat{C}$
4) $x + \hat{B} = \hat{C} \Rightarrow x = \hat{C} - \hat{B}$



Procedendo de modo análogo, obtemos:
5) $x = \hat{B} - \hat{C}$
Logo, 4), 5) $\Rightarrow x = |\hat{B} - \hat{C}|$.

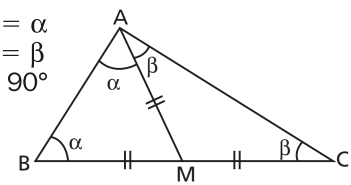


209. Hipótese / Tese

AM é mediana
AM = BM = MC $\Big\} \Rightarrow \triangle ABC$ é retângulo

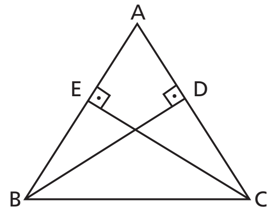
Demonstração

1) $\triangle ABM$ é isósceles $\Rightarrow A\hat{B}M = M\hat{A}B = \alpha$
2) $\triangle ACM$ é isósceles $\Rightarrow A\hat{C}M = M\hat{A}C = \beta$
3) $\triangle ABC$: $2\alpha + 2\beta = 180° \Rightarrow \alpha + \beta = 90°$
4) $\alpha + \beta = 90° \Rightarrow \hat{A} = 90°$



211. Hipótese — Tese

$$\left.\begin{array}{l}\triangle ABC \text{ é isósceles}\\ \overline{BD}\text{: altura relativa a } \overline{AC}\\ \overline{CE}\text{: altura relativa a } \overline{AB}\end{array}\right\} \Rightarrow \overline{BD} \equiv \overline{CE}$$



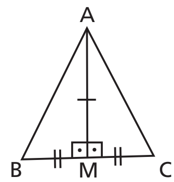
Demonstração

$$\left.\begin{array}{l}\overline{BC} \equiv \overline{CB} \text{ (comum)}\\ A\hat{B}C \equiv A\hat{C}B \text{ (}\triangle ABC \text{ isósceles)}\\ C\hat{E}B \equiv B\hat{D}C \text{ (retos)}\end{array}\right\} \xRightarrow{LAA_o} \triangle BCE \equiv \triangle CBD \Rightarrow \overline{BD} \equiv \overline{CE}$$

212.

$$\left.\begin{array}{l}\overline{AM} \text{ é lado comum}\\ A\hat{M}B \equiv A\hat{M}C \text{ (}\overline{AM} \text{ é altura)}\\ \overline{BM} \equiv \overline{MC} \text{ (}\overline{AM} \text{ é mediana)}\end{array}\right\} \xRightarrow{LAL} \triangle AMC \equiv \triangle AMB \Rightarrow$$

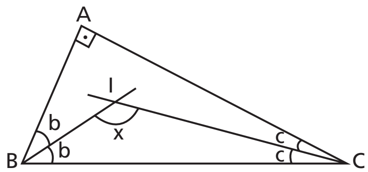
$\Rightarrow \overline{AB} \equiv \overline{AC} \Rightarrow \triangle ABC$ é isósceles



214. Conforme a figura:

1) $\triangle ABC$: $2b + 2c + 90° = 180° \Rightarrow$
$\Rightarrow b + c = 45°$

2) $\triangle IBC$: $x + b + c = 180° \Rightarrow$
$\Rightarrow x = 135°$



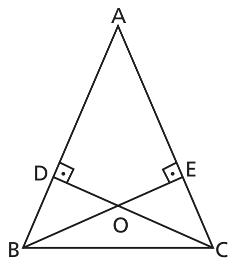
215. Hipótese — Tese

$\overline{BE} \equiv \overline{CD}$ — $\triangle ABC$ é isósceles

Demonstração

$(\overline{BE} \equiv \overline{CD}$; BC comum$)$ $\xRightarrow[\text{especial}]{\text{caso}}$

$\Rightarrow \triangle BCD \equiv \triangle CBE \Rightarrow C\hat{B}D \equiv B\hat{C}E \Rightarrow$
$\Rightarrow \triangle ABC$ é isósceles



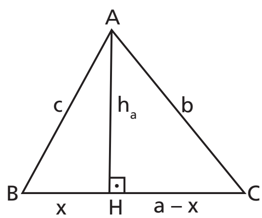
220. 1ª parte:

$\triangle AHB$: $h_a < c$

Analogamente: $h_b < a$; $h_c < b$.

Somando as desigualdades, temos:

$h_a + h_b + h_c < a + b + c$



2ª parte:

$$\left.\begin{array}{l}\triangle ABH\text{: } c < h_a + x \\ \triangle ACH\text{: } b < h_a + a - x\end{array}\right\} \Rightarrow 2h_a > b + c - a$$

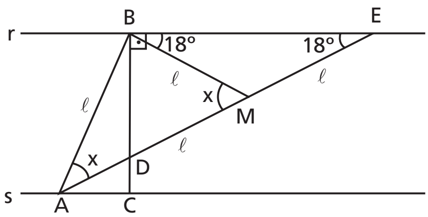
Analogamente: $2h_b > a + c - b$; $2h_c > a + b - c$.

Somando as três últimas desigualdades: $h_a + h_b + h_c > \dfrac{a + b + c}{2}$.

221. Sendo M o ponto médio de $\overline{DE}$ e indicando $AB = \ell$, temos $DM = EM = \ell$.

Note que também $BM = \ell$.

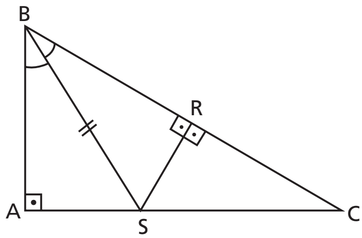
Dessa forma concluímos que os triângulos ABM e BME são isósceles. Calculando os ângulos das bases, obtemos $x = 36°$.



222. Tracemos $\overline{SR}$ tal que $\overline{SR} \perp \overline{BC}$.
Temos:

$(\overline{BS}$ comum; $S\hat{B}R \equiv S\hat{B}A$, $\hat{R} \equiv \hat{A}) \xRightarrow{LAA_o}$
$\Rightarrow \triangle BSR \equiv \triangle BSA \Rightarrow$

$$\Rightarrow \left.\begin{array}{l}\overline{AS} \equiv \overline{SR} \\ \triangle SRC \Rightarrow \overline{SR} < \overline{SC}\end{array}\right\} \Rightarrow \overline{AS} < \overline{SC}$$

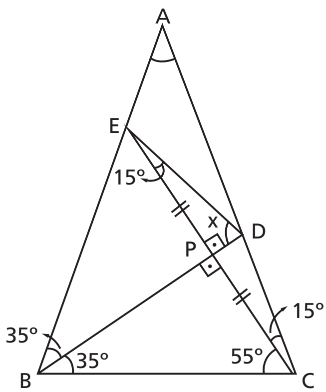


223. 1) Os ângulos da base devem medir 70° cada.
Daí, $E\hat{B}D = 35°$; $E\hat{C}B = 55°$; $B\hat{P}C = 90°$.

2) Note que $\overline{BP}$ é bissetriz e altura. Assim, o $\triangle BCE$ é isósceles e então $PC = PE$.

3) Note agora que $\overline{DP}$ é mediana e é altura no $\triangle CDE$. Então, $\triangle CDE$ é isósceles e daí:
$D\hat{E}P = 15°$.

4) Do $\triangle DEP$ tiramos $x = 75°$.

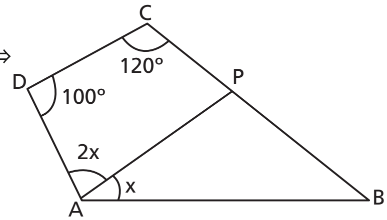


CAPÍTULO VII — Quadriláteros notáveis

226. a) $PA = PB \Rightarrow \hat{B} = x$

$100° + 120° + 3x + x = 360° \Rightarrow$

$\Rightarrow x = 35°$

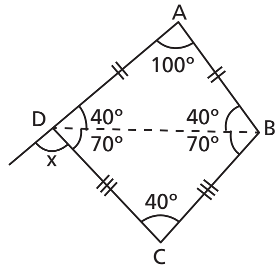


b) Traçamos $\overline{BD}$.

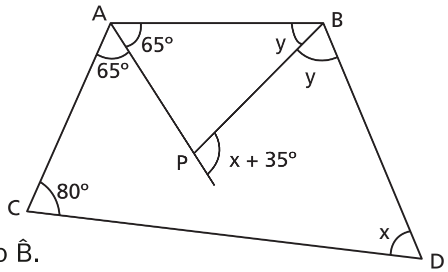
$\triangle ABD$ e $\triangle BCD$ são isósceles $\Rightarrow$

$\Rightarrow A\hat{D}B = 40°$, $C\hat{D}B = 70°$

$x + 40° + 70° = 180° \Rightarrow x = 70°$



227. a)



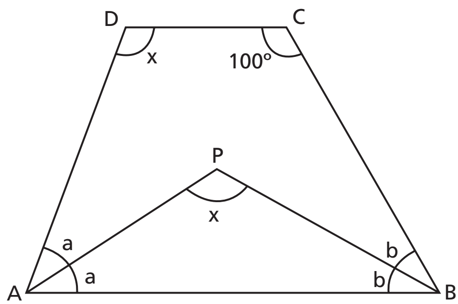
1) $\overline{AP}$ bissetriz $\Rightarrow B\hat{A}P = 65°$

2) Indiquemos por $2y$ o ângulo $\hat{B}$.

3) $\overline{BP}$ é bissetriz $\Rightarrow A\hat{B}P = P\hat{B}D = y$

4) $\left.\begin{array}{l} \triangle ABP\text{: } x + 35° = y + 65° \\ ABDC\text{: } x + 2y + 80° + 130° = 360° \end{array}\right\} \Rightarrow x = 70°$

b)



1) Marquemos os ângulos congruentes determinados pelas bissetrizes $\overline{AP}$ e $\overline{BP}$.

2) $\left.\begin{array}{l} \triangle PAB\text{: } a + b = 180° - x \\ ABCD\text{: } 2(a + b) + x + 100° = 360° \end{array}\right\} \Rightarrow x = 100°$

229. De acordo com a figura, temos:

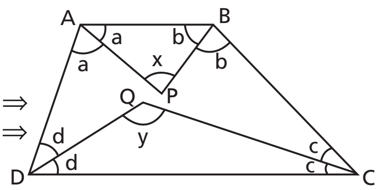
$\triangle ABP$: $a + b = 180° - x$

$\triangle QCD$: $c + d = 180° - y$

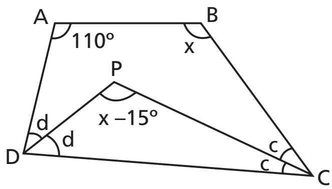
ABCD: $2(a + b) + 2(c + d) = 360° \Rightarrow$

$\Rightarrow 2(180° - x) + 2(180° - y) = 360° \Rightarrow$

$\Rightarrow x + y = 180°$



231.



1ª parte

Trapézio ABCD: $2d + 110° = 180° \Rightarrow d = 35°$

$\triangle PCD$: $c + d + (x - 15°) = 180° \Rightarrow c + 35° + x - 15° = 180° \Rightarrow$

$\Rightarrow c + x = 160°$ (1)

Trapézio ABCD: $2c + x = 180°$ (2)

(1) e (2) $\Rightarrow x = 140°$

2ª parte

$c + x = 160° \Rightarrow c + 140° = 160° \Rightarrow c = 20° \Rightarrow B\hat{C}D = 40°$

235. $AD = 20$ cm, $BQ = 12$ cm $\Rightarrow$

$\Rightarrow CQ = 8$ cm

Se $BQ = BP = 12$ cm, então

$\triangle BPQ$ é isósceles, $\hat{P} = B\hat{Q}P$ e

$B\hat{Q}P = C\hat{Q}D$ (o.p.v.).

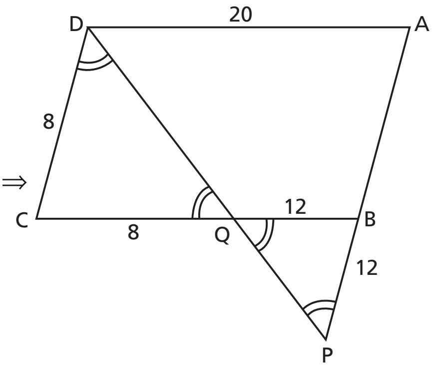
Como $\overline{AP} \,//\, \overline{CD}$, temos

$A\hat{P}Q = C\hat{D}Q$ (alternos internos) $\Rightarrow$

$\Rightarrow \triangle CQD$ é isósceles $\Rightarrow$

$\Rightarrow CQ = CD = 8$ cm.

Logo, o perímetro do paralelogramo ABCD vale 56 cm.



244. Sejam *a* e *b* os ângulos consecutivos. Temos:

$$\begin{cases} a + b = 180° \\ a - b = \dfrac{2(a + b)}{9} \end{cases} \Rightarrow (a = 110°, b = 70°).$$

Resposta: os ângulos medem 110°, 70°, 110° e 70°.

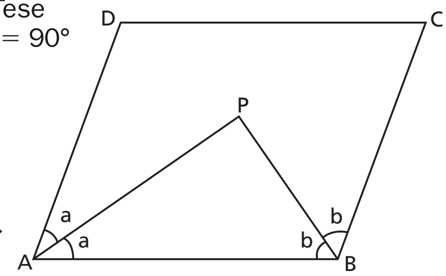
245. Hipótese Tese

$\overline{\text{ABCD}}$ é paralelogramo $\Rightarrow A\hat{P}B = 90°$

$\overline{AP}$ e $\overline{BP}$ são bissetrizes

Demonstração

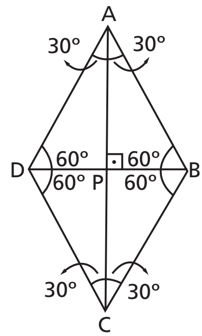
ABCD é paralelogramo $\Rightarrow$
$\Rightarrow 2a + 2b = 180° \Rightarrow$
$\Rightarrow a + b = 90°$
$\triangle PAB$: $a + b + A\hat{P}B = 180° \Rightarrow$
$\Rightarrow A\hat{P}B = 90°$



247. ABCD é losango $\Rightarrow$ as diagonais são perpendiculares

Seja $P\hat{A}B = \dfrac{1}{3} \cdot 90° = 30°$.

Então, temos: no $\triangle ABP$, $A\hat{B}P = 60°$.
Como as diagonais do losango são também bissetrizes, os ângulos do losango são: 60°, 120°, 60°, 120°.



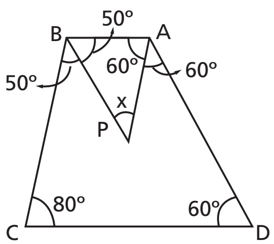
251. Os ângulos a que se refere o enunciado são adjacentes a uma mesma base, senão sua soma seria 180°. Sejam *x* e *y* os ângulos. Temos:

$$\begin{cases} x + y = 78° \\ x - y = 4° \end{cases} \Rightarrow (x = 41°, y = 37°)$$

O maior ângulo do trapézio é o suplementar de *y*, que é $180° - 37° =$ $= 143°$.
Resposta: 143°.

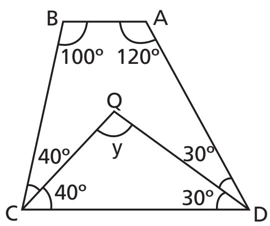
252. Seja ABCD o trapézio, com $\hat{C} = 80°$ e $\hat{D} = 60°$. Daí, $\hat{A} = 120°$ e $\hat{B} = 100°$.

1º)



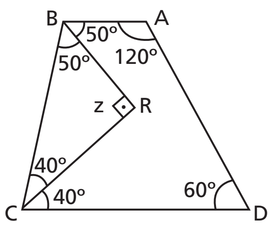
Ângulos formados pelas bissetrizes de $\hat{A}$ e $\hat{B}$:
$\triangle ABP \Rightarrow$
$\Rightarrow 50° + 60° + x =$
$= 180° \Rightarrow x = 70°$

2º)



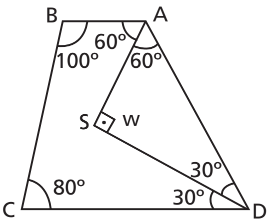
Ângulos formados pelas bissetrizes de $\hat{C}$ e $\hat{D}$:
$\triangle CDQ \Rightarrow$
$\Rightarrow 40° + 30° + y =$
$= 180° \Rightarrow y = 110°$

3º)



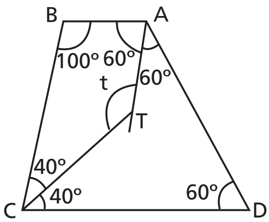
Ângulos formados pelas bissetrizes de $\hat{B}$ e $\hat{C}$:
$\triangle BCR \Rightarrow$
$\Rightarrow 50° + 40° + z =$
$= 180° \Rightarrow z = 90°$

4º)



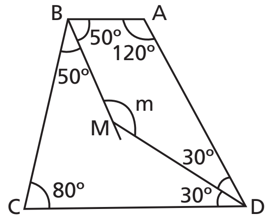
Ângulos formados pelas bissetrizes de $\hat{A}$ e $\hat{D}$:
$\triangle ADS \Rightarrow$
$\Rightarrow 60° + 30° + w =$
$= 180° \Rightarrow w = 90°$

5º)



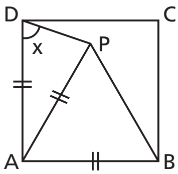
Ângulos formados pelas bissetrizes de $\hat{A}$ e $\hat{C}$:
Quadrilátero ABCT:
$60° + 100° + 40° + t =$
$= 360° \Rightarrow t = 160°$

6º)

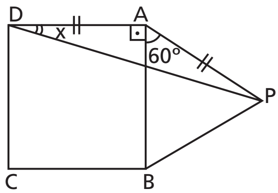


Ângulos formados pelas bissetrizes de $\hat{B}$ e $\hat{D}$:
Quadrilátero BADM:
$50° + 120° + 30° + m =$
$= 360° \Rightarrow m = 160°$

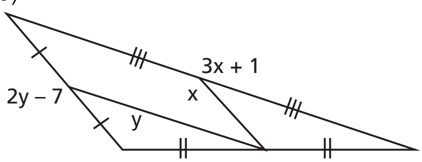
254. a) $\left.\begin{array}{l} 1)\ P\hat{A}B = 60°,\ B\hat{A}D = 90° \Rightarrow P\hat{A}D = 30° \\ 2)\ PA = AD \Rightarrow \triangle APD \text{ é isósceles} \end{array}\right\} \Rightarrow A\hat{D}P = 75°$



b) $\left.\begin{array}{l} 1)\ P\hat{A}B = 60°,\ B\hat{A}D = 90° \Rightarrow P\hat{A}D = 150° \\ 2)\ PA = AD \Rightarrow \triangle APD \text{ é isósceles} \end{array}\right\} \Rightarrow A\hat{D}P = 15°$



255. b)



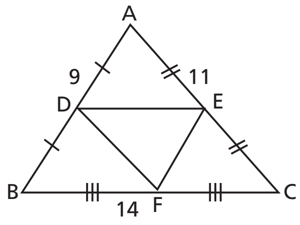
$$\left\{\begin{array}{l} x = \dfrac{2y - 7}{2} \\ y = \dfrac{3x + 1}{2} \end{array}\right. \Rightarrow x = 6;\ y = \dfrac{19}{2}$$

256. DE é base média $\Rightarrow DE = \dfrac{14}{2} = 7$

EF é base média $\Rightarrow EF = \dfrac{9}{2} = 4{,}5$

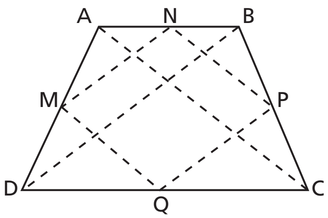
DF é base média $\Rightarrow DF = \dfrac{11}{2} = 5{,}5$

Perímetro $\triangle DEF = 7 + 4{,}5 + 5{,}5 = 17$



259. 1ª parte
M, N, P, Q são pontos médios de $\overline{AD}$, $\overline{AB}$, $\overline{BC}$, $\overline{CD}$;

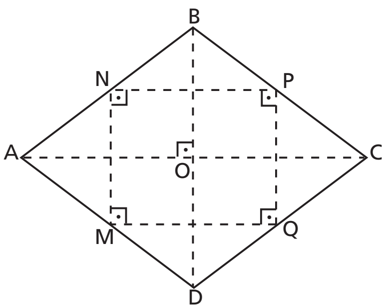
$$\left.\begin{array}{l} MN = NP = PQ = QM \\ \triangle ABC \Rightarrow AC = 2NP \\ \triangle ABD \Rightarrow BD = 2MN \end{array}\right\} \Rightarrow \overline{AC} \equiv \overline{BD}$$



2ª parte:
M, N, P, Q são pontos médios de $\overline{AD}$, $\overline{AB}$, $\overline{BC}$, $\overline{CD}$;

$$\left.\begin{array}{l} MNPQ \text{ é retângulo} \\ \triangle ABC \Rightarrow \overline{AC} \,/\!/\, \overline{NP} \\ \triangle ABD \Rightarrow \overline{BD} \,/\!/\, \overline{MN} \end{array}\right\} \Rightarrow$$

$$\Rightarrow A\hat{O}B \equiv M\hat{N}P \Rightarrow \overline{AC} \perp \overline{BD}$$



262. Sejam B a base maior e *b* a base menor. Temos:

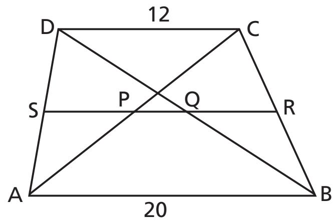
$$\begin{cases} \dfrac{B + b}{2} = 20 \\ B = \dfrac{3b}{2} \end{cases} \Rightarrow (B = 24 \text{ cm};\ b = 16 \text{ cm})$$

263. $\triangle ABC \Rightarrow PR = \dfrac{AB}{2} \Rightarrow PR = \dfrac{20}{2} \Rightarrow PR = 10 \text{ cm}$

$\triangle BCD \Rightarrow RQ = \dfrac{CD}{2} \Rightarrow RQ = \dfrac{12}{2} \Rightarrow RQ = 6 \text{ cm}$

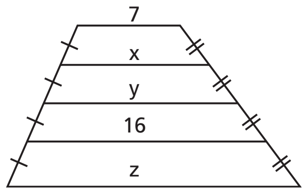
$\overline{RS}$ é base média do trapézio $\Rightarrow$

$\Rightarrow RS = \dfrac{20 + 12}{2} \Rightarrow RS = 16$ cm



264. c) $\begin{cases} x = \dfrac{y + 7}{2} \\ y = \dfrac{x + 16}{2} \end{cases} \Rightarrow (x = 10;\ y = 13)$

$16 = \dfrac{y + z}{2} \Rightarrow z = 19$

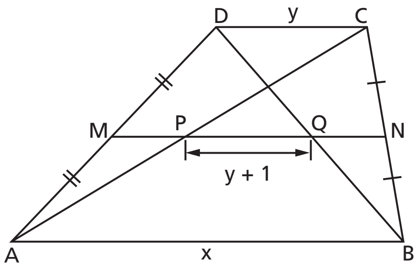


d) $\triangle ACD \Rightarrow MP = \dfrac{y}{2}$

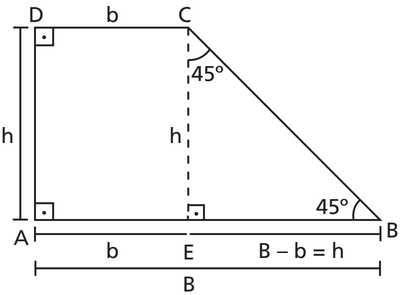
$\triangle BCD \Rightarrow NQ = \dfrac{y}{2}$

$$\left.\begin{array}{l} MN = \dfrac{y}{2} + y + 1 + \dfrac{y}{2} \Rightarrow x - 2y + 5 = 2y + 1 \\ y + 1 = \dfrac{x - y}{2} \Rightarrow x - 3y = 2 \end{array}\right\} \Rightarrow$$

$\Rightarrow (x = 20;\ y = 6)$



265. Note que no $\triangle BCE$ da figura o ângulo $\hat{C}$ mede 45°. Logo, o $\triangle BCE$ é isósceles e então BE = h. Como AE = b, temos BE = B − b. Portanto, h = B − b.

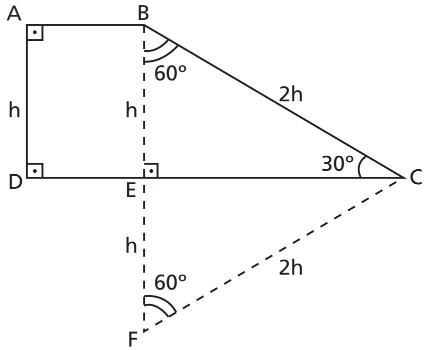


266. Seja ABCD o trapézio, com $\hat{C} =$
$= 30°$.
Tracemos $\overline{BF}$, $\overline{BF} \perp \overline{CD}$, $BF =$
$= 2h$.

$$\left.\begin{array}{l} \triangle BCE \Rightarrow \hat{B} = 60° \\ \triangle CEF \Rightarrow \hat{F} = 60° \end{array}\right\} \Rightarrow$$

$\Rightarrow \triangle BCF$ é equilátero de lado 2h

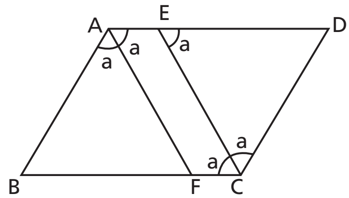
Portanto, $h = \dfrac{BC}{2}$.



267. Seja o paralelogramo ABCD, ao lado. Sejam $\overline{AF}$ e $\overline{CE}$ as bissetrizes dos ângulos obtusos.

$$\left.\begin{array}{l} \overline{AD} \,//\, \overline{BC} \\ D\hat{E}C = B\hat{C}E \text{ (alternos)} \end{array}\right\} \Rightarrow D\hat{E}C = a$$

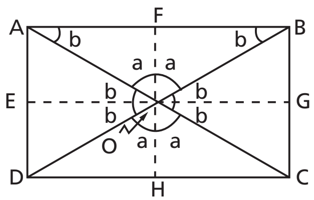
$$\left.\begin{array}{l} D\hat{E}C = a \\ E\hat{A}F = a \end{array}\right\} \Rightarrow \overline{AF} \,//\, \overline{CE}$$



268. Da figura podemos concluir que:
$4a + 4b = 360° \Rightarrow a + b = 90°$
Quadrilátero BFOG $\Rightarrow$
$\Rightarrow F\hat{O}G = a + b = 90°$ (1)

$$\left.\begin{array}{l} \triangle AOB \text{ é isósceles} \\ 2a + 2b = 180° \end{array}\right\} \Rightarrow O\hat{A}F = O\hat{B}F = b$$

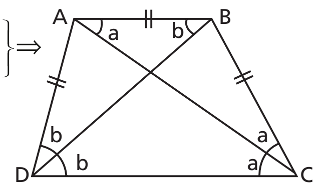
$\triangle BOF \Rightarrow \hat{F} = 90°$ (2)
(1), (2) $\Rightarrow \overline{EG} \,//\, \overline{AB}$
Analogamente, $\overline{FH} \,//\, \overline{AD}$.



269. Seja $B\hat{A}C = a$.

$$\left.\begin{array}{l} B\hat{A}C, A\hat{C}D \text{ alternos internos} \Rightarrow A\hat{C}D = a \\ \triangle ABC \text{ é isósceles de base } \overline{AC} \Rightarrow B\hat{C}A = a \end{array}\right\} \Rightarrow$$

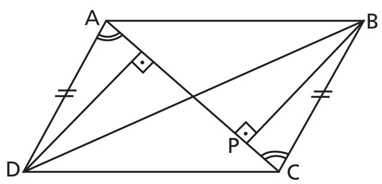
$\Rightarrow \overline{AC}$ é bissetriz do ângulo $\hat{C}$
Analogamente, $\overline{BD}$ é bissetriz de $\hat{D}$.



270. Seja o paralelogramo ABCD.

$$\left.\begin{array}{l} \overline{AD} \equiv \overline{BC} \text{ (lados opostos)} \\ D\hat{A}C \equiv B\hat{C}A \text{ (alternos)} \\ A\hat{Q}D \equiv B\hat{P}C \text{ (por construção)} \end{array}\right\} \xrightarrow{LAA_o}$$

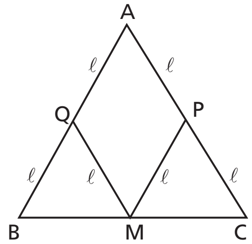
$\Rightarrow \triangle AQD \equiv \triangle CPB \Rightarrow \overline{BP} \equiv \overline{DQ}$



271. Inicialmente observemos que P é ponto médio de $\overline{AC}$ e Q é ponto médio de $\overline{AB}$.

$$\left.\begin{array}{l} AQ = BQ = AP = PC = \ell \\ MQ \text{ é base média} \Rightarrow MQ = \ell \\ MP \text{ é base média} \Rightarrow MP = \ell \end{array}\right\} \Rightarrow$$

$\Rightarrow$ APMQ é losango



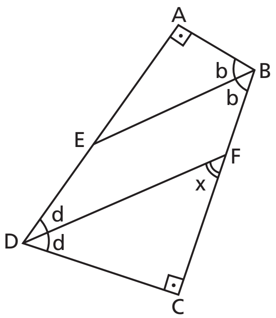
272. Seja ABCD o quadrilátero com $\hat{A} = \hat{C} = 90°$, $\overline{BE}$ bissetriz de $\hat{B}$, $\overline{DF}$ bissetriz de $\hat{D}$.

Temos:

$$\left.\begin{array}{l} \triangle CDF \Rightarrow d + x = 90° \\ ABCD \Rightarrow 2b + 2d = 180° \end{array}\right\} \Rightarrow b = x$$

b e *x* são correspondentes $\Rightarrow$

$\Rightarrow \overline{BE} \mathbin{/\!/} \overline{DF}$



273. Unimos E com M, ponto médio de $\overline{BC}$. Temos:

$$\left.\begin{array}{l} C\hat{M}E \equiv B\hat{M}G \text{ (o.p.v.)} \\ \overline{CM} \equiv \overline{BM} \\ \hat{C} \equiv \hat{B} \text{ (retos)} \end{array}\right\} \xrightarrow{ALA} \triangle CEM \equiv \triangle BGM \Rightarrow (\overline{EM} \equiv \overline{MG}, \overline{EC} \equiv \overline{BG})$$

Além disso, como $\overline{BC} + \overline{CE} = \overline{AE}$, temos:

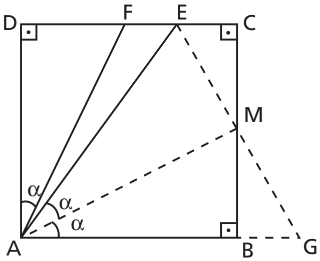
$\overline{EC} \equiv \overline{BG} \Rightarrow \overline{AG} \equiv \overline{AB} + \overline{BG} \equiv \overline{BC} + \overline{CE} \equiv \overline{AE}$

Então:

$(\overline{EM} \equiv \overline{MG}, \overline{AG} \equiv \overline{AE}, \overline{AM} \text{ comum}) \xrightarrow{LLL} \triangle AME \equiv \triangle AMG \Rightarrow G\hat{A}M = E\hat{A}M = \alpha$

$(\overline{BM} \equiv \overline{DF}, \overline{AB} \equiv \overline{AD}, \hat{B} = \hat{D}) \xrightarrow{LAL} \triangle ABM \equiv \triangle ADF \Rightarrow B\hat{A}M = D\hat{A}F = \alpha$

Logo, $B\hat{A}E = 2\alpha = 2 \cdot F\hat{A}D$.



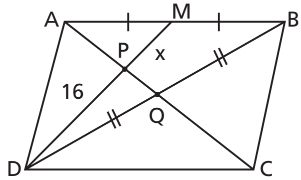
CAPÍTULO VIII — Pontos notáveis do triângulo

278. Tracemos a diagonal $\overline{BD}$.

$\left.\begin{array}{l}\text{ABCD é paralelogramo} \Rightarrow \overline{BQ} \equiv \overline{DQ} \\ \text{M é ponto médio de } \overline{AB} \Rightarrow \overline{AM} \equiv \overline{MB}\end{array}\right\} \Rightarrow$

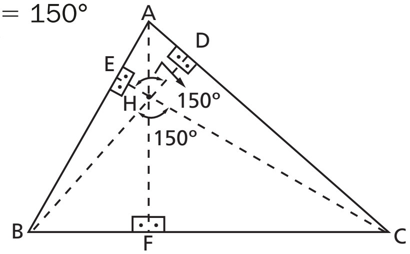
$\Rightarrow$ P é baricentro do $\triangle ABD \Rightarrow$

$\Rightarrow x = \dfrac{DP}{2} \Rightarrow x = \dfrac{16}{2} \Rightarrow x = 8$



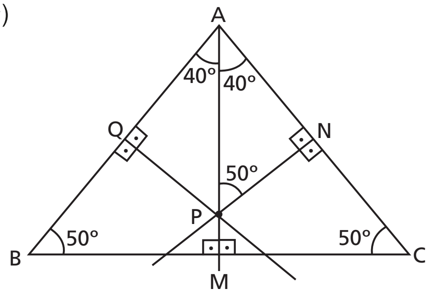
279. 1) $D\hat{H}E$ e $B\hat{H}C$ são o.p.v. $\Rightarrow D\hat{H}E = 150°$

2) Quadrilátero ADHE $\Rightarrow \hat{A} = 30°$



282. Temos duas possibilidades:

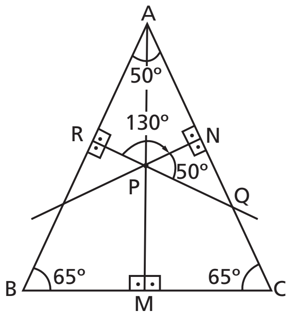
1ª)



$A\hat{P}N = 50° \Rightarrow P\hat{A}N = 40° = \hat{A} = 80°$

Então, $\hat{B} = \hat{C} = 50°$.

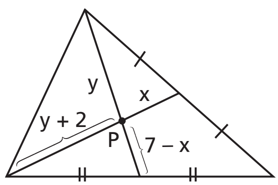
2ª)



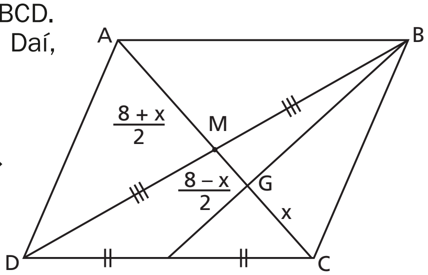
$N\hat{P}Q = 50° \Rightarrow N\hat{P}R =$
$= 130° \Rightarrow \hat{A} = 50°$

Então, $\hat{B} = \hat{C} = 65°$.

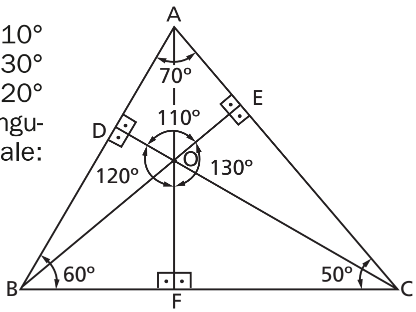
283. a) P é baricentro $\Rightarrow \begin{cases} y + 2 = 2x \\ y = 2(7 - x) \end{cases} \Rightarrow (x = 4, y = 6)$



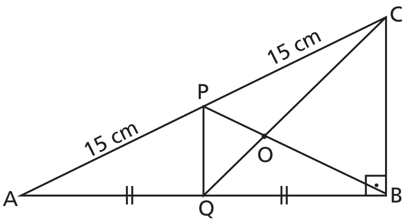
b) Tracemos a diagonal $\overline{BD}$. Seja $\overline{BD} \cap \overline{AC} = \{M\}$.
Note que G é baricentro do $\triangle BCD$. A diagonal $\overline{AC}$ mede $8 + x$. Daí,
$AM = MC = \dfrac{8 + x}{2}$.
$MG = MC - GC = \dfrac{8 + x}{2} - x \Rightarrow$
$\Rightarrow MG = \dfrac{8 - x}{2}$
G é baricentro do $\triangle BCD \Rightarrow$
$\Rightarrow GC = 2 \cdot MG \Rightarrow x = 8 - x \Rightarrow$
$\Rightarrow x = 4$



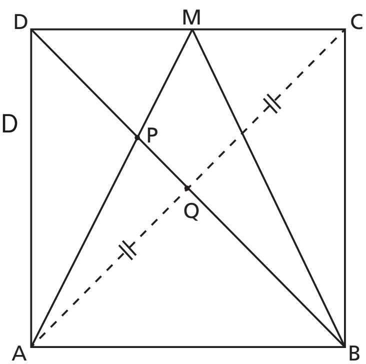
286. Quadrilátero ADOE $\Rightarrow$ DÔE $= 110°$
Quadrilátero CEOF $\Rightarrow$ EÔF $= 130°$
Quadrilátero BDOF $\Rightarrow$ DÔF $= 120°$
A razão entre os dois maiores ângulos formados pelas alturas vale:
$\dfrac{130°}{120°} = \dfrac{13}{12}$ ou $\dfrac{120°}{130°} = \dfrac{12}{13}$



288. PQ é base média do $\triangle ABC \Rightarrow AP =$
$= PC = 15$ cm.
$\left.\begin{array}{l}\triangle ABC \text{ é retângulo} \\ \overline{BP} \text{ é mediana relativa à} \\ \text{hipotenusa}\end{array}\right\} \Rightarrow$
$\Rightarrow BP = 15$ cm
O é baricentro do $\triangle ABC \Rightarrow$
$\Rightarrow PO = 5$ cm.



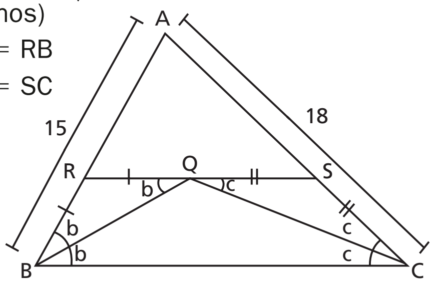
289. Tracemos a diagonal $\overline{AC}$, que intercepta $\overline{BD}$ em Q. Temos:
$\left.\begin{array}{l}AQ = CQ \\ DM = MC\end{array}\right\} \Rightarrow$ P é baricentro do $\triangle ACD$
$AM = AB = 15 \Rightarrow AP = \dfrac{2}{3} \cdot 15 \Rightarrow$
$\Rightarrow AP = 10$



290. $\overline{RS} \parallel \overline{BC} \Rightarrow \begin{cases} R\hat{Q}B = Q\hat{B}C \text{ (alternos)} \\ S\hat{Q}C = Q\hat{C}B \text{ (alternos)} \end{cases} \Rightarrow$

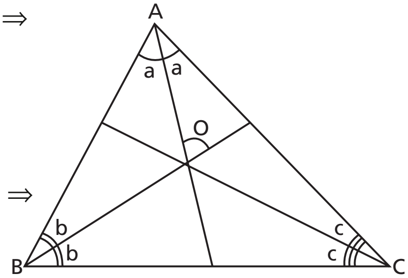
$\Rightarrow \begin{cases} \triangle RBQ \text{ é isósceles} \\ \triangle SCQ \text{ é isósceles} \end{cases} \Rightarrow \begin{cases} RQ = RB \\ QS = SC \end{cases}$

Temos:
Perímetro do $\triangle ARS =$
$= (AR + RQ) + (QS + AS) =$
$= (AR + RB) + (SC + AS) =$
$= 15 + 18 = 33$ cm



291. Para facilitar, sejam
$\hat{A} = 2a$, $\hat{B} = 2b$ e $\hat{C} = 2c$.
$\triangle ABC \Rightarrow 2a + 2b + 2c = 180° \Rightarrow$
$\Rightarrow a + b + c = 90°$
Daí:
$a + b = 90° - c$
$a + c = 90° - b$
$b + c = 90° - a$
$\triangle AOB \Rightarrow A\hat{O}B = 180° - (a + b) \Rightarrow$
$\Rightarrow A\hat{O}B = 180° - (90° - c) \Rightarrow$
$\Rightarrow A\hat{O}B = 90° + c \Rightarrow$

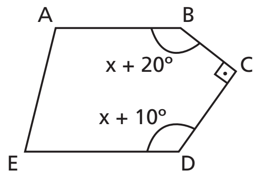
$\Rightarrow A\hat{O}B = 90° + \dfrac{\hat{C}}{2}$



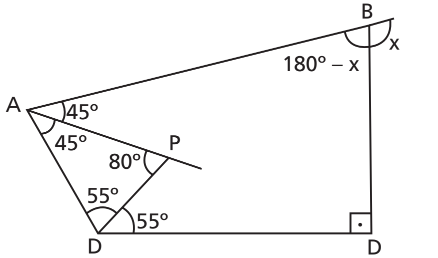
Analogamente, $A\hat{O}C = 90° + \dfrac{\hat{B}}{2}$; $B\hat{O}C = 90° + \dfrac{\hat{A}}{2}$

CAPÍTULO IX — Polígonos

293. e) $\overline{AB} \parallel \overline{ED} \Rightarrow \hat{A} + \hat{E} = 180°$
ABCDE é pentágono $\Rightarrow$
$\Rightarrow \hat{A} + \hat{B} + \hat{C} + \hat{D} + \hat{E} = 540° \Rightarrow$
$\Rightarrow (\hat{A} + \hat{E}) + \hat{B} + \hat{C} + \hat{D} = 540° \Rightarrow$
$\Rightarrow 180° + (x + 20°) + 90° + (x + 10°) =$
$= 540° \Rightarrow x = 120°$

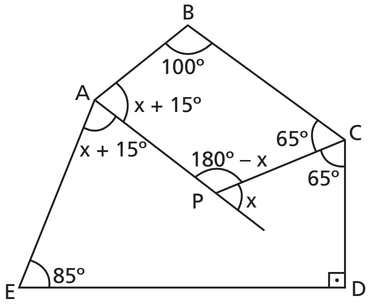


294. a) Quadrilátero ABCD:
$\hat{A} + \hat{B} + \hat{C} + \hat{D} = 360° \Rightarrow$
$\Rightarrow 90° + 110° + 90° +$
$+ 180° - x = 360° \Rightarrow$
$\Rightarrow x = 110°$



b) Quadrilátero ABCP:
$\cancel{x} + 15° + \hat{B} + 65° +$
$+ 180° - \cancel{x} = 360° \Rightarrow$
$\Rightarrow \hat{B} = 100°$

Pentágono ABCDE:
$\hat{A} + \hat{B} + \hat{C} + \hat{D} + \hat{E} = 540° \Rightarrow$
$\Rightarrow 2x + 30° + 100° + 130° +$
$+ 90° + 85° = 540° \Rightarrow$
$\Rightarrow x = 52°30'$

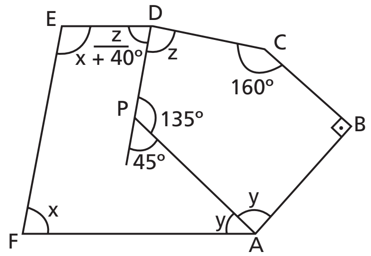


c) Análogo ao item *b*.

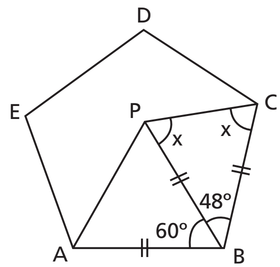
d) Sejam $\hat{A} = 2y$ e $\hat{D} = 2z$. Temos o que segue:

Pentágono ABCDP $\Rightarrow$
$\Rightarrow y + 90° + 160° + z + 135° =$
$= 540° \Rightarrow y + z = 155°$

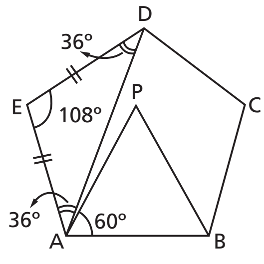
Hexágono ABCDEF $\Rightarrow$
$\Rightarrow 2y + 90° + 160° + 2z + x +$
$+ 40° + x = 720° \Rightarrow$
$\Rightarrow 2(y + z) + 2x + 290° = 720° \Rightarrow$
$\Rightarrow x = 60°$



297. a) Note que o $\triangle BPC$ é isósceles, pois $BP = BC$.
O ângulo interno a_i do pentágono mede $a_i = \dfrac{540°}{5} = 108°$.
$A\hat{B}P = 60° \Rightarrow P\hat{B}C = 48° \Rightarrow$
$\Rightarrow x = 66°$



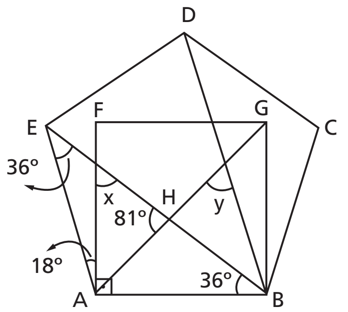
b) $\triangle ABP$ é equilátero $\Rightarrow B\hat{A}P = 60°$
$\left.\begin{array}{l}\hat{E} = 108° \\ \triangle ADE \text{ é isósceles}\end{array}\right\} \Rightarrow D\hat{A}E = 36°$
$D\hat{A}E + B\hat{A}P + x = 108° \Rightarrow$
$\Rightarrow 96° + x = 108° \Rightarrow x = 12°$



298. a) $B\hat{A}F = 90° \Rightarrow F\hat{A}E = 18°$
$\triangle ABE$ é isósceles $\Rightarrow A\hat{E}B = 36°$
x é externo ao $\triangle ABE \Rightarrow x = 36° + 18° \Rightarrow x = 54°$
$G\hat{A}F = 45°$, $x = 54° \Rightarrow E\hat{H}A = 81° = G\hat{H}B$

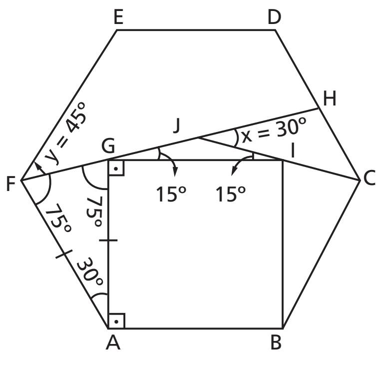
$$\left.\begin{array}{l}\triangle CBD \text{ é isósceles} \Rightarrow C\hat{B}D = 36° \\ E\hat{B}A = A\hat{E}B = 36°\end{array}\right\} \Rightarrow D\hat{B}E = 36°$$

$y + D\hat{B}E + G\hat{H}B = 180° \Rightarrow y + 36° + 81° = 180° \Rightarrow y = 63°$



b)
$$\left.\begin{array}{l}AF = AG \Rightarrow \triangle AFG \text{ é isósceles} \\ B\hat{A}G = 90° \Rightarrow F\hat{A}G = 30°\end{array}\right\} \Rightarrow A\hat{F}G = 75° \Rightarrow y = 45°$$

Note que $F\hat{G}A = 75°$, $A\hat{G}I = 90°$ e, então: $H\hat{G}I = 15°$.
Analogamente, $J\hat{I}G = 15°$.
$\triangle GIJ \Rightarrow x = 15° + 15° \Rightarrow x = 30°$



311. De cada vértice partem $n - 3$ diagonais. Logo, $n - 3 = 25$ e, então, $n = 28$.
Resposta: o polígono possui 28 lados.

320. Seja 2x o ângulo interno.

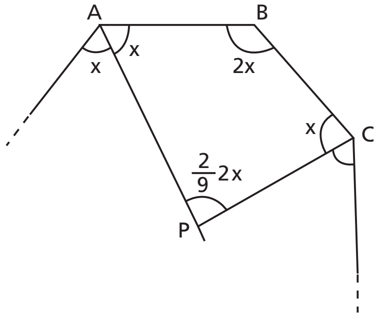
Quadrilátero ABCP $\Rightarrow$

$\Rightarrow x + 2x + x + \dfrac{2}{9} \cdot 2x = 360° \Rightarrow$

$\Rightarrow x = 81° \Rightarrow a_i = 162° \Rightarrow$

$\Rightarrow a_e = 18° \Rightarrow$

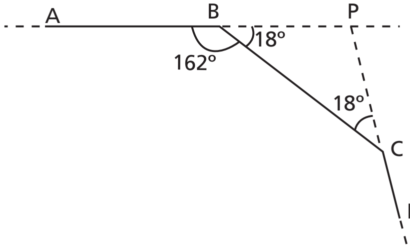
$\Rightarrow \dfrac{360°}{n} = 18° \Rightarrow n = 20$



321. $n = 20 \Rightarrow a_i = 162° \Rightarrow a_e = 18°$

$\triangle PBC \Rightarrow \hat{P} = 180° - 18° - 18° \Rightarrow$

$\Rightarrow \hat{P} = 144°$



322. Observando o quadrilátero MBNP, temos:

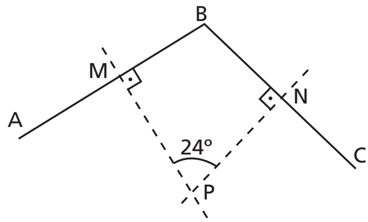
$\hat{B} = 156° \Rightarrow a_i = 156° \Rightarrow$

$\Rightarrow a_e = 24° \Rightarrow$

$\Rightarrow \dfrac{360°}{n} = 24° \Rightarrow n = 15$

$d = \dfrac{n(n-3)}{2} \Rightarrow$

$\Rightarrow d = \dfrac{15(15-3)}{2} \Rightarrow d = 90$



323. Sendo $d = \dfrac{n(n-3)}{2}$, temos:

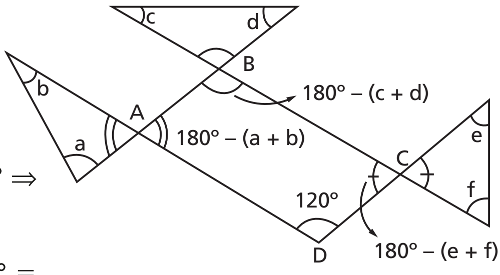
$d + 21 = \dfrac{(n+3)\cdot(n+3-3)}{2} \Rightarrow \dfrac{n(n-3)}{2} + 21 = \dfrac{(n+3)\cdot n}{2} \Rightarrow$

$\Rightarrow \dfrac{(n+3)n}{2} - \dfrac{n(n-3)}{2} = 21 \Rightarrow n[(n+3) - (n-3)] = 42 \Rightarrow$

$\Rightarrow 6n = 42 \Rightarrow n = 7$

$d = \dfrac{n(n-3)}{2} \Rightarrow d = \dfrac{7\cdot(7-3)}{2} \Rightarrow d = 14$

324. Quadrilátero ABCD:



$$\left.\begin{aligned}\hat{A} &= 180° - (a + b)\\ \hat{B} &= 180° - (c + d)\\ \hat{C} &= 180° - (e + f)\\ \hat{D} &= 120°\end{aligned}\right\} \Rightarrow$$

$\Rightarrow \hat{A} + \hat{B} + \hat{C} + \hat{D} = 360° \Rightarrow$
$\Rightarrow 180° - (a + b) +$
$+ 180° - (c + d) +$
$+ 180° - (e + f) + 120° =$
$= 360° \Rightarrow$
$\Rightarrow a + b + c + d + e + f = 300°$

328. $n_1 = n;\ n_2 = n + 1;\ n_3 = n + 2$

$$\frac{n(n-3)}{2} + \frac{(n+1)(n+1-3)}{2} + \frac{(n+2)(n+2-3)}{2} = 28 \Rightarrow$$

$\Rightarrow n^2 - n - 20 = 0 \Rightarrow n = -4$ (não serve) ou $n = 5$

O polígono com maior número de lados é o que tem mais diagonais.

Logo, $n_3 = 5 + 2 \Rightarrow n_3 = 7$.

331. $$\frac{(n+1-2)\cdot 180°}{n+1} - \frac{(n-2)\cdot 180°}{n} = 5° \Rightarrow$$

$$\Rightarrow \frac{(n-1)\cdot 180°}{n+1} - \frac{(n-2)\cdot 180°}{n} = 5° \Rightarrow$$

$\Rightarrow n(n-1)\cdot 180° - (n+1)(n-2)\cdot 180° = 5° \cdot n(n+1) \Rightarrow$
$\Rightarrow n^2 + n - 72 = 0 \Rightarrow (n = -9$ ou $n = 8) \Rightarrow n = 8$

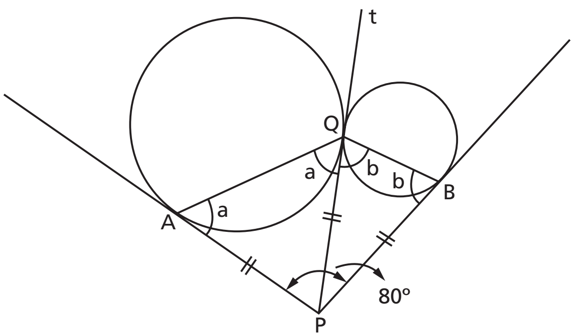
CAPÍTULO X — Circunferência e círculo

342. (1) $R_A + R_B = 7$; (2) $R_A + R_C = 5$; (3) $R_B + R_C = 6$

Somando (1), (2) e (3) temos $R_A + R_B + R_C = 9$. (4)

Fazendo (4) − (1), vem $R_C = 2$; (4) − (2) vem $R_B = 4$; e (4) − (3) vem $R_A = 3$.

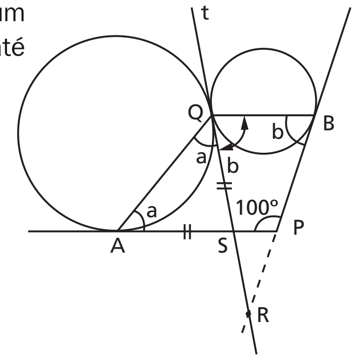
343. a)



$$\left.\begin{array}{l} PA = PQ \Rightarrow \triangle PAQ \text{ é isósceles} \\ PB = PQ \Rightarrow \triangle PBQ \text{ é isósceles} \\ A\hat{P}B = 80° \end{array}\right\} \text{quadrilátero } APBQ \Rightarrow$$

$$\left.\begin{array}{r} \Rightarrow 2a + 2b = 280° \Rightarrow a + b = 140° \\ A\hat{Q}B = a + b \end{array}\right\} \Rightarrow A\hat{Q}B = 140°$$

b) Traçamos a reta *t*, tangente comum pelo ponto Q. Prolongamos $\overline{BP}$ até interceptar a reta *t* em R.



Note que $t \cap \overline{AP} = \{S\}$.

$$\left.\begin{array}{l} SA = SQ \Rightarrow \triangle SAQ \text{ é isósceles} \\ RQ = RB \Rightarrow \triangle RQB \text{ é isósceles} \\ A\hat{P}B = 100° \end{array}\right\} \text{quadrilátero } APBQ \Rightarrow$$

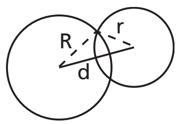
$$\left.\begin{array}{r} \Rightarrow 2a + 2b = 260° \Rightarrow a + b = 130° \\ A\hat{Q}B = a + b \end{array}\right\} \Rightarrow A\hat{Q}B = 130°$$

353.

$$\left.\begin{array}{l} R - r < d < R + r \\ d = 20 \text{ cm}; r = 11 \text{ cm} \end{array}\right\} \Rightarrow R - 11 < 20 < R + 11 \Rightarrow 9 < R < 31$$

$$\left.\begin{array}{l} R \text{ é múltiplo de } 6 \\ 9 < R < 31 \end{array}\right\} \Rightarrow$$

$\Rightarrow$ (R = 12 cm ou R = 18 cm ou R = 24 cm ou R = 30 cm)



354. Sejam R_A, R_B e R_C os raios das circunferências de centros A, B e C, respectivamente. Temos:

$R_A + R_B = 12$ (1)

$R_C - R_A = 17$ (2)

$R_C - R_B = 13$ (3)

$(2) + (3) \Rightarrow 2 \cdot R_C - (R_A + R_B) = 30 \Rightarrow 2R_C - 12 = 30 \Rightarrow$

$\Rightarrow R_C = 21 \text{ m} \Rightarrow R_A = 4 \text{ m} \Rightarrow R_B = 8 \text{ m}$

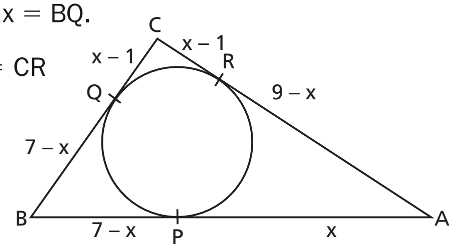
Resposta: $R_A = 4$ m, $R_B = 8$ m, $R_C = 21$ m.

355. Seja $AP = x$. Então, $PB = 7 - x = BQ$.

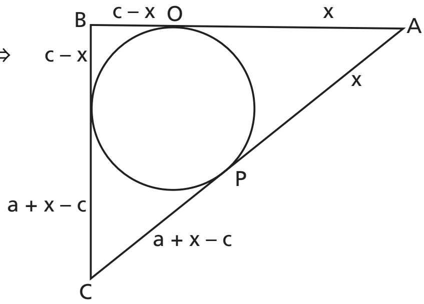
$$\left.\begin{array}{l} BQ = 7 - x \\ BC = 6 \end{array}\right\} \Rightarrow QC = x - 1 = CR$$

$$\left.\begin{array}{l} CR = x - 1 \\ AC = 8 \end{array}\right\} \Rightarrow AR = 9 - x$$

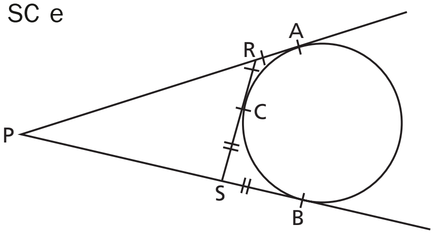
Mas: $AR = AP \Rightarrow 9 - x = x \Rightarrow$
$\Rightarrow x = 4{,}5$



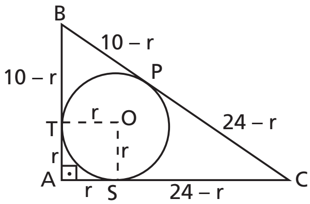
359. Temos $a + b + c = 2p \Rightarrow$
$\Rightarrow b + c = 2p - a$ (1)
Seja $AP = x \Rightarrow AO = x \Rightarrow$
$\Rightarrow OB = c - x \Rightarrow BR = c - x \Rightarrow$
$\Rightarrow RC = a + x - c \Rightarrow$
$\Rightarrow CP = a + x - c$
Como $CP + AP = b$, temos
$a + x - c + x = b \Rightarrow$
$\Rightarrow 2x = b + c - a$
Utilizando (1), segue que
$2x = 2p - a - a \Rightarrow x = p - a$



361. Note que $RA = RC$, que $SB = SC$ e que $PA = PB$.
Temos:
perímetro $\triangle PRS =$
$= (PR + RC) + (SC + PS) =$
$= (PR + RA) + (SB + PS) =$
$= PA + PB = 10 + 10 = 20$ cm



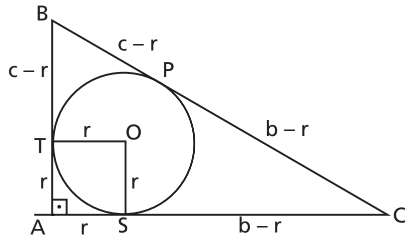
364. Temos $BC = 26$ cm (Pitágoras).
De acordo com a figura:
$(10 - r) + (24 - r) = 26 \Rightarrow$
$\Rightarrow r = 4$ cm



366. De acordo com as medidas indicadas na figura:

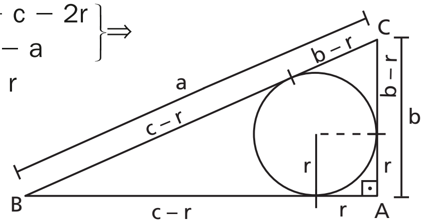
$(c - r) + (b - r) = a \Rightarrow$

$\Rightarrow r = \dfrac{b + c - a}{2}$



367. $\left.\begin{array}{l} a = (b - r) + (c - r) \Rightarrow a = b + c - 2r \\ a + b + c = 2p \Rightarrow b + c = 2p - a \end{array}\right\} \Rightarrow$

$\Rightarrow a = 2p - a - 2r \Rightarrow a = p - r$



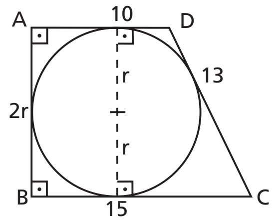
370. Observe que a altura do trapézio ($\overline{AB}$) tem medida igual a 2r.

ABCD é circunscrito $\Rightarrow$

$\Rightarrow AB + CD = AD + BC$

Então:

$2r + 13 = 10 + 15 \Rightarrow r = 6$



371. Sejam *a* e *b* dois lados opostos e *c* e *d* os outros dois lados opostos.

Temos:

$a - b = 8$ (1); $c - d = 4$ (2); $a + b = c + d$ (3); $a + b + c + d = 56$ (4)

Substituindo (3) em (4):

$a + b + a + b = 56 \Rightarrow a + b = 28$ (5)

(5) e (1) $\Rightarrow$ ($a = 18$ cm, $b = 10$ cm)

(3) $\Rightarrow c + d = a + b \Rightarrow c + d = 28$ (6)

(6) e (2) $\Rightarrow$ ($c = 16$ cm, $d = 12$ cm)

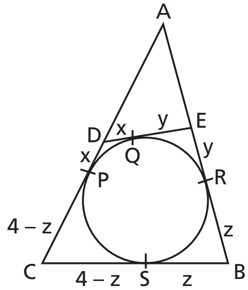
372. $\left.\begin{array}{l} BC = 4 \\ \text{perímetro } \triangle ABC = 10 \end{array}\right\} \Rightarrow AB + AC = 6$ (1)

$DP = x \Rightarrow DQ = x$

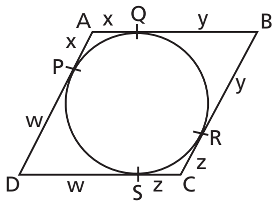
$EQ = y \Rightarrow ER = y$

$BR = z \Rightarrow (BS = z, SC = 4 - z = CP)$

$(1) \Rightarrow AB + AC = 6 \Rightarrow$
$\Rightarrow AE + ER + RB + AD +$
$+ DP + PC = 6 \Rightarrow$
$\Rightarrow AE + y + z + AD +$
$+ x + 4 - z = 6 \Rightarrow$
$\Rightarrow AE + y + AD + x =$
$= 2$ cm, em que
$AE + y + AD + x$ é o
perímetro do $\triangle ADE$



374.



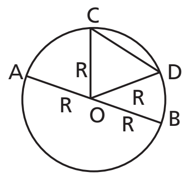
Hipótese: ABCD é paralelogramo circunscrito
Tese: ABCD é losango

Demonstração
Basta mostrar que AB = BC.

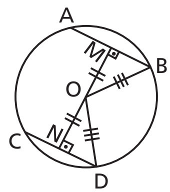
ABCD é paralelogramo $\Rightarrow \begin{cases} AB = CD \Rightarrow x + y = z + w \ (1) \\ AD = BC \Rightarrow x + w = y + z \ (2) \end{cases}$

Somando membro a membro (1) e (2), obtemos:
$2x + y + w = 2z + y + w \Rightarrow x = z$
Então:
$AB = x + y = z + y = BC \Rightarrow AB = BC = CD = AD \Rightarrow$ ABCD é losango.

375. Sendo O o centro, $\overline{AB}$ o diâmetro e $\overline{CD}$ uma corda qualquer que não passa pelo centro, considerando o triângulo COD, vem:
$CD < OC + OD \Rightarrow CD < R + R \Rightarrow$
$\Rightarrow CD < 2R \Rightarrow CD < AB$

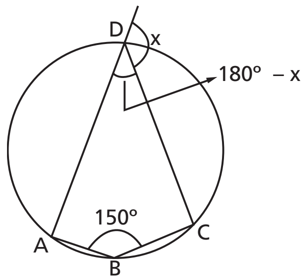


376. Sejam $\overline{AB}$ e $\overline{CD}$ as cordas tais que $\overline{MO} \equiv \overline{NO}$, em que M é ponto médio de $\overline{AB}$, N é ponto médio de $\overline{CD}$ e O é o centro da circunferência.
Temos:



$\left.\begin{array}{l} \overline{MO} \equiv \overline{NO} \text{ (hipótese)} \\ \overline{OB} \equiv \overline{OD} \text{ (raios)} \\ \triangle MBO, \triangle NDO \text{ retângulos} \end{array}\right\} \Rightarrow \triangle MBO \equiv \triangle NDO \Rightarrow \overline{MB} \equiv \overline{ND} \Rightarrow \overline{AB} \equiv \overline{CD}$

CAPÍTULO XI — Ângulos na circunferência



382. c) $C\hat{D}A = 180° - x \Rightarrow$

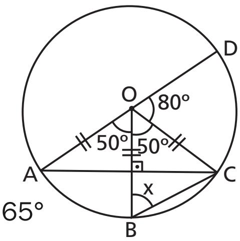
$\Rightarrow \overset{\frown}{ABC} = 2 \cdot (180° - x)$

$A\hat{B}C = 150° \Rightarrow \overset{\frown}{ADC} = 300°$

$\overset{\frown}{ABC} + \overset{\frown}{ADC} = 360° \Rightarrow$

$\Rightarrow 2(180° - x) + 300° = 360° \Rightarrow$

$\Rightarrow x = 150°$



384. a) $A\hat{O}C = 100°$

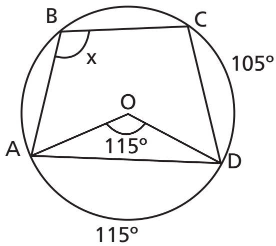
$\overline{OA} = \overline{OC}$ (raios) $\Rightarrow \triangle AOC$ é isósceles

($\overline{OB} \perp \overline{AC}$; $\triangle AOC$ isósceles) $\Rightarrow$

$\Rightarrow \overline{OB}$ é também bissetriz $\Rightarrow$

$\Rightarrow A\hat{O}B = B\hat{O}C = 50°$

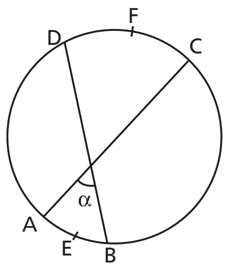
$$\left.\begin{array}{r} OB = OC \text{ (raios)} \Rightarrow \triangle BOC \text{ isósceles} \\ B\hat{O}C = 50° \end{array}\right\} \Rightarrow x = 65°$$



385. $A\hat{O}D = 115° \Rightarrow \overset{\frown}{AD} = 115°$

$$x = \frac{\overset{\frown}{ADC}}{2} \Rightarrow x = \frac{\overset{\frown}{AD} + \overset{\frown}{DC}}{2} \Rightarrow$$

$$\Rightarrow x = \frac{115° + 105°}{2} \Rightarrow x = 110°$$

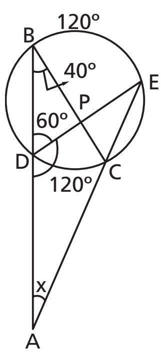


386. $$\alpha = \frac{\overset{\frown}{CFD} + \overset{\frown}{AEB}}{2} \Rightarrow$$

$$\Rightarrow 70° = \frac{\overset{\frown}{AEB} + 50° + \overset{\frown}{AEB}}{2} \Rightarrow$$

$\Rightarrow \overset{\frown}{AEB} = 45°$

$\overset{\frown}{CFD} = \overset{\frown}{AEB} + 50° \Rightarrow \overset{\frown}{CFD} = 95°$



388. b) $A\hat{B}C = 40° \Rightarrow \overset{\frown}{CD} = 80°$

$A\hat{D}P = 120° \Rightarrow P\hat{D}B = 60° \Rightarrow$

$\Rightarrow \overset{\frown}{BE} = 120°$

$$x = \frac{\overset{\frown}{BE} - \overset{\frown}{CD}}{2} \Rightarrow$$

$$\Rightarrow x = \frac{120° - 80°}{2} \Rightarrow$$

$\Rightarrow x = 20°$

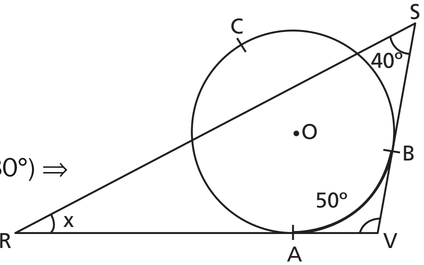
389. b) $A\hat{V}B = \dfrac{\overset{\frown}{ACB} - \overset{\frown}{AB}}{2} \Rightarrow$

$\Rightarrow A\hat{V}B = \dfrac{310° - 50°}{2} \Rightarrow$

$\Rightarrow A\hat{V}B = 130°$

$\triangle RVS \Rightarrow (\hat{S} = 40°;\ \hat{V} = 130°) \Rightarrow$

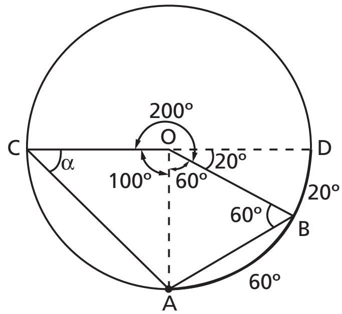
$\Rightarrow x = 10°$



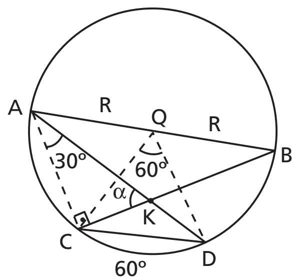
391. $B\hat{O}C = 160°$. Prolongamos $\overline{CO}$ até interceptar a circunferência em D.
Temos, então, $B\hat{O}D = 20°$.
$B\hat{O}D = 20° \Rightarrow \overset{\frown}{DB} = 20°$
Unindo O e A e usando o fato de $O\hat{B}A =$ $= 60°$, obtemos $\triangle AOB$ isósceles.
Daí, $A\hat{O}B = 60°$ e $\overset{\frown}{AB} = 60°$.
Então:

$\alpha = \dfrac{\overset{\frown}{AB} + \overset{\frown}{BD}}{2} \Rightarrow$

$\Rightarrow \alpha = \dfrac{60° + 20°}{2} \Rightarrow \alpha = 40°$



393. Unimos o ponto A com o ponto C.
Note que $A\hat{C}B = 90°$.
Unimos C com Q e Q com D. Temos:
$(CD = R,\ CQ = R,\ QD = R) \Rightarrow$
$\Rightarrow \triangle CQD$ é equilátero.
Então:
$\overset{\frown}{CD} = 60° \Rightarrow C\hat{A}D = 30°$
$(\triangle ACK,\ C = 90°) \Rightarrow \alpha = 60°$



395. a) $\triangle AOD$ é isósceles $\Rightarrow$

$\Rightarrow (O\hat{A}D = y,\ A\hat{O}D = 180° - 2y)$

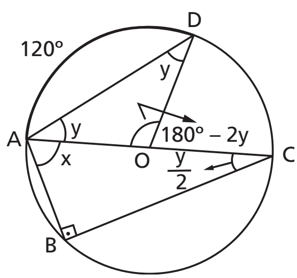
$A\hat{O}D = \overset{\frown}{AD} \Rightarrow 180° - 2y = 120° \Rightarrow$

$\Rightarrow y = 30°$

$\triangle ABC$ é retângulo em B, então:

$x + \dfrac{y}{2} = 90° \Rightarrow$

$\Rightarrow x + 15° = 90° \Rightarrow x = 75°$



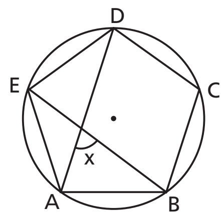
b) ABCDE é pentágono regular. Então:

$\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD} = \overset{\frown}{DE} = \overset{\frown}{AE} = 72°$

Daí:

$$x = \frac{\overset{\frown}{AB} + \overset{\frown}{DE}}{2} \Rightarrow$$

$$\Rightarrow x = \frac{72° + 72°}{2} \Rightarrow x = 72°$$



396. Unimos o centro O com o ponto C e com o ponto A:

$\overline{OB} \perp \overline{AC} \Rightarrow$ M é ponto médio de $\overline{AC}$.

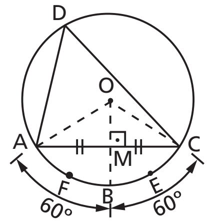
$\triangle OMA \equiv \triangle OMC$ (LAL) $\Rightarrow$

$\Rightarrow A\hat{O}B = B\hat{O}C = 60°$

$A\hat{O}B = 60° \Rightarrow \overset{\frown}{AFB} = 60°$

$\overset{\frown}{AFB} = 60° \Rightarrow \overset{\frown}{ABC} = 120° \Rightarrow$

$\Rightarrow A\hat{D}C = 60°$



397. Consideremos o triângulo PQR da figura.

Seja $\overset{\frown}{QR} = x$. Calculemos x:

$$80° = \frac{\overset{\frown}{QPR} - \overset{\frown}{QR}}{2} \Rightarrow$$

$$\Rightarrow 80° = \frac{360° - x - x}{2} \Rightarrow$$

$\Rightarrow x = 100°$

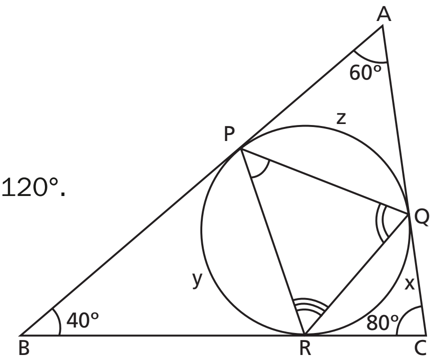
Analogamente, $y = 140°$ e $z = 120°$.

Daí:

$$\hat{P} = \frac{x}{2} \Rightarrow \hat{P} = 50°$$

$$\hat{Q} = \frac{y}{2} \Rightarrow \hat{Q} = 70°$$

$$\hat{R} = \frac{z}{2} \Rightarrow \hat{R} = 60°$$



398. $\overset{\frown}{AB}$, $\overset{\frown}{BC}$ e $\overset{\frown}{AC}$ serem proporcionais a 2, 9 e 7 quer dizer que $\overset{\frown}{AB}$, $\overset{\frown}{BC}$ e $\overset{\frown}{AC}$ são da forma 2k, 9k, 7k.

Então:

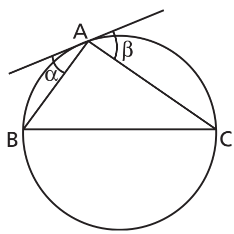
$2k + 9k + 7k = 360° \Rightarrow k = 20° \Rightarrow$

$\Rightarrow \overset{\frown}{AB} = 40°$; $\overset{\frown}{BC}$ 180°; $\overset{\frown}{AC} = 140°$

Daí:

$$\alpha = \frac{\overset{\frown}{AB}}{2} \Rightarrow \alpha = 20°;\ \beta = \frac{\overset{\frown}{AC}}{2} \Rightarrow \beta = 70°$$

A razão entre α e β é $\frac{2}{7}$.



401. $B\widehat{Q}C = x \Rightarrow \overset{\frown}{BC} = 360° - x$

$28° = \dfrac{360° - x - x}{2} \Rightarrow$

$\Rightarrow x = 152° \Rightarrow B\hat{O}C = 152°$

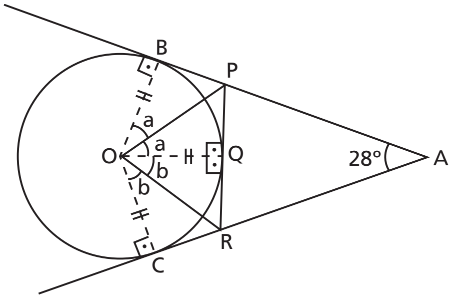
$\triangle BOP \equiv \triangle QOP$ (caso especial) $\Rightarrow B\hat{O}P = Q\hat{O}P = a$

$\triangle COR \equiv \triangle QOR$ (caso especial) $\Rightarrow C\hat{O}R = Q\hat{O}R = b$

Temos:

$2a + 2b = 152° \Rightarrow a + b = 76°$

Como $P\hat{O}R = a + b$, temos $P\hat{O}R = 76°$.



402. $\overline{AM}$ é lado do triângulo equilátero inscrito $\Rightarrow \overset{\frown}{AM} = \dfrac{360°}{3} \Rightarrow \overset{\frown}{AM} = 120°$

$\overline{BN}$ é lado do quadrado inscrito $\Rightarrow \overset{\frown}{BN} = \dfrac{360°}{4} \Rightarrow \overset{\frown}{BN} = 90°$

$(\overset{\frown}{AM} = 120°, \overset{\frown}{AMB} = 180°) \Rightarrow$

$\Rightarrow \overset{\frown}{MB} = 60°$

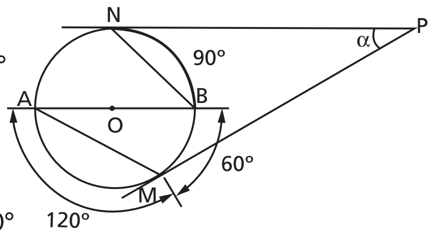
$(\overset{\frown}{MB} = 60°, \overset{\frown}{NB} = 90°) \Rightarrow$

$\Rightarrow \overset{\frown}{NBM} = 150° \Rightarrow \overset{\frown}{NAM} = 210°$

Então:

$\alpha = \dfrac{\overset{\frown}{NAM} - \overset{\frown}{NBM}}{2} \Rightarrow$

$\Rightarrow \alpha = \dfrac{210° - 150°}{2} \Rightarrow \alpha = 30°$



404. $\hat{A} = \hat{B} = \hat{C} = 60° \Rightarrow$

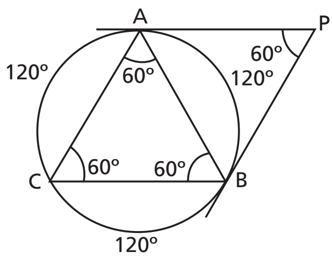
$\Rightarrow \overset{\frown}{AB} = \overset{\frown}{AC} = \overset{\frown}{BC} = 120°$

Temos:

$A\hat{P}B = \dfrac{\overset{\frown}{ACB} - \overset{\frown}{AB}}{2} \Rightarrow$

$\Rightarrow A\hat{P}B = \dfrac{240° - 120°}{2} \Rightarrow$

$\Rightarrow A\hat{P}B = 60°$

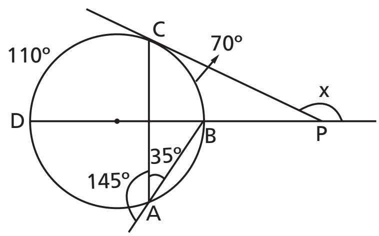


405. a) $B\hat{A}C = 35° \Rightarrow \overset{\frown}{BC} = 70° \Rightarrow$

$\Rightarrow \overset{\frown}{CD} = 110°$

$C\hat{P}D = \dfrac{110° - 70°}{2} \Rightarrow$

$\Rightarrow C\hat{P}D = 20° \Rightarrow x = 160°$

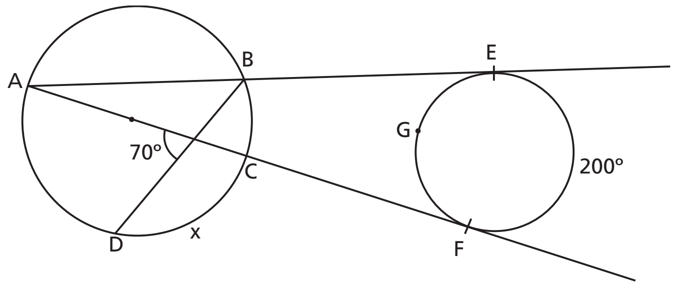


b) $\widehat{EGF} = 160° \Rightarrow B\hat{A}C = \dfrac{200° - 160°}{2} \Rightarrow B\hat{A}C = 20°$

$B\hat{A}C = 20° \Rightarrow \widehat{BC} = 40°$

$70° = \dfrac{\widehat{AD} + \widehat{BC}}{2} \Rightarrow 70° = \dfrac{\widehat{AD} + 40°}{2} \Rightarrow \widehat{AD} = 100°$

$\widehat{AD} = 100° \Rightarrow x = 80°$



407. Hipótese Tese

$r \parallel s \Rightarrow m(\widehat{AB}) = m(\widehat{CD})$

Demonstração

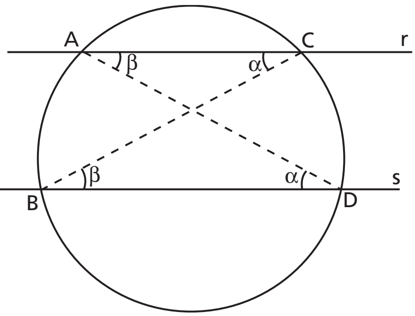
$A\hat{C}B = A\hat{D}B = \alpha$ (pois subtendem o mesmo arco $\widehat{AB}$)

Analogamente, $C\hat{A}D = C\hat{B}D = \beta$.

$A\hat{C}B$, $C\hat{B}D$ são alternos $\Rightarrow$

$\Rightarrow A\hat{C}B = C\hat{B}D \Rightarrow \alpha = \beta$

$\alpha = \beta \Rightarrow m(\widehat{AB}) = m(\widehat{CD})$



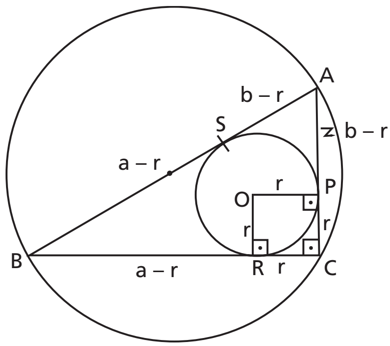
409. Sendo a hipotenusa igual ao diâmetro (2R) da circunferência circunscrita e CPOR um quadrado, temos:

$\left.\begin{aligned} BR &= BS = a - r \\ AP &= AS = b - r \end{aligned}\right\} \Rightarrow$

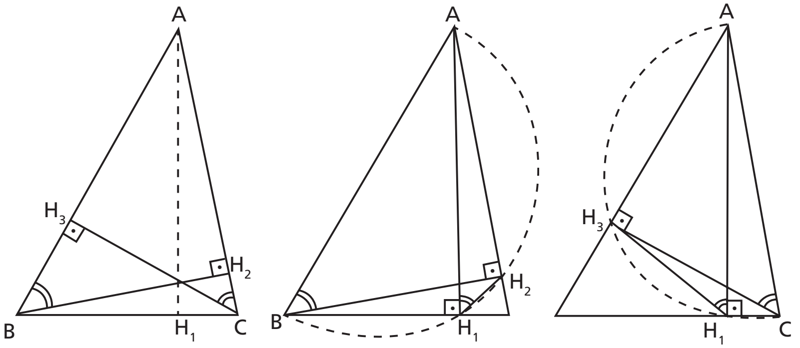
$\Rightarrow AB = a + b - 2r \Rightarrow$

$\Rightarrow a + b - 2r = 2R \Rightarrow$

$\Rightarrow a + b = 2(R + r)$



412.

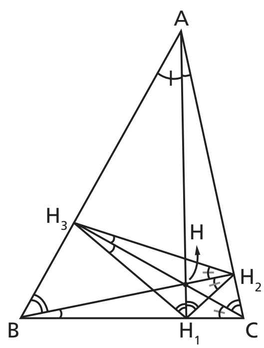


1) $A\hat{B}H_2 \equiv A\hat{C}H_3$, pois possuem lados respectivamente perpendiculares.

2) A circunferência de diâmetro $\overline{AB}$ passa por H_1 e H_2, pois $A\hat{H}_1B = A\hat{H}_2B = 90°$. Então, $A\hat{B}H_2 \equiv A\hat{H}_1H_2$, pois subtendem o mesmo arco $\overset{\frown}{AH_2}$ na circunferência de diâmetro $\overline{AB}$.

3) Analogamente ao passo 2), temos $A\hat{C}H_3 \equiv A\hat{H}_1H_3$, pois subtendem o mesmo arco $\overset{\frown}{AH_3}$ na circunferência de diâmetro $\overline{AC}$.

De 1), 2) e 3) concluímos que: $\overline{AH_1}$ é bissetriz do ângulo $H_3\hat{H}_1H_2$. Procedendo de modo análogo aos passos 1), 2) e 3), teremos $H_1\hat{H}_3C \equiv C\hat{H}_3H_2$ e $H_3\hat{H}_2B \equiv B\hat{H}_2H_1$ e, portanto, o ponto H é incentro do $\triangle H_1H_2H_3$.



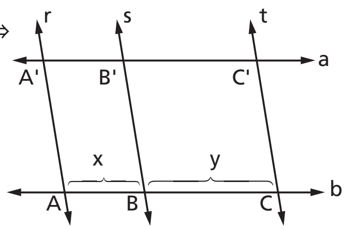
CAPÍTULO XII — Teorema de Tales

419. $\left.\begin{array}{l}\overline{A'C'} \parallel \overline{AC} \\ \overline{AA'} \parallel \overline{CC'}\end{array}\right\} \Rightarrow ACC'A'$ é paralelogramo $\Rightarrow$

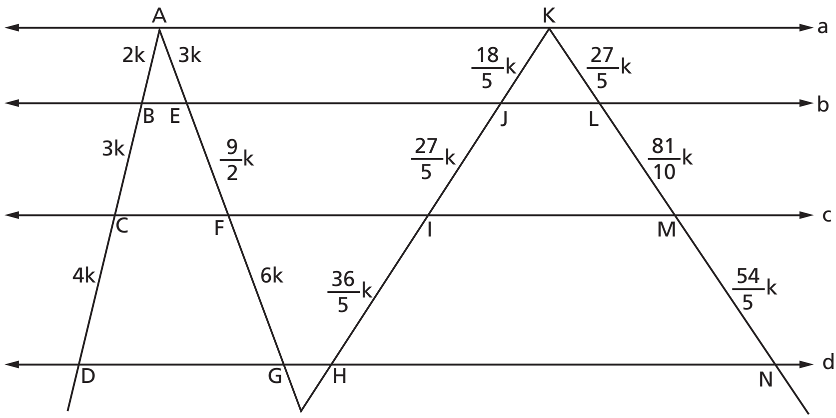
$\Rightarrow A'C' = AC = 30$ cm. Daí:

$\left(\dfrac{x}{y} = \dfrac{2}{3}, x + y = 30\right) \Rightarrow$

$\Rightarrow (x = 12 \text{ cm}, y = 18 \text{ cm})$



424.



AB, BC e CD são proporcionais a 2, 3 e 4, isto é, AB, BC e CD são da forma 2k, 3k e 4k, respectivamente. Temos:

1) $\dfrac{AE}{AB} = \dfrac{3}{2} \Rightarrow \dfrac{AE}{2k} = \dfrac{3}{2} \Rightarrow AE = 3k$

$$\frac{AB}{BC} = \frac{AE}{EF} \Rightarrow \frac{2k}{3k} = \frac{3k}{EF} \Rightarrow EF = \frac{9}{2}k$$

$$\frac{BC}{CD} = \frac{EF}{FG} \Rightarrow \frac{3k}{4k} = \frac{\frac{9}{2}k}{FG} \Rightarrow FG = 6k$$

$$\frac{JK}{AB} = \frac{9}{5} \Rightarrow \frac{JK}{2k} = \frac{9}{5} \Rightarrow JK = \frac{18}{5}k$$

2) Analogamente, encontramos:

$JI = \frac{27}{5}k$; $IH = \frac{36}{5}k$

$KL = \frac{27}{5}k$, $LM = \frac{81}{10}k$ e $MN = \frac{54}{5}k$

3) $AD + AG + HK + KN = 180 \Rightarrow$

$\Rightarrow \left(2 + 3 + 4 + 3 + \frac{9}{2} + 6 + \frac{36}{5} + \frac{27}{5} + \frac{18}{5} + \frac{27}{5} + \frac{81}{10} + \frac{54}{5}\right)k =$

$= 180 \Rightarrow k = \frac{20}{7}$

4) $k = \frac{20}{7} \Rightarrow \left(EF = \frac{90}{7}\text{ cm, } LM = \frac{162}{7}\text{ cm, } CD = \frac{80}{7}\text{ cm}\right)$

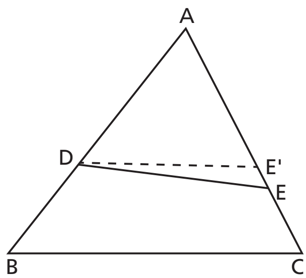
427. Hipótese Tese

$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \overline{DE} \mathbin{/\!/} \overline{BC}$

Demonstração

Tomemos E' em $\overline{AC}$, $\overline{DE'} \mathbin{/\!/} \overline{BC}$.

Temos:



$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow$

$\Rightarrow \frac{AD + DB}{DB} = \frac{AE + EC}{EC} \Rightarrow$

$\Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$ (1)

Teorema de Tales $\Rightarrow$

$\Rightarrow \frac{AB}{DB} = \frac{AC}{E'C}$ (2)

(1) e (2) $\Rightarrow EC = E'C \Rightarrow E = E' = \overline{DE} \mathbin{/\!/} \overline{BC}$

Teorema das bissetrizes

434. a) $\left.\begin{array}{l}\text{perímetro } \triangle ABC = 75\text{ m} \\ BS = 10\text{ m, } AC = 30\text{ m}\end{array}\right\} \Rightarrow AB + SC = 35\text{ m}$

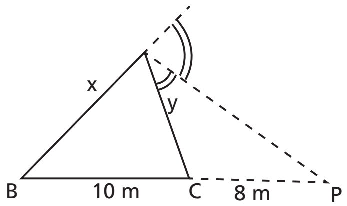
Sejam $AB = x$, $SC = y$. Temos:

$$\begin{cases} \frac{10}{x} = \frac{y}{30} \\ x + y = 35 \end{cases} \Rightarrow \begin{cases} xy = 300 \\ x + y = 35 \end{cases} \Rightarrow \begin{cases} x = 20\text{ m e } y = 15\text{ m} \\ \text{ou} \\ x = 15\text{ m e } y = 20\text{ m} \end{cases} \Rightarrow$$

$\Rightarrow (AB = 15\text{ m ou } AB = 20\text{ m})$

b) Sejam AB = x e AC = y. Temos:

$$\left.\begin{array}{l} \text{perímetro } \triangle ABC = 23 \Rightarrow x + y = 13 \\ \text{AP é bissetriz externa} \Rightarrow \dfrac{18}{x} = \dfrac{8}{y} \end{array}\right\} \Rightarrow x = 9 \text{ m}$$



437. Sejam CP = x, AB = y, AC = z.

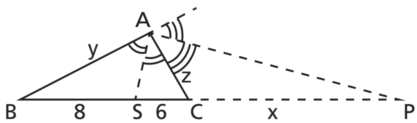
Temos:

Teo. biss. int. $\Rightarrow \dfrac{8}{y} = \dfrac{6}{z} \Rightarrow$

$\Rightarrow z = \dfrac{3}{4}y$

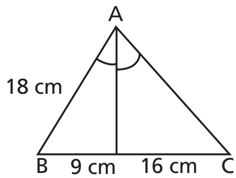
Teo. biss. ext. $\Rightarrow \dfrac{14 + x}{y} = \dfrac{x}{z} \Rightarrow$

$\Rightarrow \dfrac{14 + x}{\cancel{y}} = \dfrac{x}{\dfrac{3}{4}\cancel{y}} \Rightarrow x = 42 \text{ m}$



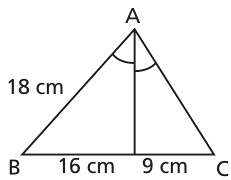
438. Temos duas possibilidades:

1ª)



$\dfrac{9}{18} = \dfrac{16}{AC} \Rightarrow AC = 32 \text{ cm}$

2ª)



$\dfrac{16}{18} = \dfrac{9}{AC} \Rightarrow AC = \dfrac{81}{8} \text{ cm}$

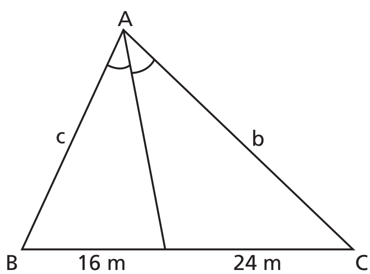
439. Note que BC = 40 m.

Perímetro $\triangle ABC = 100 \text{ m} \Rightarrow$

$\Rightarrow AB + AC = 60 \text{ m} \Rightarrow$

$\Rightarrow c + b = 60$

$$\left.\begin{array}{l} c + b = 60 \\ \dfrac{c}{16} = \dfrac{b}{24} \end{array}\right\} \Rightarrow (c = 24 \text{ e } b = 36)$$

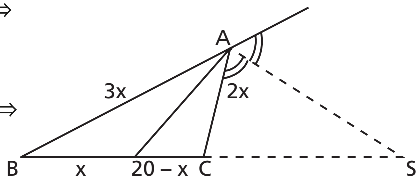


441. Teo. biss. int. $\Rightarrow \dfrac{x}{3x} = \dfrac{20 - x}{2x} \Rightarrow$

$\Rightarrow x = 12$ cm

Teo. biss. ext. $\Rightarrow \dfrac{20 + y}{3x} = \dfrac{y}{2x} \Rightarrow$

$\Rightarrow \dfrac{20 + y}{36} = \dfrac{y}{24} \Rightarrow y = 40$ cm



443. O centro do círculo é o incentro do $\triangle ABC$.

Sejam E, F, G os pontos de tangência da circunferência com os lados $\overline{BC}$, $\overline{AB}$ e $\overline{AC}$, respectivamente.

Temos:

$AF = AG = 3$; $CG = CE = x$;

$BE = BF = 6$

$(BD = 7; BE = 6) \Rightarrow DE = 1 \Rightarrow$

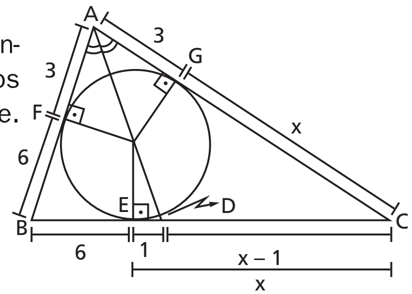
$\Rightarrow CD = x - 1$

O centro do círculo inscrito é incentro do $\triangle ABC$, donde tiramos $\overline{AD}$ bissetriz de $\hat{A}$.

Então:

$\dfrac{BD}{AB} = \dfrac{CD}{AC} \Rightarrow \dfrac{7}{9} = \dfrac{x - 1}{3 + x} \Rightarrow$

$\Rightarrow x = 15$



444. $\left.\begin{array}{l} BC = 5 \text{ cm} \\ \text{Perímetro } \triangle ABC = 15 \text{ cm} \end{array}\right\} \Rightarrow$

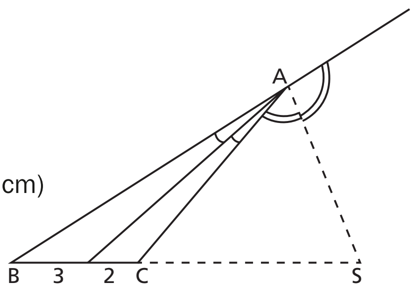
$\Rightarrow AB + AC = 10$ cm (1)

Teo. biss. int. $\Rightarrow \dfrac{3}{AB} = \dfrac{2}{AC}$ (2)

(1) e (2) $\Rightarrow$ ($AB = 6$ cm, $AC = 4$ cm)

Teo. biss. int. $\Rightarrow \dfrac{BS}{AB} = \dfrac{CS}{AC} \Rightarrow$

$\Rightarrow \dfrac{5 + CS}{6} = \dfrac{CS}{4} \Rightarrow CS = 10$ cm



CAPÍTULO XIII — Semelhança de triângulos e potência de ponto

Semelhança de triângulos

454. $2p = 8{,}4 + 15{,}6 + 18 \Rightarrow 2p = 42$ cm

$$k = \frac{2p}{2p'} \Rightarrow k = \frac{42}{35} \Rightarrow k = \frac{6}{5}$$

Seja ℓ o maior lado do segundo triângulo. Temos:

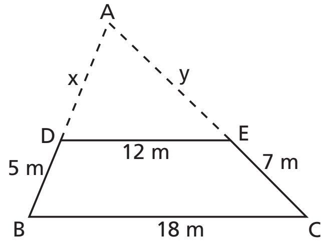
$$\frac{18}{\ell} = k \Rightarrow \frac{18}{\ell} = \frac{6}{5} \Rightarrow \ell = 15 \text{ cm}$$

458. $\overline{DE} \parallel \overline{BC} \Rightarrow \triangle ABC \sim \triangle ADE \Rightarrow$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE} \Rightarrow$$

$$\Rightarrow \frac{x+5}{x} = \frac{y+7}{y} = \frac{18}{12} \Rightarrow$$

$\Rightarrow (x = 10 \text{ m}, y = 14 \text{ m})$



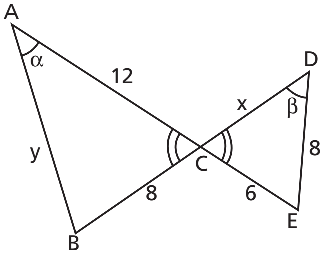
Casos ou critérios de semelhança

460. a) $\left.\begin{array}{l}\alpha = \beta \\ A\hat{C}B \equiv B\hat{C}E\end{array}\right\} \Rightarrow \triangle ABC \sim \triangle DEC \Rightarrow$

$$\Rightarrow \frac{AB}{DE} = \frac{AC}{DC} = \frac{BC}{EC} \Rightarrow$$

$$\Rightarrow \frac{y}{8} = \frac{12}{x} = \frac{8}{6} \Rightarrow$$

$$\Rightarrow \left(x = 9,\ y = \frac{32}{3}\right)$$

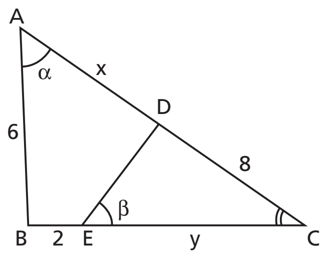


b) $\left.\begin{array}{l}\alpha = \beta \\ A\hat{C}B \equiv D\hat{C}E\end{array}\right\} \Rightarrow \triangle ABC \sim \triangle EDC \Rightarrow$

$$\Rightarrow \frac{AB}{ED} = \frac{AC}{EC} = \frac{BC}{DC} \Rightarrow$$

$$\Rightarrow \frac{6}{4} = \frac{x+8}{y} = \frac{y+2}{8} \Rightarrow$$

$\Rightarrow (x = 7, y = 10)$



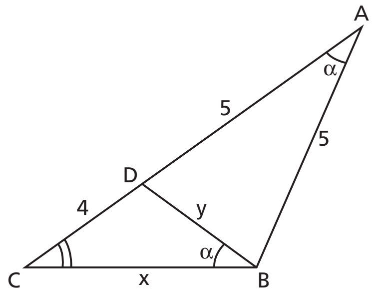
461. a) $\left.\begin{array}{l} B\hat{A}C \equiv C\hat{B}D \\ A\hat{C}B \equiv B\hat{C}D \text{ (comum)} \end{array}\right\} \Rightarrow$

$\Rightarrow \triangle ABC \sim \triangle BDC \Rightarrow$

$\Rightarrow \dfrac{AB}{BD} = \dfrac{AC}{BC} = \dfrac{BC}{DC} \Rightarrow$

$\Rightarrow \dfrac{5}{y} = \dfrac{9}{x} = \dfrac{x}{4} \Rightarrow$

$\Rightarrow \left(x = 6, y = \dfrac{10}{3}\right)$



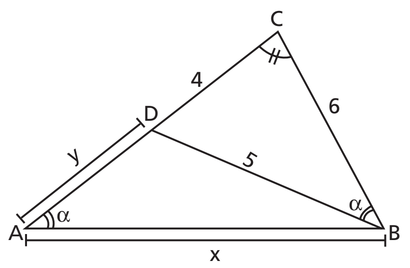
b) $\left.\begin{array}{l} B\hat{A}C \equiv C\hat{B}D \\ A\hat{C}B \equiv B\hat{C}D \text{ (comum)} \end{array}\right\} \Rightarrow$

$\Rightarrow \triangle ABC \sim \triangle BDC \Rightarrow$

$\Rightarrow \dfrac{AB}{BD} = \dfrac{AC}{BC} = \dfrac{BC}{DC} \Rightarrow$

$\Rightarrow \dfrac{x}{5} = \dfrac{y + 4}{6} = \dfrac{6}{4} \Rightarrow$

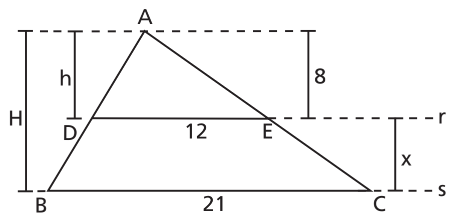
$\Rightarrow \left(x = \dfrac{15}{2}, y = 5\right)$



462. a) $r \parallel s \Rightarrow \triangle ABC \sim \triangle ADE \Rightarrow$

$\Rightarrow \dfrac{BC}{DE} = \dfrac{H}{h} \Rightarrow$

$\Rightarrow \dfrac{21}{12} = \dfrac{8 + x}{8} \Rightarrow x = 6$



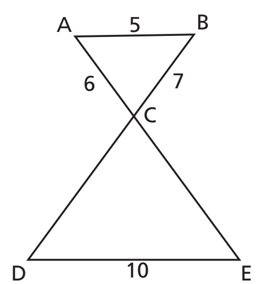
b) Análogo ao item *a*.

464. a) $\left.\begin{array}{l} \overline{AB} \parallel \overline{DE} \Rightarrow B\hat{A}C = D\hat{E}C \text{ (alternos)} \\ A\hat{C}B \equiv E\hat{C}D \text{ (o.p.v.)} \end{array}\right\} \Rightarrow \triangle ABC \sim \triangle EDC$

b) Da semelhança do item *a*, temos:

$\dfrac{AB}{DE} = \dfrac{BC}{CD} \Rightarrow \dfrac{5}{10} = \dfrac{7}{CD} \Rightarrow$

$\Rightarrow CD = 14$

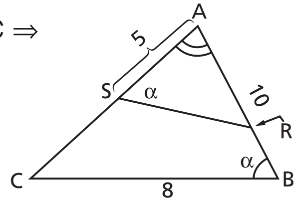


465.

$$\left.\begin{array}{l} A\hat{S}R \equiv A\hat{B}C \text{ (iguais a } \alpha) \\ S\hat{A}R \equiv B\hat{A}C \text{ (comum)} \end{array}\right\} \Rightarrow \triangle SAR \sim \triangle BAC \Rightarrow$$

$$\Rightarrow \frac{SR}{BC} = \frac{AS}{AB} \Rightarrow \frac{x}{8} = \frac{5}{10} \Rightarrow$$

$$\Rightarrow x = 4$$

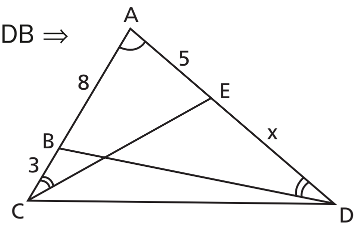


467.

$$\left.\begin{array}{l} A\hat{C}E \equiv A\hat{D}B \text{ (dado)} \\ C\hat{A}E \equiv D\hat{A}B \text{ (comum)} \end{array}\right\} \Rightarrow \triangle ACE \sim \triangle ADB \Rightarrow$$

$$\Rightarrow \frac{AC}{AD} = \frac{AE}{AB} \Rightarrow \frac{11}{x + 5} = \frac{5}{8} \Rightarrow$$

$$\Rightarrow x = \frac{63}{5} \text{ cm}$$



468.

a) $$\left.\begin{array}{l} B\hat{A}C \equiv C\hat{D}E \text{ (retos)} \\ A\hat{C}B \equiv D\hat{C}E \text{ (comum)} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \triangle ABC \sim \triangle DEC \Rightarrow$$

$$\Rightarrow \frac{AB}{DE} = \frac{AC}{DC} \Rightarrow$$

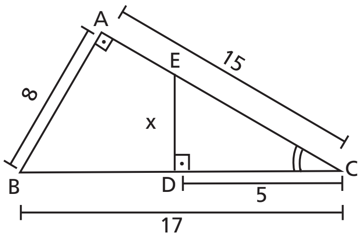
$$\Rightarrow \frac{8}{x} = \frac{15}{5} \Rightarrow x = \frac{8}{3}$$

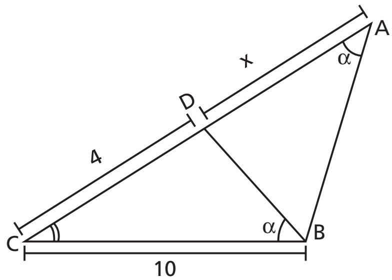
b) $$\left.\begin{array}{l} B\hat{A}C \equiv C\hat{B}D \\ A\hat{C}B \equiv D\hat{C}B \end{array}\right\} \Rightarrow$$

$$\Rightarrow \triangle ABC \sim \triangle BDC \Rightarrow$$

$$\Rightarrow \frac{AC}{BC} = \frac{BC}{DC} \Rightarrow$$

$$\Rightarrow \frac{x + 4}{10} = \frac{10}{4} \Rightarrow x = 21$$





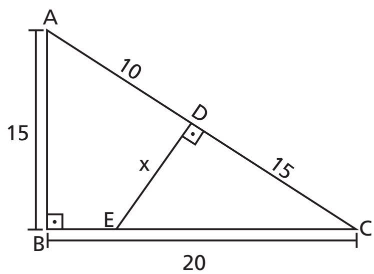
469.

$$\left.\begin{array}{l} C\hat{D}E \equiv A\hat{B}C \text{ (retos)} \\ D\hat{C}E \equiv A\hat{C}B \text{ (comum)} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \triangle DCE \sim \triangle BCA \Rightarrow$$

$$\Rightarrow \frac{DE}{AB} = \frac{CD}{BC} \Rightarrow$$

$$\Rightarrow \frac{x}{15} = \frac{15}{20} \Rightarrow x = \frac{45}{4}$$

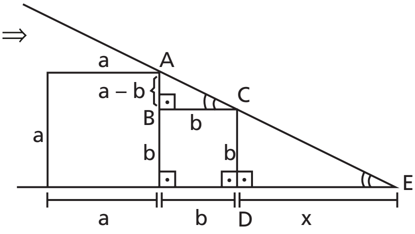


470. $\left.\begin{array}{l} A\hat{B}C \equiv C\hat{D}E \text{ (retos)} \\ A\hat{C}B \equiv C\hat{E}D \text{ (correspondentes)} \end{array}\right\} \Rightarrow$

$\Rightarrow \triangle ABC \sim \triangle CDE \Rightarrow$

$\Rightarrow \dfrac{AB}{CD} = \dfrac{BC}{DE} \Rightarrow$

$\Rightarrow \dfrac{a - b}{b} = \dfrac{b}{x} \Rightarrow x = \dfrac{b^2}{a - b}$



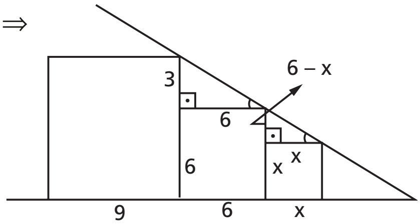
471. $\left.\begin{array}{l} A\hat{B}C \equiv C\hat{D}E \text{ (retos)} \\ A\hat{C}B \equiv C\hat{E}D \text{ (correspondentes)} \end{array}\right\} \Rightarrow$

$\Rightarrow \triangle ABC \sim \triangle CDE \Rightarrow$

$\Rightarrow \dfrac{AB}{CD} + \dfrac{BC}{DE} \Rightarrow$

$\Rightarrow \dfrac{3}{6 - x} = \dfrac{6}{x} \Rightarrow x = 4$

$2p = 4x \Rightarrow 2p = 16$



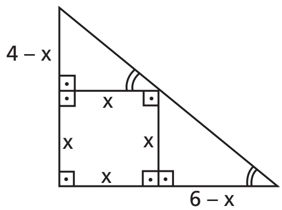
472. Seja *x* o lado do quadrado. Temos:

$\left.\begin{array}{l} C\hat{E}D \equiv C\hat{A}B \text{ (retos)} \\ C\hat{D}E \equiv D\hat{B}F \text{ (correspondentes)} \end{array}\right\} \Rightarrow$

$\Rightarrow \triangle CDE \sim \triangle DBF \Rightarrow$

$\Rightarrow \dfrac{DE}{BF} = \dfrac{CE}{DF} \Rightarrow$

$\Rightarrow \dfrac{x}{6 - x} = \dfrac{4 - x}{x} \Rightarrow x = \dfrac{12}{5}$



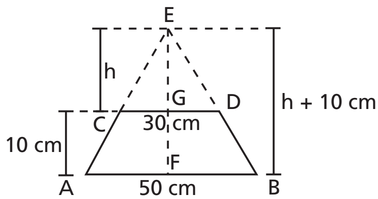
474. ABCD trapézio $\Rightarrow \overline{AB} \,//\, \overline{CD} \Rightarrow$

$\Rightarrow \triangle EAB \sim \triangle ECD \Rightarrow$

$\Rightarrow \dfrac{EF}{EG} = \dfrac{AB}{CD} \Rightarrow$

$\Rightarrow \dfrac{h + 10}{h} = \dfrac{50}{30} \Rightarrow h = 15 \text{ cm} \Rightarrow$

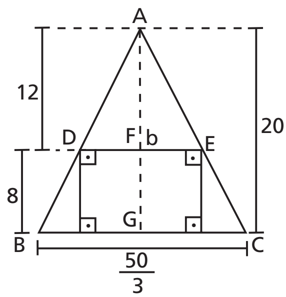
$\Rightarrow$ (EG $= 15$ cm, EF $= 25$ cm)



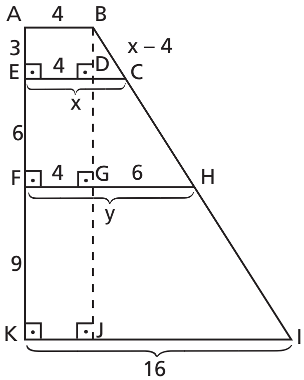
475. $\overline{DE} \,//\, \overline{BC} \Rightarrow \triangle ADE \sim \triangle ABC \Rightarrow$

$\Rightarrow \dfrac{DE}{BC} = \dfrac{AF}{AG} \Rightarrow$

$\Rightarrow \dfrac{b}{\frac{50}{3}} = \dfrac{12}{20} \Rightarrow b = 10 \text{ cm}$



476. Note que y é base média do trapézio.



Daí:

$$y = \frac{4 + 16}{2} \Rightarrow y = 10$$

Tracemos $\overline{BJ}$, com $\overline{BJ} \perp \overline{KI}$. Temos:

$ED = FG = JK = 4$.

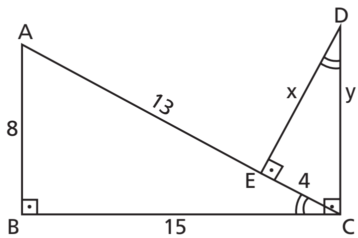
Então obtemos: $CD = x - 4$ e $GH = 6$.

$\overline{CD} \,/\!/\, \overline{GH} \Rightarrow \triangle BCD \sim \triangle BHG \Rightarrow$

$$\Rightarrow \frac{CD}{HG} = \frac{BD}{BG} \Rightarrow$$

$$\Rightarrow \frac{x - 4}{6} = \frac{3}{9} \Rightarrow x = 6$$

477. $(AC = 17, EC = 4) \Rightarrow AE = 13$



$A\hat{C}B$ e $E\hat{D}C$ possuem lados respectivamente perpendiculares. Daí:

$$\left.\begin{array}{l} A\hat{C}B \equiv E\hat{D}C \\ A\hat{B}C \equiv D\hat{E}C \text{ (retos)} \end{array}\right\} \Rightarrow$$

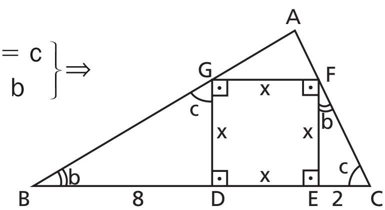
$$\Rightarrow \triangle ABC \sim \triangle CED \Rightarrow$$

$$\Rightarrow \frac{AB}{CE} = \frac{AC}{CD} = \frac{BC}{ED} \Rightarrow$$

$$\Rightarrow \frac{8}{4} = \frac{17}{y} = \frac{15}{x} \Rightarrow$$

$$\Rightarrow \left(x = \frac{15}{2}, y = \frac{17}{2}\right)$$

478. Sejam $A\hat{B}C = b$, $A\hat{C}B = c$. Então:



$\triangle ABC \Rightarrow b + c = 90°$

$$\left.\begin{array}{l} \triangle BGD \Rightarrow b + B\hat{G}D = 90° \Rightarrow B\hat{G}D = c \\ \triangle CFE \Rightarrow c + C\hat{F}E = 90° \Rightarrow C\hat{F}E = b \end{array}\right\} \Rightarrow$$

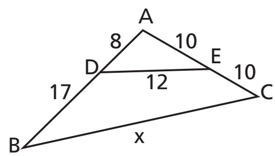
$$\Rightarrow \triangle BGD \sim \triangle FCE \Rightarrow \frac{BD}{FE} = \frac{GD}{CE} \Rightarrow$$

$$\Rightarrow \frac{8}{x} = \frac{x}{2} \Rightarrow x = 4 \text{ cm}$$

Logo, o perímetro do quadrado é igual a 16 cm.

480.

$$\left.\begin{array}{l} \dfrac{AB}{AE} = \dfrac{AC}{AD} \\ B\hat{A}C \text{ (comum)} \end{array}\right\} \xrightarrow[\text{semelhança}]{\text{caso LAL}} \triangle ABC \sim \triangle AED \Rightarrow$$



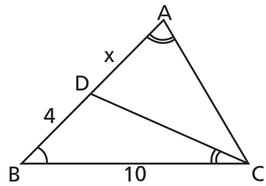
$$\Rightarrow \frac{AB}{AE} = \frac{BC}{ED} \Rightarrow \frac{25}{10} = \frac{x}{12} \Rightarrow x = 30$$

481. $\left.\begin{array}{l} B\hat{A}C \equiv B\hat{C}D \\ A\hat{B}C \text{ (comum)} \end{array}\right\} \Rightarrow$

$\Rightarrow \triangle ABC \sim \triangle CBD \Rightarrow$

$\Rightarrow \dfrac{AB}{CB} = \dfrac{BC}{BD} \Rightarrow$

$\Rightarrow \dfrac{x + 4}{10} = \dfrac{10}{4} \Rightarrow x = 21$

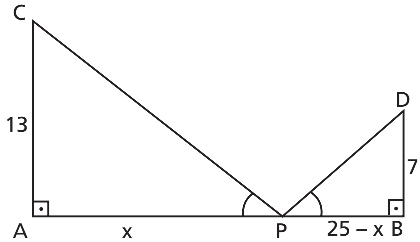


482. $\left.\begin{array}{l} B\hat{A}C \equiv A\hat{B}D \text{ (retos)} \\ A\hat{P}C \equiv B\hat{P}D \end{array}\right\} \Rightarrow$

$\Rightarrow \triangle APC \sim \triangle BPD \Rightarrow$

$\Rightarrow \dfrac{AP}{BP} = \dfrac{AC}{BD} \Rightarrow$

$\Rightarrow \dfrac{x}{25 - x} = \dfrac{13}{7} \Rightarrow x = \dfrac{65}{4}$ cm



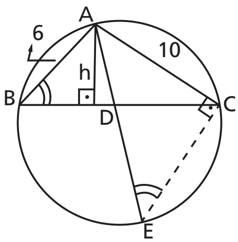
483. Unimos os pontos C e E.

$\overline{AE}$ é diâmetro $\Rightarrow A\hat{C}E = 90°$ (1)

$A\hat{B}D$ e $A\hat{E}C$ subtendem o mesmo arco $\overset{\frown}{AC} \Rightarrow A\hat{B}D \equiv A\hat{E}C$ (2)

(1) e (2) $\Rightarrow \triangle ABD \sim \triangle AEC \Rightarrow$

$\Rightarrow \dfrac{6}{30} = \dfrac{h}{10} \Rightarrow h = 2$ cm



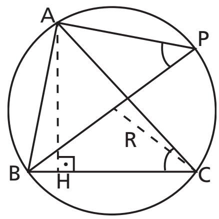
484. Tracemos o diâmetro $\overline{BP}$ e unamos P com A.

$\left.\begin{array}{l} A\hat{P}B \equiv A\hat{C}B \text{ (subtendem o arco } \overset{\frown}{AB}\text{)} \\ B\hat{A}P \equiv A\hat{H}C \text{ (retos)} \end{array}\right\} \Rightarrow$

$\Rightarrow \triangle APB \sim \triangle HCA \Rightarrow$

$\Rightarrow \dfrac{AB}{HA} = \dfrac{PB}{CA} \Rightarrow$

$\Rightarrow \dfrac{4}{3} = \dfrac{2R}{6} \Rightarrow R = 4$



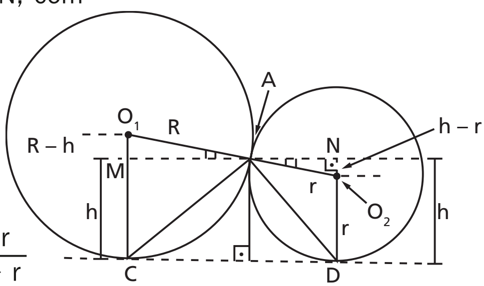
485. Pelo ponto A tracemos $\overline{MN}$, com $\overline{MN} \parallel \overline{CD}$.

$\left.\begin{array}{l} O_1\hat{A}M \equiv O_2\hat{A}N \text{ (o.p.v.)} \\ O_1\hat{M}A \equiv O_2\hat{N}A \text{ (retos)} \end{array}\right\} \Rightarrow$

$\Rightarrow \triangle O_1MA \sim \triangle O_2NA \Rightarrow$

$\Rightarrow \dfrac{O_1A}{O_2A} = \dfrac{O_1M}{O_2N} \Rightarrow$

$\Rightarrow \dfrac{R}{r} = \dfrac{R - h}{h - r} \Rightarrow h = \dfrac{2Rr}{R + r}$



486. $\left.\begin{array}{l} G\hat{O}B \equiv E\hat{O}D \text{ (o.p.v.)} \\ \overline{OD} \equiv \overline{OB} \\ A\hat{B}O \equiv A\hat{D}O \equiv C\hat{D}O \end{array}\right\} \xRightarrow{\text{ALA}}$

$\Rightarrow \triangle BGO \equiv \triangle BEO \Rightarrow$

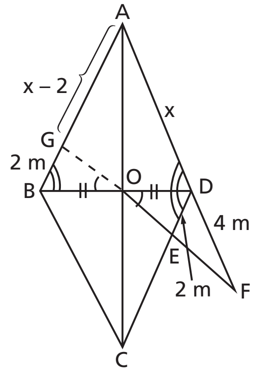
$\Rightarrow BG = DE = 2\text{ m}$

$DE = 2\text{ m} \Rightarrow AG = x - 2$

$\overline{DE} \,//\, \overline{AG} \Rightarrow \triangle FAG \sim \triangle FDE \Rightarrow$

$\Rightarrow \dfrac{FA}{FD} = \dfrac{AG}{DE} \Rightarrow$

$\Rightarrow \dfrac{x + 4}{4} = \dfrac{x - 2}{2} \Rightarrow x = 8\text{ m}$



487. Tracemos a bissetriz interna $\overline{AS}$.

Temos o que segue:

$A\hat{C}S = S\hat{A}C = y \Rightarrow$

$\Rightarrow \triangle ACS$ isósceles $\Rightarrow AS = SC = k$

$(BC = x,\ SC = k) \Rightarrow BS = x - k$

$A\hat{S}B$ é externo ao $\triangle ACS \Rightarrow A\hat{S}B =$

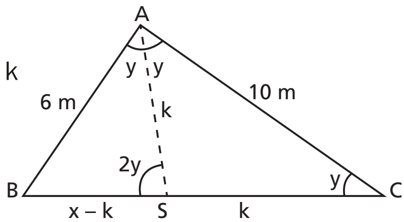
$= 2y$

$\left.\begin{array}{l} B\hat{A}S \equiv A\hat{C}B \\ A\hat{S}B \equiv B\hat{A}C \end{array}\right\} \Rightarrow \triangle ABS \sim \triangle CBA \Rightarrow$

$\Rightarrow \dfrac{6}{x} = \dfrac{k}{10} = \dfrac{x - k}{6} \Rightarrow \begin{cases} xk = 60 \ (1) \\ 10x - 10k = 6k \ (2) \end{cases}$

$(2) \Rightarrow 10x = 16k \Rightarrow k = \dfrac{5}{8}x \ (3)$

$(3) \text{ em } (1) \Rightarrow x \cdot \dfrac{5}{8}x = 60 \Rightarrow x = 4\sqrt{6}\text{ m}$



488. Unimos A e B com Q. Temos o que segue:

$\left.\begin{array}{l} Q\hat{A}H \equiv P\hat{B}Q \text{ (subtendem } \overset{\frown}{QB}) \\ Q\hat{H}A \equiv Q\hat{S}B \text{ (retos)} \end{array}\right\} \Rightarrow$

$\Rightarrow \triangle QAH \sim \triangle QBS \Rightarrow$

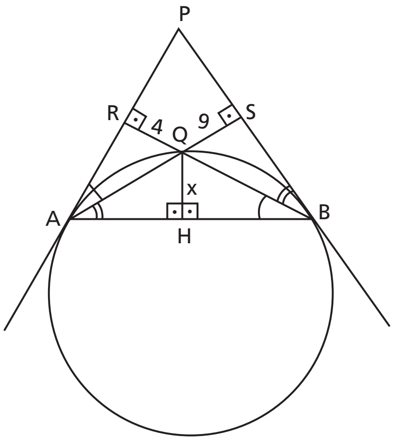
$\Rightarrow \dfrac{QH}{QS} = \dfrac{QA}{QB} \Rightarrow$

$\Rightarrow \dfrac{x}{9} = \dfrac{QA}{QB} \ (1)$

$\left.\begin{array}{l} Q\hat{B}A \equiv R\hat{A}Q \text{ (subtendem } \overset{\frown}{AQ}) \\ Q\hat{H}B \equiv Q\hat{R}A \text{ (retos)} \end{array}\right\} \Rightarrow$

$\Rightarrow \triangle QHB \sim \triangle QRA \Rightarrow$

$\Rightarrow \dfrac{QH}{QR} = \dfrac{QB}{QA} \Rightarrow$

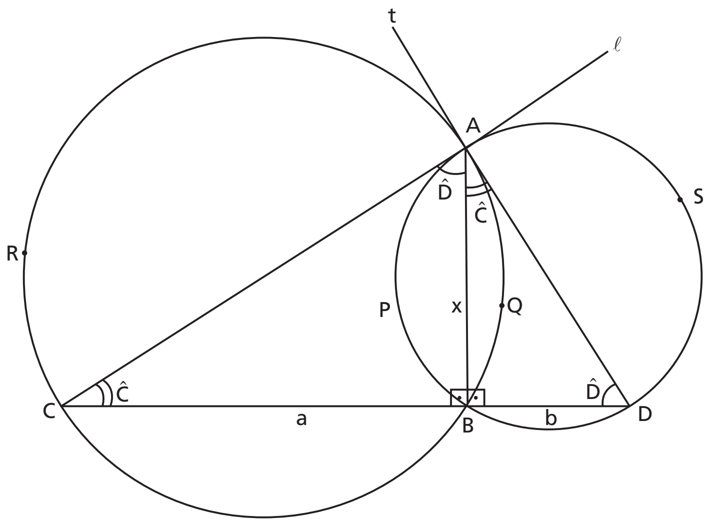


$$\Rightarrow \frac{x}{4} = \frac{QB}{QA} \Rightarrow \frac{4}{x} = \frac{QA}{QB} \text{ (2)}$$

$$\text{(1) e (2)} \Rightarrow \frac{x}{9} = \frac{4}{x} \Rightarrow x = 6$$

489. 1) $\hat{D} = \dfrac{\overset{\frown}{ARC} - \overset{\frown}{AQB}}{2} \Rightarrow \hat{D} = \dfrac{\overset{\frown}{ARC}}{2} - \dfrac{\overset{\frown}{AQB}}{2} \Rightarrow \hat{D} = \dfrac{\overset{\frown}{ARC}}{2} - \hat{C} \Rightarrow$

$\Rightarrow \hat{C} + \hat{D} = \dfrac{\overset{\frown}{ARC}}{2}$



2) $\hat{C} = \dfrac{\overset{\frown}{ASD} - \overset{\frown}{APB}}{2} \Rightarrow \hat{C} = \dfrac{\overset{\frown}{ASD}}{2} - \dfrac{\overset{\frown}{APB}}{2} \Rightarrow \hat{C} = \dfrac{\overset{\frown}{ASD}}{2} - \hat{D} \Rightarrow$

$\Rightarrow \hat{C} + \hat{D} = \dfrac{\overset{\frown}{ASD}}{2}$

1) e 2) $\Rightarrow \overset{\frown}{ARC} = \overset{\frown}{ASD} \Rightarrow A\hat{B}C = A\hat{B}D = 90° \Rightarrow \begin{cases} \overline{AC} \text{ é diâmetro} \\ \overline{AD} \text{ é diâmetro} \end{cases}$

3) Como *t* e ℓ são tangentes, temos $C\hat{A}D = 90°$. Então:

$\triangle ACD \Rightarrow \hat{C} + \hat{D} = 90°$. Daí:

$(\triangle ABC \Rightarrow B\hat{A}C = \hat{D}; \triangle ABD \Rightarrow B\hat{A}D = \hat{C}) \Rightarrow \triangle ABC \sim \triangle DBA \Rightarrow$

$\Rightarrow \dfrac{AB}{DB} = \dfrac{BC}{BA} \Rightarrow \dfrac{x}{a} = \dfrac{b}{x} \Rightarrow x^2 = ab \Rightarrow x = \sqrt{ab}$

Potência de ponto

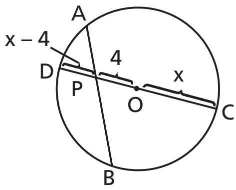
495. a) $(\overline{PA}) \times (\overline{PB}) = (\overline{PC}) \times (\overline{PD}) \Rightarrow$

$\Rightarrow 3 \cdot 8 = (x + 4) \cdot (x - 4) \Rightarrow$

$\Rightarrow x^2 - 16 = 24 \Rightarrow$

$\Rightarrow x^2 = 40 \Rightarrow x = 2\sqrt{10}$

Resposta: $2\sqrt{10}$.



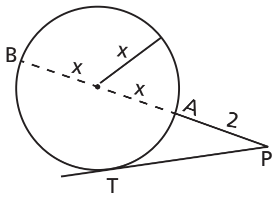
b) $(\overline{PT})^2 = (\overline{PA}) \times (\overline{PB}) \Rightarrow$

$\Rightarrow x^2 = 2 \cdot (2 + 2x) \Rightarrow$

$\Rightarrow x = 2(1 - \sqrt{2})$ (não serve)

ou $x = 2(1 + \sqrt{2})$

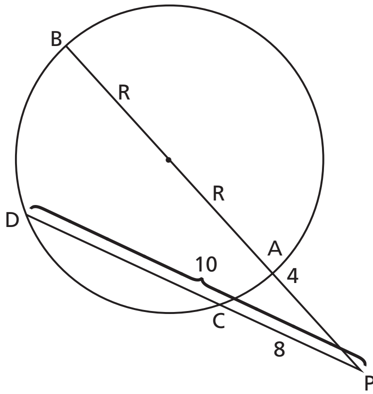
Resposta: $2(1 + \sqrt{2})$.



496. a) $(\overline{PA}) \times (\overline{PB}) = (\overline{PC}) \times (\overline{PD}) \Rightarrow$

$\Rightarrow 4 \cdot (4 + 2R) = 8 \cdot 10 \Rightarrow$

$\Rightarrow R = 16$



b) $(\overline{PA}) \times (\overline{PB}) = (\overline{PC}) \times (\overline{PE}) \Rightarrow$

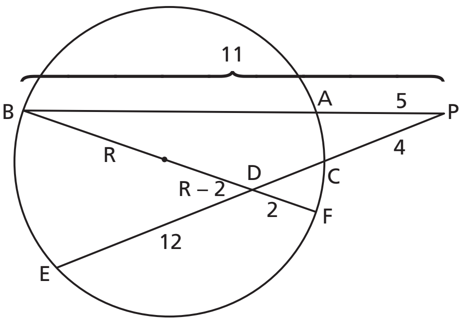
$\Rightarrow 5 \cdot 11 = 4 \cdot (16 + \overline{CD}) \Rightarrow$

$\Rightarrow \overline{CD} = 4$

$(\overline{CD}) \times (\overline{DE}) = (\overline{DF}) \times (\overline{DB}) \Rightarrow$

$\Rightarrow 4 \cdot 12 = 2 \cdot (2R - 2) \Rightarrow$

$\Rightarrow R = 13$



500. Temos: $d_A = 10$, $d_B = 3$, $d_C = 6$, $r = 6$.

Pot A $= |d_A^2 - r^2| \Rightarrow$

$\Rightarrow$ Pot A $= |10^2 - 6^2| \Rightarrow$ Pot A $= 64$

Pot B $= |d_B^2 - r^2| \Rightarrow$

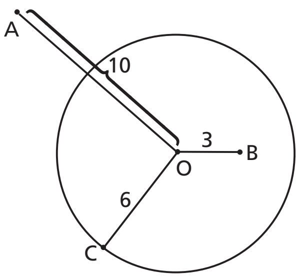
$\Rightarrow$ Pot B $= |3^2 - 6^2| \Rightarrow$ Pot B $= 27$

Pot C $= |d_C^2 - r^2| \Rightarrow$

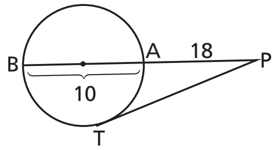
$\Rightarrow$ Pot C $= |6^2 - 6^2| \Rightarrow$ Pot C $= 0$

Logo,

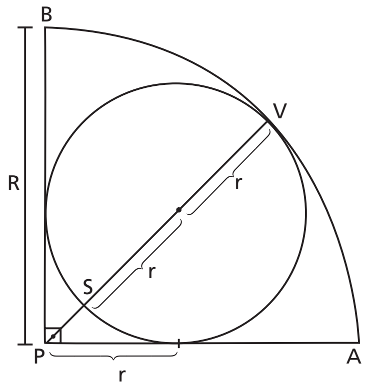
Pot A + Pot B + Pot C = 91.



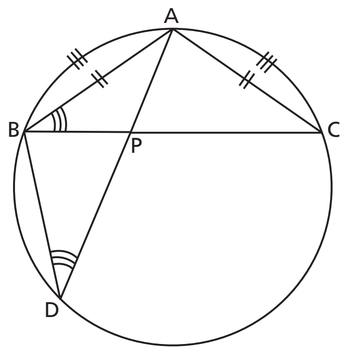
501. $(PT)^2 = (PA) \times (PB) \Rightarrow$
$\Rightarrow (PT)^2 = 18 \cdot 28 \Rightarrow$
$\Rightarrow PT = 6\sqrt{14}$



502. $(PV = R, SV = 2r) \Rightarrow PS = R - 2r$
Potência de ponto $\Rightarrow$
$\Rightarrow (PT)^2 = (PV) \cdot (PS) \Rightarrow$
$\Rightarrow r^2 = R(R - 2r) \Rightarrow$
$\Rightarrow r^2 + 2Rr - R^2 = 0 \Rightarrow$
$\Rightarrow r^2 + 2Rr + R^2 - R^2 - R^2 = 0 \Rightarrow$
$\Rightarrow r^2 + 2Rr + R^2 - 2R^2 = 0 \Rightarrow$
$\Rightarrow (r + R)^2 = 2R^2 \Rightarrow$
$\Rightarrow r + R = \sqrt{2}R \Rightarrow$
$\Rightarrow r = (\sqrt{2} - 1)R$



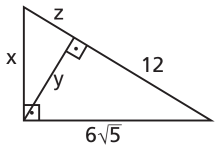
505. $\left.\begin{array}{l} \overline{AB} \equiv \overline{AC} \Rightarrow \overset{\frown}{AB} \equiv \overset{\frown}{AC} \Rightarrow A\hat{D}B \equiv A\hat{B}P \\ B\hat{A}D \equiv B\hat{A}P \text{ (comum)} \end{array}\right\} \Rightarrow$
$\Rightarrow \triangle ABD \sim \triangle APB$



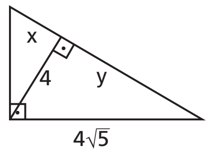
CAPÍTULO XIV — Triângulos retângulos

Relações métricas

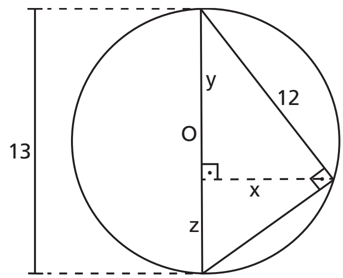
514. a) $(6\sqrt{5})^2 = 12^2 + y^2 \Rightarrow y = 6$
$y^2 = 12 \cdot z \Rightarrow 36 = 12 \cdot z \Rightarrow$
$\Rightarrow z = 3$
$x^2 = y^2 + z^2 \Rightarrow x^2 = 36 + 9 \Rightarrow$
$\Rightarrow x = 3\sqrt{5}$



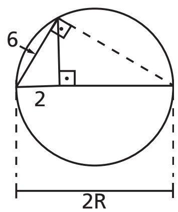
b) $y^2 + 4^2 = (4\sqrt{5})^2 \Rightarrow$
$\Rightarrow y^2 = 80 - 16 \Rightarrow$
$\Rightarrow y = 8$
$4^2 = x \cdot y \Rightarrow 16 = x \cdot 8 \Rightarrow$
$\Rightarrow x = 2$



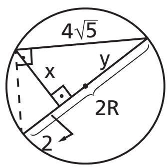
515. $x^2 + 12^2 = 13^2 \Rightarrow x = 5$
$12^2 = 13 \cdot y \Rightarrow y = \dfrac{144}{13}$
$x^2 = 13 \cdot z \Rightarrow 25 = 13z \Rightarrow$
$\Rightarrow z = \dfrac{25}{13}$
$12 \cdot x = 13 \cdot t \Rightarrow 12 \cdot 5 = 13 \cdot t \Rightarrow$
$\Rightarrow t = \dfrac{60}{13}$



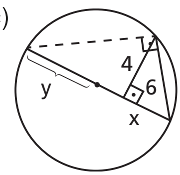
516. a)



b)



c)

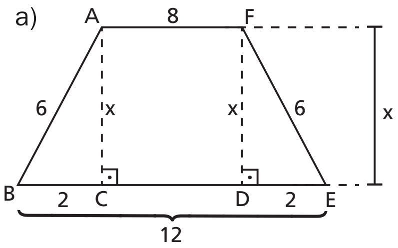


a) $6^2 = 2R \cdot 2 \Rightarrow$
$\Rightarrow R = 9$

b) $\begin{cases} x^2 + y^2 = 80 \\ x^2 = 2y \end{cases} \Rightarrow$
$\Rightarrow y^2 + 2y - 80 = 0 \Rightarrow$
$\Rightarrow y = -10$ (não serve) ou $y = 8$
Mas $y = 2R - 2$. Daí:
$2R - 2 = 8 \Rightarrow$
$\Rightarrow R = 5$

c) $x^2 + 4^2 = 6^2 \Rightarrow x = 2\sqrt{5}$
$4^2 = x \cdot y \Rightarrow$
$\Rightarrow 16 = 2\sqrt{5} \cdot y \Rightarrow$
$\Rightarrow y = \dfrac{8\sqrt{5}}{5}$
$x + y = 2R \Rightarrow$
$\Rightarrow 2\sqrt{5} + \dfrac{8\sqrt{5}}{5} = 2R \Rightarrow$
$\Rightarrow R = \dfrac{9\sqrt{5}}{5}$

519. a)



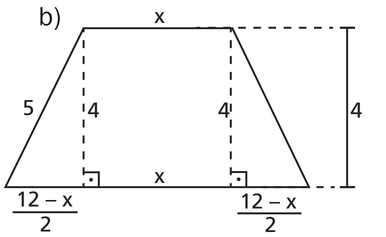
Note que

$\triangle ABC \equiv \triangle FED$ (caso especial) $\Rightarrow$

$\Rightarrow BC = DE = 2$. Daí:

$x^2 + 2^2 = 6^2 \Rightarrow x = 4\sqrt{2}$

b)

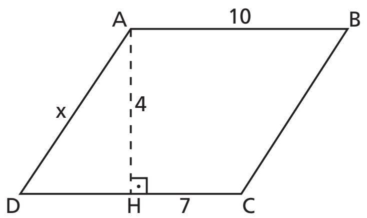


$\left(\dfrac{12 - x}{2}\right)^2 + 4^2 = 5^2 \Rightarrow$

$\Rightarrow \left(\dfrac{12 - x}{2}\right)^2 = 9 \Rightarrow$

$\Rightarrow \dfrac{12 - x}{2} = 3 \Rightarrow x = 6$

522. a)

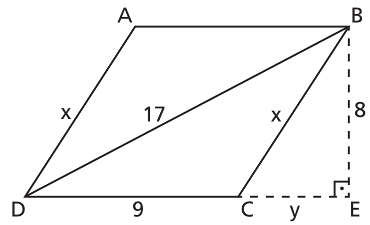


ABCD paralelogramo $\Rightarrow$

$\Rightarrow AB = CD = 10 \Rightarrow HD = 3$

$\triangle AHD$: $x^2 = 3^2 + 4^2 \Rightarrow x = 5$

b)



ABCD paralelogramo $\Rightarrow$

$\Rightarrow AD = BC = x$

$\triangle BED$: $(9 + y)^2 + 8^2 = 17^2 \Rightarrow$

$\Rightarrow (9 + y)^2 = 225 \Rightarrow$

$\Rightarrow 9 + y = 15 \Rightarrow y = 6$

$\triangle BEC$: $x^2 = 8^2 + y^2 \Rightarrow$

$\Rightarrow x^2 = 64 + 36 \Rightarrow x = 10$

523. Da figura temos:

$(EF = AB = 10,\ CD = 20,\ DE = x) \Rightarrow$

$\Rightarrow (CF = 10 - x)$

$$\left.\begin{array}{l}\triangle ADE:\ h^2 + x^2 = 64\\ \triangle BCF:\ h^2 + (10 - x)^2 = 84\end{array}\right\} \Rightarrow$$

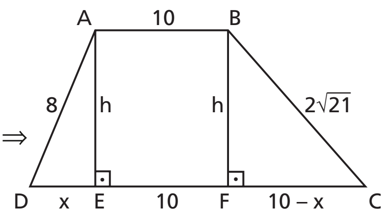
$$\Rightarrow \left.\begin{array}{l}h^2 = 64 - x^2\\ h^2 = 84 - (10 - x)^2\end{array}\right\} \Rightarrow$$

$\Rightarrow 64 - x^2 = 84 - 100 + 20x - x^2 \Rightarrow$

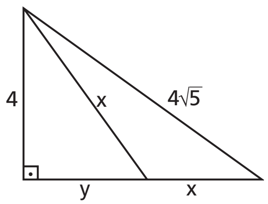
$\Rightarrow x = 4$

$h^2 + x^2 = 64 \Rightarrow h^2 + 16 = 64 \Rightarrow$

$\Rightarrow h = 4\sqrt{3}$

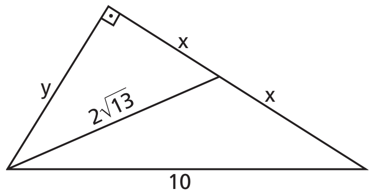


524. a)



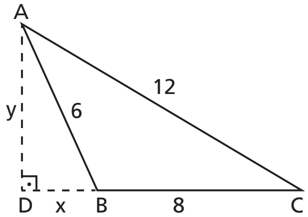
$(x + y)^2 + 4^2 = (4\sqrt{5})^2 \Rightarrow$
$\Rightarrow x + y = 8$ (1)
$x^2 = y^2 + 4^2 \Rightarrow x^2 - y^2 = 16 \Rightarrow$
$\Rightarrow (x + y)(x - y) = 16 \Rightarrow$
$\Rightarrow 8(x - y) = 16 \Rightarrow$
$\Rightarrow x - y = 2$ (2)
(1) e (2) $\Rightarrow x = 5$

b)



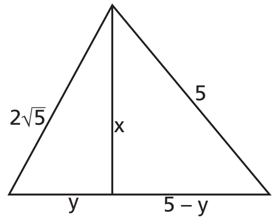
$x^2 + y^2 = (2\sqrt{13})^2 = 52$
$(2x)^2 + y^2 = 10^2 \Rightarrow$
$\Rightarrow 4x^2 + 52 - x^2 = 100 \Rightarrow$
$\Rightarrow x = 4$

c)



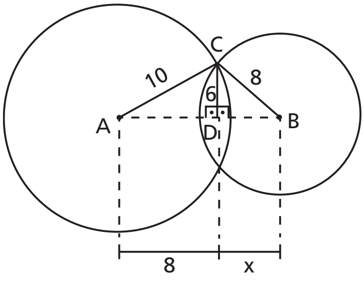
$\begin{cases} y^2 = 36 - x^2 \\ y^2 = 144 - (x + 8)^2 \end{cases} \Rightarrow$
$\Rightarrow 36 - x^2 = 144 - (x + 8)^2 \Rightarrow$
$\Rightarrow x = \dfrac{11}{4}$

d)



$\begin{cases} x^2 + y^2 = 20 \\ x^2 + (5 - y)^2 = 25 \end{cases} \Rightarrow$
$\Rightarrow 20 - y^2 = 25 - (5 - y)^2 \Rightarrow$
$\Rightarrow y = 2 \Rightarrow x = 4$

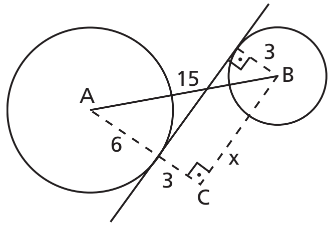
525. c) $\triangle ACD$: $(AC = 10, AD = 8) \xRightarrow{\text{Pit.}} CD = 6$
$\triangle BCD \Rightarrow (BC = 8, CD = 6) \xRightarrow{\text{Pit.}} x = 2\sqrt{7}$



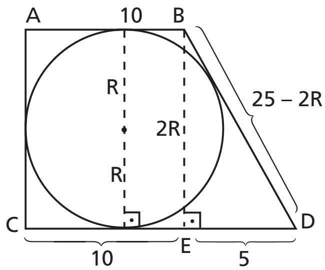
527. b) Tracemos $\overline{AC}$ pelo ponto de contato da circunferência maior com a reta tangente.

Temos:

$\triangle ABC$: $(AC = 9,\ AB = 15,$
$BC = x) \Rightarrow$
$\Rightarrow x^2 = 15^2 - 9^2 \Rightarrow x = 12$

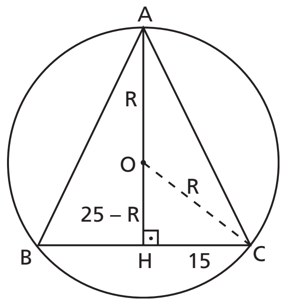


528. a)



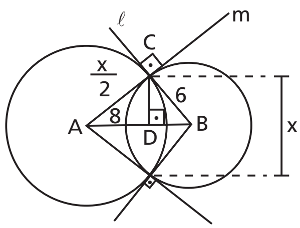
ABCD é circunscrível $\Rightarrow$
$\Rightarrow AB + CD = AC + BD \Rightarrow$
$\Rightarrow 10 + 15 = 2R + BD \Rightarrow$
$\Rightarrow BD = 25 - 2R$
$\triangle BED \Rightarrow$
$\Rightarrow (25 - 2R)^2 = (2R)^2 + 5^2 \Rightarrow$
$\Rightarrow R = 6$ m

b)



$(AH = 25,\ OA = R) \Rightarrow$
$\Rightarrow OH = 25 - R$
$\triangle OHC \Rightarrow$
$\Rightarrow R^2 = (25 - R)^2 + 15^2 \Rightarrow$
$\Rightarrow R = 17$ m

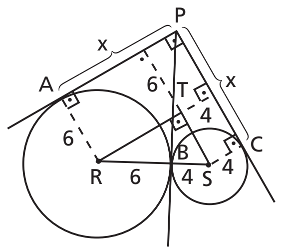
529. a)



Note que as retas ℓ e m são tangentes e, portanto, perpendiculares aos raios nos pontos de contato. Daí:

$\triangle ABC$ é retângulo em C $\xRightarrow{\text{Pit.}}$
$\Rightarrow AB^2 = 8^2 + 6^2 \Rightarrow AB = 10$
Rel. métricas $\Rightarrow$
$\Rightarrow 8 \cdot 6 = 10 \cdot \dfrac{x}{2} \Rightarrow x = 9{,}6$

b)



$PA = PB = PC = x$ (tangentes a partir de P). Daí:

$\triangle RST$: $(RS = 10,\ RT = x - 4,$
$ST = x - 6)$
Teorema de Pitágoras:
$10^2 = (x - 6)^2 + (x - 4)^2 \Rightarrow$
$\Rightarrow x^2 - 10x - 24 = 0 \Rightarrow$
$\Rightarrow x = -2$ (não serve) ou $x = 12$

530. b) Unimos o centro com os pontos de tangência e obtemos o quadrado POQA.

$(AB = 8,\ BC = 4\sqrt{13} \xrightarrow{\text{Pit.}}$
$\Rightarrow AC = 12$

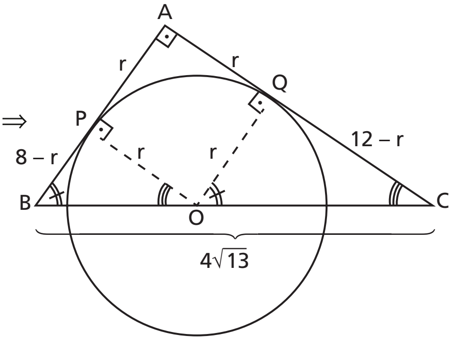
$$\left.\begin{array}{l} AC \parallel OP \Rightarrow A\hat{C}O \equiv P\hat{O}B \\ AB \parallel OQ \Rightarrow A\hat{B}O \equiv Q\hat{O}C \end{array}\right\} \Rightarrow$$

$\Rightarrow \triangle PBO \sim \triangle QOC \Rightarrow$

$$\Rightarrow \frac{PB}{QO} = \frac{PO}{QC} \Rightarrow$$

$$\Rightarrow \frac{8 - r}{r} = \frac{r}{12 - r} \Rightarrow$$

$\Rightarrow r = 4{,}8$ m



531. b) AS é bissetriz $\Rightarrow \dfrac{y}{x} = \dfrac{5}{10} \Rightarrow$

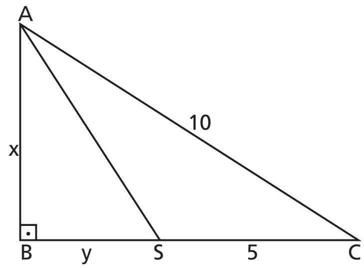
$\Rightarrow x = 2y$

$\triangle ABC \Rightarrow AC^2 = AB^2 + BC^2 \Rightarrow$

$\Rightarrow 100 = (2y)^2 + (y + 5)^2 \Rightarrow$

$\Rightarrow y = -5$ (não serve) ou $y = 3$

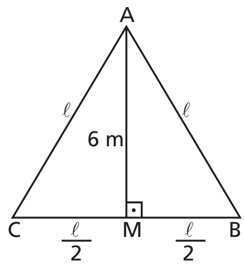
$y = 3 \Rightarrow x = 6$



536. Aplicando o teorema de Pitágoras no $\triangle AMB$:

$$6^2 + \left(\frac{\ell}{2}\right)^2 = \ell^2 \Rightarrow \ell = 4\sqrt{3}$$

$$2p = 3\ell \Rightarrow 2p = 12\sqrt{3}\ \text{m}$$

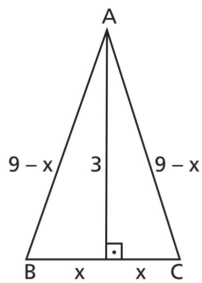


537. Para facilitar os cálculos, seja a base $BC = 2x$.

$2p = 18 \Rightarrow AB = AC = 9 - x$.

$\triangle AMC$: $x^2 + 3^2 = (9 - x)^2 \Rightarrow$

$\Rightarrow x = 4 \Rightarrow BC = 2x \Rightarrow BC \Rightarrow 8$ m



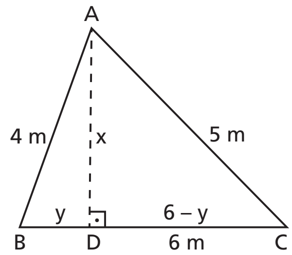
538. A menor altura é relativa ao maior lado.
Triângulo de lados 4 m, 5 m e 6 m é acutângulo.
Temos:

$$\left.\begin{array}{l}\triangle ABD: x^2 + y^2 = 16 \\ \triangle ACD: x^2 + (6 - y)^2 = 25\end{array}\right\} \Rightarrow$$

$\Rightarrow 16 - y^2 = 25 - (6 - y)^2 \Rightarrow$

$\Rightarrow y = \dfrac{9}{4} \Rightarrow x^2 + \left(\dfrac{9}{4}\right)^2 = 16 \Rightarrow$

$\Rightarrow x = \dfrac{5\sqrt{7}}{4}$ m



539.

$$\left.\begin{array}{l}\triangle AHC \Rightarrow (10 - x)^2 + h^2 = 100 \\ \triangle BHC \Rightarrow x^2 + h^2 = 144\end{array}\right\} \Rightarrow$$

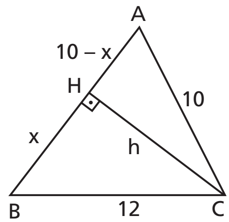
$\Rightarrow 100 - (10 - x)^2 = 144 - x^2 \Rightarrow$

$\Rightarrow x^2 - (10 - x)^2 = 100 \Rightarrow$

$\Rightarrow x = \dfrac{36}{5}$

$x^2 + h^2 = 144 \Rightarrow h^2 = 144 - \dfrac{36^2}{5^2} \Rightarrow$

$\Rightarrow h = 9{,}6$ m



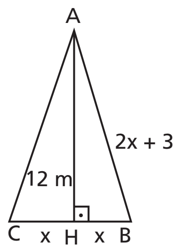
543. Seja 2x a medida da base. Temos que os lados congruentes devem medir 2x + 3, cada um. Aplicando Pitágoras no $\triangle AHB$:

$(2x + 3)^2 = x^2 + 12^2 \Rightarrow$

$\Rightarrow x^2 + 4x - 45 = 0 \Rightarrow$

$\Rightarrow (x = -9$ (não serve) ou $x = 5)$

$x = 5 \Rightarrow$ base $= 2x = 10$ m.



544. Sendo 2D e 2d as medidas das diagonais e ℓ a medida do lado do losango, temos:

$2p = 68 \Rightarrow \ell = \dfrac{68}{4} \Rightarrow \ell = 17$ m

$$\left\{\begin{array}{l}2D - 2d = 14 \\ D^2 + d^2 = 17^2\end{array}\right. \Rightarrow$$

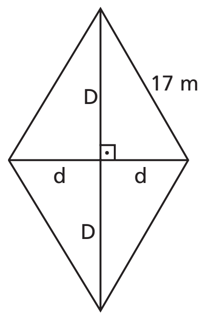
$$\Rightarrow \left\{\begin{array}{l}d = D - 7 \\ D^2 + d^2 = 289\end{array}\right. \Rightarrow$$

$\Rightarrow D^2 + (D - 7)^2 = 289 \Rightarrow$

$\Rightarrow D^2 - 7x - 120 = 0 \Rightarrow$

$\Rightarrow (D = -8$ (não serve) ou $D = 15$ m)

$(D = 15 \text{ m} \Rightarrow d = 8 \text{ m}) \Rightarrow (2D = 30 \text{ m}, 2d = 16 \text{ m})$



545. Seja ABCD o trapézio retângulo.

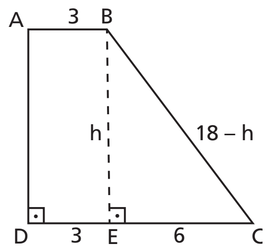
Temos:

$AB + BC + CD + AD = 30 \Rightarrow$
$\Rightarrow AD + BC = 18$ m
$AD = h \Rightarrow h + BC = 12 \Rightarrow$
$\Rightarrow BC = 18 - h$.

Traçando $\overline{BE}$, $BE \perp CD$, temos:

$\left.\begin{array}{r} DE = AB = 3 \\ CD = 9 \end{array}\right\} \Rightarrow CE = 6$ m

$\triangle BCE$: $(18 - h)^2 = h^2 + 6^2 \Rightarrow$
$\Rightarrow h = 8$ m



550. Sejam *b* e *c* as medidas dos catetos.

Temos:

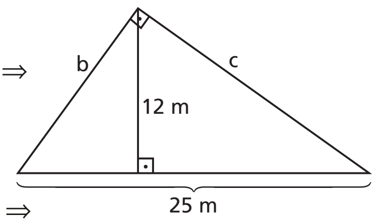
$\begin{cases} b^2 + c^2 = 625 \\ bc = 12 \cdot 25 \end{cases} \Rightarrow \begin{cases} b^2 + c^2 = 625 \\ bc = 300 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} b^2 + c^2 = 625 \ (1) \\ 2bc = 600 \ (2) \end{cases}$

$(1) + (2) \Rightarrow b^2 + 2bc = c^2 = 1225 \Rightarrow$
$\Rightarrow (b + c)^2 = 1225 \Rightarrow$
$\Rightarrow b + c = 35 \ (3)$

(3) e $(2) \Rightarrow b^2 - 35b + 300 = 0 \Rightarrow \begin{cases} b = 20 \Rightarrow c = 15 \\ \quad \text{ou} \\ b = 15 \Rightarrow c = 20 \end{cases}$

Resposta: os catetos medem 15 m e 20 m.



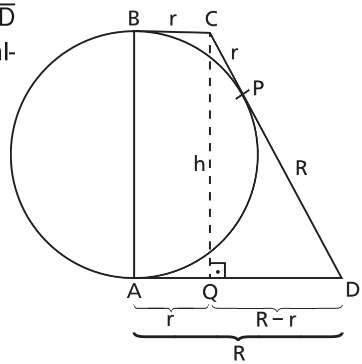
556. Seja P o ponto de tangência de $\overline{CD}$ com a circunferência e tracemos a altura $\overline{CQ}$. Temos:

$\left.\begin{array}{l} BC = CP = r; \ AD = DP = R \\ AD = R, \ AQ = r \Rightarrow QD = R - r \end{array}\right\} \Rightarrow$

$\Rightarrow CD = R + r$

$\triangle CQD$: $(R + r)^2 = h^2 + (R - r)^2 \Rightarrow$

$h = 2\sqrt{Rr}$



557. *a*: hipotenusa, *b*, *c*: catetos.

Temos:

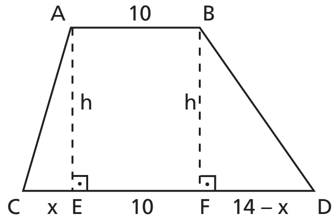
$\begin{cases} a^2 + b^2 + c^2 = 200 \ (1) \\ a^2 = b^2 + c^2 \ (2) \end{cases}$

(1) em $(2) \Rightarrow a^2 = 200 - a^2 \Rightarrow a = 10$

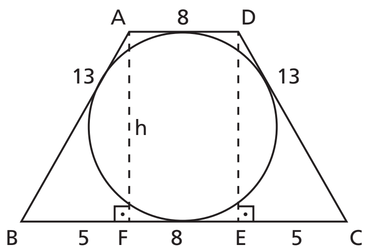
559. Considerando a figura, note que
$AB = EF = 10$ cm. Temos:
$(CE = x, EF = 10, CD = 24) \Rightarrow DF = 14 - x$

$$\left.\begin{array}{l}\triangle ACE: x^2 + h^2 = 169 \Rightarrow h^2 = 169 - x^2 \\ \triangle BDF: (14 - x)^2 + h^2 = 225 \Rightarrow h^2 = 225 - (14 - x)^2\end{array}\right\} \Rightarrow$$

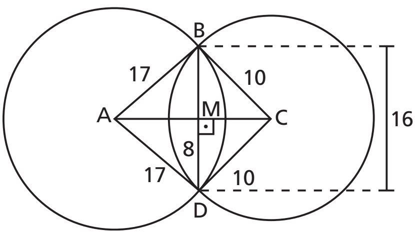
$\Rightarrow 169 - x^2 = 225 - (14 - x)^2 \Rightarrow x = 5$ cm
$\triangle ACE: x^2 + h^2 = 169 \Rightarrow 5^2 + h^2 = 169 \Rightarrow h \Rightarrow 12$ cm



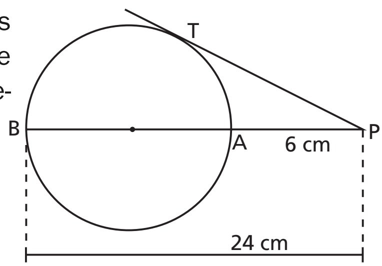
560. Trapézio é isósceles $\Rightarrow$
$\Rightarrow AB = CD = 13$ cm
Trapézio é circunscrito $\Rightarrow$
$\Rightarrow AD + BC = AB + CD \Rightarrow$
$\Rightarrow AD = 8$ cm
Traçando as alturas AF e DE, temos:
$(EF = 8, BC = 18, BF = CE) \Rightarrow$
$\Rightarrow BF = CE = 5$ cm
$\triangle ABF: 5^2 + h^2 = 13^2 \Rightarrow h = 12$ cm



561. $\triangle ABM: AM^2 + 8^2 = 17^2 \Rightarrow$
$\Rightarrow AM = 15$ cm
$\triangle BMC: MC^2 + 8^2 = 10^2 \Rightarrow$
$\Rightarrow MC = 6$ cm
$AC = AM + MC \Rightarrow AC = 21$ cm



562. Considerando as medidas indicadas na figura e aplicando potência de ponto ao ponto P em relação a λ, temos:
$(PT)^2 = (PA) \times (PB) \Rightarrow$
$\Rightarrow (PT)^2 = 6 \cdot 24 \Rightarrow (PT) = 12$ cm



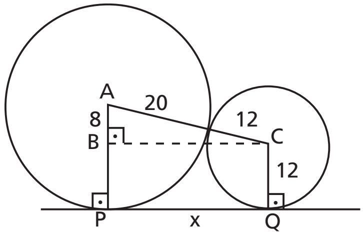
563. Traçando os raios pelos pontos de tangência e $\overline{BC} \,//\, \overline{PQ}$, em que C é o centro da circunferência menor, obtemos o triângulo ABC. Daí:

$AB^2 + BC^2 = AC^2 \Rightarrow$

$\Rightarrow 8^2 + BC^2 = 32^2 \Rightarrow$

$\Rightarrow BC = 8\sqrt{15}$ cm

$PQ = BC = 8\sqrt{15}$ cm



565. Seja ABCD o trapézio isósceles circunscritível, conforme figura ao lado. Sejam *x* e *y* as bases. Temos:

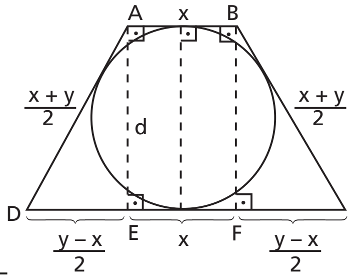
$AB = EF = x$; $DE = FC = \dfrac{y - x}{2}$ e

$AD = BC = \dfrac{x + y}{2}$

Sendo *d* o diâmetro, no $\triangle ADE$, vem:

$$\left(\frac{x + y}{2}\right)^2 = d^2 + \left(\frac{y - x}{2}\right)^2 \Rightarrow$$

$$\Rightarrow d^2 = \left(\frac{x + y}{2}\right)^2 - \left(\frac{y - x}{2}\right)^2 \Rightarrow d = \sqrt{xy}.$$



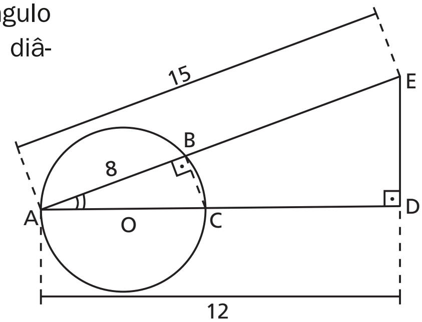
566. Unindo B com C obtemos o triângulo ABC, retângulo em B, pois $\overline{AC}$ é diâmetro. Daí:

$$\left.\begin{array}{l} \hat{B} \equiv \hat{D} \text{ (retos)} \\ B\hat{A}C \equiv E\hat{A}D \text{ (comum)} \end{array}\right\} \Rightarrow$$

$\Rightarrow \triangle ABC \sim \triangle ADE \Rightarrow$

$$\Rightarrow \frac{AC}{AE} = \frac{AB}{AD} \Rightarrow$$

$$\Rightarrow \frac{2R}{15} = \frac{8}{12} \Rightarrow R = 5 \text{ cm}$$



567. Seja D o ponto de tangência da circunferência com o lado $\overline{AB}$. Tracemos o raio $\overline{OD}$. Temos:

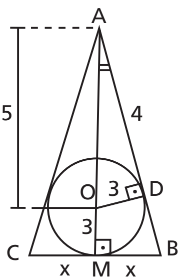
$\triangle ADO \Rightarrow OD^2 + DA^2 = OA^2 \Rightarrow$

$\Rightarrow 3^2 + DA^2 = 5^2 \Rightarrow DA = 4$ cm

$$\left.\begin{array}{l} A\hat{D}O \equiv A\hat{M}B \text{ (retos)} \\ O\hat{A}D \equiv B\hat{A}M \text{ (comum)} \end{array}\right\} \Rightarrow$$

$\Rightarrow \triangle AMB \sim \triangle ADO \Rightarrow$

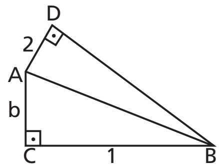
$$\Rightarrow \frac{MB}{DO} = \frac{AM}{AD} \Rightarrow \frac{x}{3} = \frac{8}{4} \Rightarrow x = 6 \Rightarrow BC = 12 \text{ cm}$$



568. $\left.\begin{array}{l}\triangle ABC \Rightarrow AB^2 = b^2 + 1 \\ \triangle ABD \Rightarrow AB^2 = 4 + BD^2\end{array}\right\} \Rightarrow$

$\Rightarrow BD^2 + 4 = b^2 + 1 \Rightarrow$

$\Rightarrow BD = \sqrt{b^2 - 3},\ b > \sqrt{3}$



569. $\triangle ACD \Rightarrow AC^2 + AD^2 = CD^2 \Rightarrow$

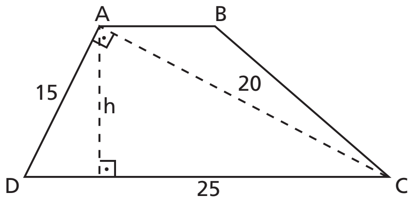
$\Rightarrow AC^2 + 15^2 = 25^2 \Rightarrow$

$\Rightarrow AC = 20$ cm

Relações métricas no $\triangle ACD$:

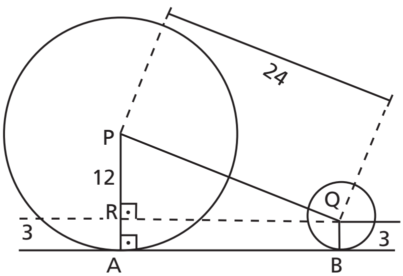
$AC \cdot AD = CD \cdot h \Rightarrow$

$\Rightarrow 20 \cdot 15 = 25 \cdot h \Rightarrow h = 12$ cm



571. Temos duas possibilidades:

1ª)



Sejam P e Q os centros das circunferências. Traçamos $\overline{QR}$, $\overline{QR} // \overline{AB}$ e os raios $\overline{PA}$ e $\overline{QB}$.

Note RA = QB = 3 cm. Como PA = 15 cm, segue-se PR = = 12 cm.

Então:

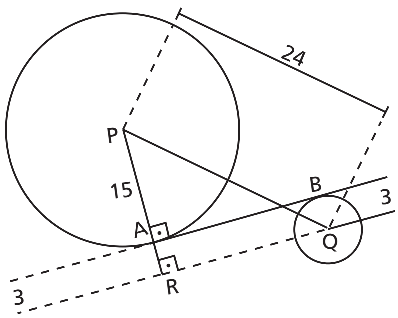
$\triangle PQR$: $PR^2 + RQ^2 = PQ^2 \Rightarrow$

$\Rightarrow 12^2 + RQ^2 = 24^2 \Rightarrow$

$\Rightarrow RQ^2 = 432 \Rightarrow RQ = 12\sqrt{3}$ cm

$AB = RQ = 12\sqrt{3}$ cm

2ª)



Neste caso traçamos $\overline{PR}$ tal que $\overline{PR} // \overline{BQ}$ e $\overline{QR}$ tal que $\overline{QR} // \overline{AB}$.

Note RA = BQ = 3 cm. Como PA = 15 cm, segue-se PR = = 18 cm.

Então:

$\triangle PQR$: $PR^2 + RQ^2 = PQ^2 \Rightarrow$

$\Rightarrow 18^2 + RQ^2 = 24^2 \Rightarrow$

$\Rightarrow RQ^2 = 252 \Rightarrow RQ = 6\sqrt{7}$ cm

$AB = RQ = 6\sqrt{7}$ cm

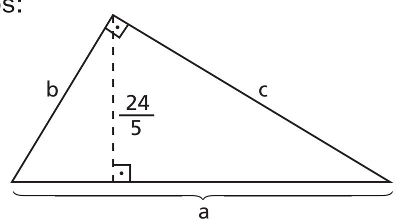
572. *a*: hipotenusa; *b*, *c*: catetos. Temos:

$2p = 24 \Rightarrow a + b + c = 24 \Rightarrow$

$\Rightarrow b + c = 24 - a$ (1)

Rel. métricas $\Rightarrow b \cdot c = a \cdot \dfrac{24}{5} \Rightarrow$

$\Rightarrow b \cdot c = \dfrac{24}{5} \cdot a$ (2)



Teorema de Pitágoras $\Rightarrow$

$\Rightarrow b^2 + c^2 = a^2$ (3)

$(1) \Rightarrow (b + c)^2 = (24 - a)^2 \Rightarrow \underbrace{b^2 + c^2}_{(3)} + \underbrace{2bc}_{(2)} = 576 - 48a + a^2 \Rightarrow$

$\Rightarrow a^2 + 2 \cdot \dfrac{24}{5} a = 576 - 48a + a^2 \Rightarrow a = 10$ m

573. Considere o triângulo PQR, em que P, Q e R são os centros das três circunferências que se tangenciam externamente.

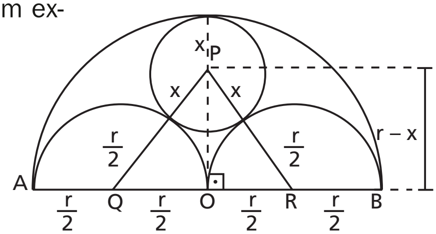
Seja *x* o raio a determinar.

Note que PO = r − x. Então:

$\triangle$OPR:

$\left(x + \dfrac{r}{2}\right)^2 = (r - x)^2 + \left(\dfrac{r}{2}\right)^2 \Rightarrow$

$\Rightarrow x = \dfrac{r}{3}$



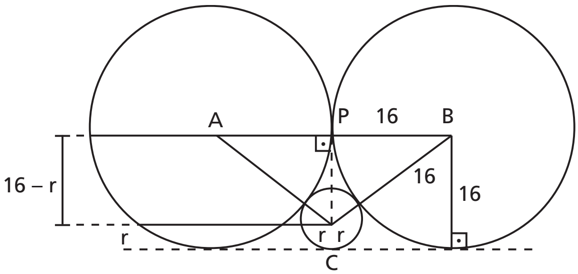
574. Sejam ABC o triângulo que obtemos ao unir os centros dos círculos e P o ponto de tangência entre os dois círculos de mesmo raio. Temos:

$\triangle BPC: BC^2 = BP^2 + PC^2 \Rightarrow (16 + r)^2 = 16^2 + (16 - r)^2 \Rightarrow$

$\Rightarrow (16 + r)^2 - (16 - r)^2 = 256 \Rightarrow$

$\Rightarrow (16 + \cancel{r} + 16 - \cancel{r})(\cancel{16} + r - \cancel{16} + r) = 256 \Rightarrow$

$\Rightarrow 32 \cdot (2r) = 256 \Rightarrow r = 4$



575. Sejam ABCD o quadrado e EFGHIJLM o octógono regular.

Temos:

$(AD = 1, AF = x, DE = x) \Rightarrow$

$\Rightarrow EF = 1 - 2x$

EFGHIJLM é regular $\Rightarrow$

$\Rightarrow EF = FG = 1 - 2x$

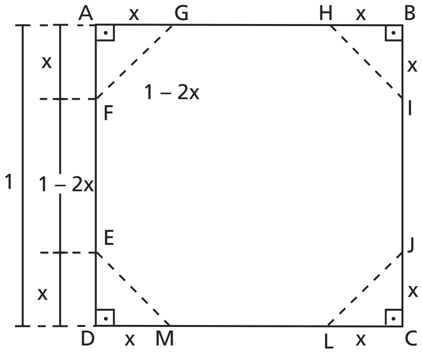
$\triangle AFG: (1 - 2x)^2 = x^2 + x^2 \Rightarrow$

$\Rightarrow (1 - 2x)^2 = 2x^2 \Rightarrow$

$\Rightarrow 1 - 2x = x\sqrt{2} \Rightarrow$

$\Rightarrow x(\sqrt{2} + 2) = 1 \Rightarrow$

$\Rightarrow x = \dfrac{2 - \sqrt{2}}{2}$



576. Traçamos os raios pelos pontos de tangência e obtemos o trapézio retângulo EPQF.

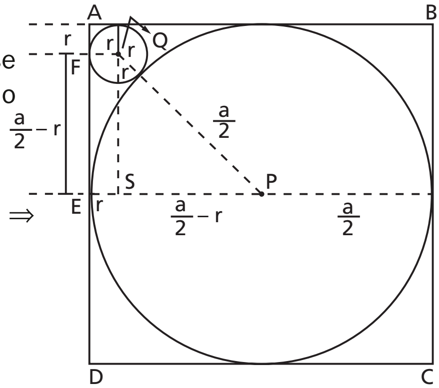
Traçamos a altura $\overline{QS}$ desse trapézio, obtemos o triângulo retângulo QSP.

Daí:

$\left(\dfrac{a}{2} + r\right)^2 = \left(\dfrac{a}{2} - r\right)^2 + \left(\dfrac{a}{2} - r\right)^2 \Rightarrow$

$\Rightarrow \dfrac{a}{2} + r = \left(\dfrac{a}{2} - r\right) \cdot \sqrt{2} \Rightarrow$

$\Rightarrow r = \dfrac{\left(3 - 2\sqrt{2}\right) \cdot a}{2}$

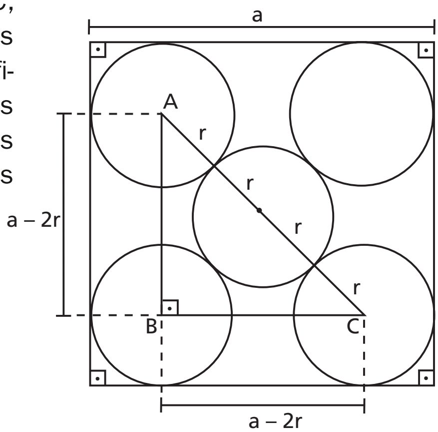


577. Construímos o triângulo ABC, de lados $\overline{AB}$ e $\overline{BC}$ paralelos aos lados do quadrado, conforme figura ao lado. Considerando as medidas indicadas, podemos aplicar o teorema de Pitágoras ao $\triangle ABC$:

$(4r)^2 = 2 \cdot (a - 2r)^2 \Rightarrow$

$\Rightarrow 4r = (a - 2r)\sqrt{2} \Rightarrow$

$\Rightarrow r = \dfrac{\left(\sqrt{2} - 1\right) \cdot a}{2}$



578. Construímos o triângulo OPB, tal que $\overline{OP} // \overline{AC}$, $\overline{PB} // \overline{DE}$.

Note que OB = R, OP = R − 12 e PB = 54 − R.

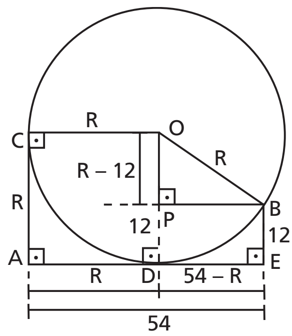
$\triangle OPB \Rightarrow$

$\Rightarrow R^2 = (R - 12)^2 + (54 - R)^2 \Rightarrow$

$\Rightarrow R^2 - 132R + 3060 = 0 \Rightarrow$

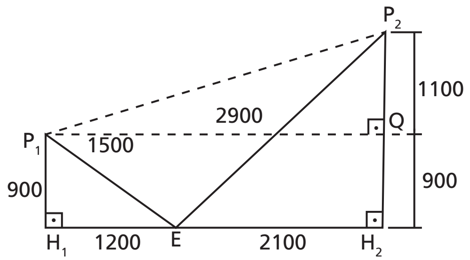
$\Rightarrow R = 102$ cm (não serve) ou

$R = 30$ cm



579. Temos duas possibilidades:

1ª) E está entre as montanhas.



$\triangle P_1H_1E \xrightarrow{\text{Pitágoras}} H_1E = 1200$ m

$\triangle P_2H_2E \xrightarrow{\text{Pitágoras}} H_2E = 2100$ m

$\triangle P_1P_2Q \Rightarrow$

$\Rightarrow (P_1Q = H_1H_2 = 3300$ m;

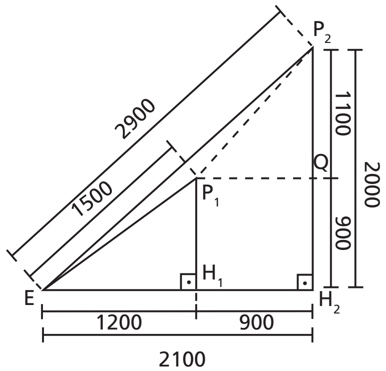
$P_2Q = 1100$ m)

Aplicando o teorema de Pitágoras neste último triângulo, temos:

$(P_1P_2)^2 = 3300^2 + 1100^2 \Rightarrow$

$\Rightarrow P_1P_2 \cong 3478$ m

2ª) A montanha menor está entre E e a maior.



$(P_1H_1 = 900,\ P_2H_2 = 2000) \Rightarrow$

$\Rightarrow P_2Q = 1100$ m

$\left.\begin{array}{l} \triangle P_1H_1E \xrightarrow{\text{Pitágoras}} EH_1 = 1200 \text{ m} \\ \triangle P_2H_2E \xrightarrow{\text{Pitágoras}} EH_2 = 2100 \text{ m} \end{array}\right\} \Rightarrow$

$\Rightarrow H_1H_2 = 900$ m

$\triangle P_1P_2Q \Rightarrow$

$\Rightarrow (P_1Q = H_1H_2 = 900$ m;

$P_2Q = 1100$ m)

Aplicando o teorema de Pitágoras ao $\triangle P_1P_2Q$:

$(P_1P_2)^2 = 1100^2 + 900^2 \Rightarrow$

$\Rightarrow P_1P_2 \cong 1421$ m

580. Considere o ponto Z, interseção de $\overline{PQ}$ com a circunferência menor. Temos:

$ZS = SQ = ST = 3 \text{ cm} \Rightarrow$

$\Rightarrow (ZQ = 6 \text{ cm})$

$(ZQ = 6 \text{ cm}, PQ = 8 \text{ cm}) \Rightarrow$

$\Rightarrow PZ = 2 \text{ cm} \Rightarrow PS = 5 \text{ cm}$

$\triangle PST \Rightarrow (PT)^2 + 3^2 = 5^2 \Rightarrow$

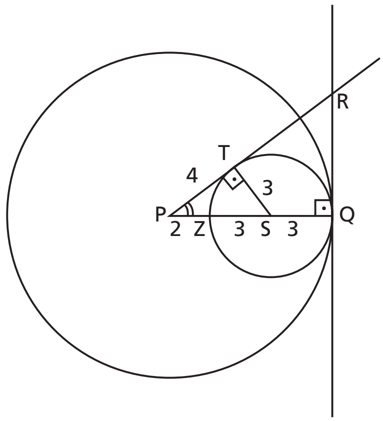
$\Rightarrow PT = 4 \text{ cm}$

$$\left.\begin{array}{l} \hat{T} \equiv \hat{Q} \text{ (retos)} \\ T\hat{P}S \equiv R\hat{P}Q \text{ (comum)} \end{array}\right\} \Rightarrow$$

$\Rightarrow \triangle PST \sim \triangle PRQ \Rightarrow$

$$\Rightarrow \frac{PT}{PQ} = \frac{ST}{RQ} \Rightarrow \frac{4}{8} = \frac{3}{RQ} \Rightarrow$$

$\Rightarrow RQ = 6 \text{ cm}$



581. Considere o triângulo isósceles OAB, $\overline{OM}$ sua altura relativa à base, P o centro do círculo inscrito no setor OAB, S, T pontos de tangência entre a circunferência e raios OA e OB, respectivamente. Temos:

$$AB = \frac{R}{2} \Rightarrow AM = MB = \frac{R}{4}$$

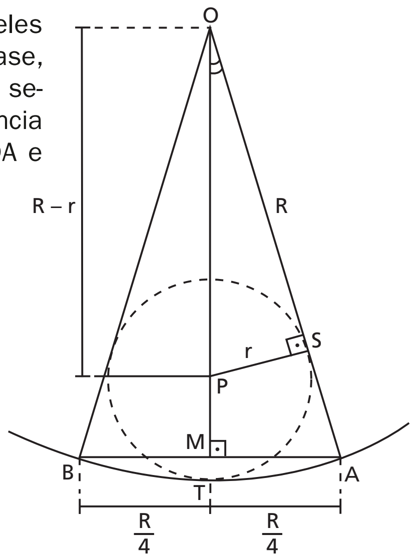
$OT = R \Rightarrow OP = R - r$

$$\left.\begin{array}{l} \hat{S} \equiv \hat{M} \text{ (retos)} \\ S\hat{O}P \equiv A\hat{O}M \text{ (comum)} \end{array}\right\} \Rightarrow$$

$\Rightarrow \triangle SOP \sim \triangle MOA \Rightarrow$

$$\Rightarrow \frac{SP}{MA} = \frac{OP}{OA} \Rightarrow$$

$$\Rightarrow \frac{r}{\frac{R}{4}} = \frac{R - r}{R} \Rightarrow r = \frac{R}{5}$$



582. De acordo com a figura, temos:

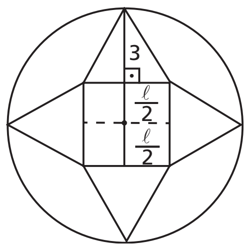
$$3 + \frac{\ell}{2} = R \Rightarrow$$

$$\Rightarrow 3 + \frac{\ell}{2} = 3(\sqrt{2} + 2) \Rightarrow$$

$\Rightarrow \ell = 6(\sqrt{2} + 1) \text{ cm}$

Sendo 2p o perímetro do quadrado, vem:

$2p = 4 \cdot \ell \Rightarrow 2p = 24(\sqrt{2} + 1) \text{ cm}$



583. Cálculo da diagonal $\overline{AC}$:

$\triangle ACD$: $AC^2 = AD^2 + CD^2 \Rightarrow$

$\Rightarrow AC^2 = a^2 + a^2 \Rightarrow$

$\Rightarrow AC = a\sqrt{2} = CE$

$AM = \dfrac{1}{3} \cdot AC \Rightarrow AM = \dfrac{a\sqrt{2}}{3} \Rightarrow$

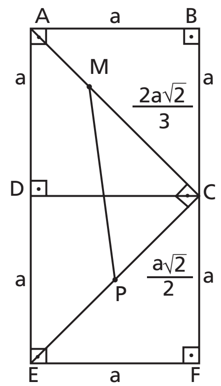
$\Rightarrow MC = \dfrac{2}{3} \cdot a\sqrt{2}$

$EP = \dfrac{1}{2}\,CE \Rightarrow EP = PC = \dfrac{a\sqrt{2}}{2}$

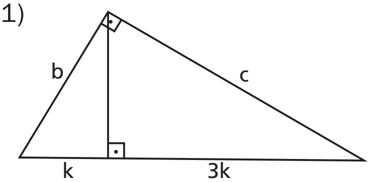
$(A\hat{C}D = 45°,\ E\hat{C}D = 45°) \Rightarrow$

$\Rightarrow A\hat{C}E = 90°$

$\triangle ACE$: $(MP)^2 = \left(\dfrac{2a\sqrt{2}}{3}\right)^2 + \left(\dfrac{a\sqrt{2}}{2}\right)^2 \Rightarrow MP = \dfrac{5\sqrt{2}}{6}a$



584. 1)



Sendo *m* e *n* as projeções proporcionais a 1 e 3, temos:

$m = k$ e $n = 3k$.

Rel. métricas $\Rightarrow$

$\Rightarrow b^2 = 4k \cdot k \Rightarrow b = 2k$

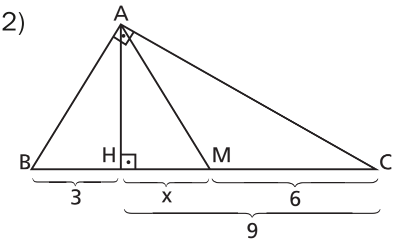
$c^2 = 4k \cdot 3k \Rightarrow c = 2\sqrt{3}k$

Dado $\Rightarrow 2p = 18 + 6\sqrt{3} \Rightarrow$

$\Rightarrow 2k + 4k + 2\sqrt{3}k = 18 + 6\sqrt{3} \Rightarrow$

$\Rightarrow k = 3$

2)



AH: altura; AM: mediana

Se $k = 3$, temos:

$(BH = 3,\ HC = 9) \Rightarrow$

$\Rightarrow (BC = 12,\ MC = 6)$

$(HC = 9,\ MC = 6) \Rightarrow$

$\Rightarrow HM = 3$ m

585. $\left.\begin{array}{l} \triangle AHM \xRightarrow{\text{Pitágoras}} HM = 2\text{ cm} \\ \text{AM é mediana} \Rightarrow AM = MC = MB = 4\text{ cm} \end{array}\right\} \Rightarrow BH = 2\text{ cm}$

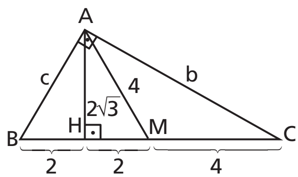
Rel. métricas no $\triangle ABC$:

$\begin{cases} b^2 = 8 \cdot 6 \Rightarrow b = 4\sqrt{3}\text{ cm} \\ c^2 = 8 \cdot 2 \Rightarrow c = 4\text{ cm} \end{cases}$

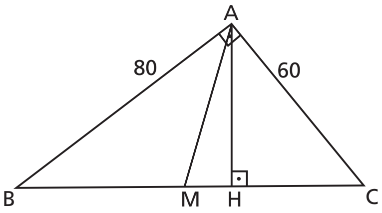
Logo:

$2p = 4 + 8 + 4\sqrt{3} \Rightarrow$

$\Rightarrow 2p = 4(3 + \sqrt{3})$ cm.

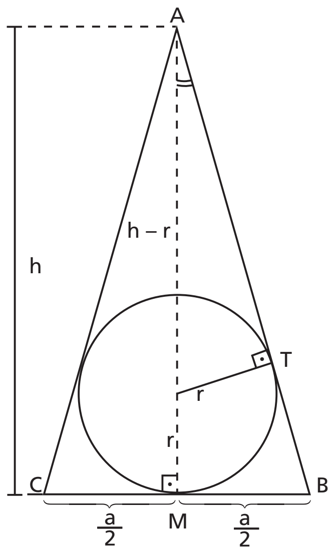


586. $(BC)^2 = (AB)^2 + (AC)^2 \Rightarrow$
$\Rightarrow (BC)^2 = 80^2 + 60^2 \Rightarrow$
$\Rightarrow (BC) = 100$ cm
$(AB) \times (AC) = (BC) \cdot (AH) \Rightarrow$
$\Rightarrow 80 \cdot 60 = 100 \cdot (AH) \Rightarrow$
$\Rightarrow (AH) = 48$ cm
$(AM) = \dfrac{(BC)}{2} \Rightarrow (AM) = \dfrac{100}{2} \Rightarrow$
$\Rightarrow (AM) = 50$ cm



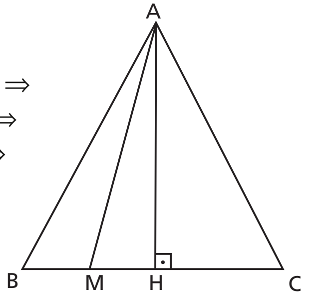
$(AB)^2 = (BC) \cdot (HB) \Rightarrow 80^2 = (100)^2(HB) \Rightarrow HB = 64$ cm
$(AC)^2 = (BC) \cdot (HC) \Rightarrow 60^2 = (100)^2(HC) \Rightarrow HC = 36$ cm
$(MH) = (BC) - (BM) - (HC) \Rightarrow (MH) = 100 - 50 - 36 \Rightarrow (MH) =$
$= 14$ cm

587. Na figura, sejam ABC o triângulo isósceles de base BC, O o centro da circunferência inscrita e T o ponto de tangência desta com o lado $\overline{AB}$.
Temos:
$\triangle ATO \Rightarrow (AT)^2 + (OT)^2 = (AO)^2 \Rightarrow$
$\Rightarrow (AT)^2 + r^2 = (h - r)^2 \Rightarrow$
$\Rightarrow AT = \sqrt{h(h - 2r)}$
$$\left.\begin{array}{l} A\hat{T}O \equiv A\hat{M}B \text{ (retos)} \\ T\hat{A}O \equiv M\hat{A}B \text{ (comum)} \end{array}\right\} \Rightarrow$$
$\Rightarrow \triangle TAO \sim \triangle MAB \Rightarrow$
$$\Rightarrow \frac{AT}{AM} = \frac{TO}{MB} \Rightarrow$$
$$\Rightarrow \frac{\sqrt{h(h - 2r)}}{h} = \frac{r}{\frac{a}{2}} \Rightarrow$$
$$\Rightarrow \frac{h(h - 2r)}{h^2} = \frac{r^2}{\frac{a^2}{4}} \Rightarrow$$
$$\Rightarrow h = \frac{2a^2 \cdot r}{a^2 - 4r^2}$$
(Note que devemos ter $a \neq 2r$.)

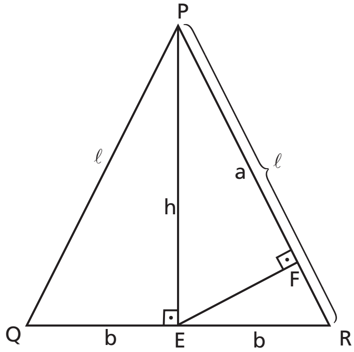


589. Na figura, ABC é isósceles de base BC e AH é altura relativa à base.
Temos:
$\triangle AMH$: $(AM)^2 = (AH)^2 + (MH)^2$ (1)
$\triangle ABH$: $(AH)^2 = (AB)^2 - (BH)^2$ (2)

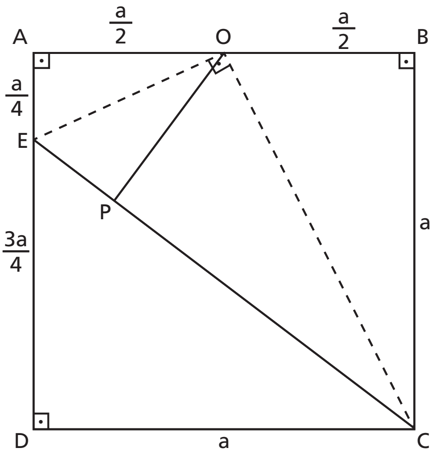
(2) em (1)
$\Rightarrow (AM)^2 = (AB)^2 - (BH)^2 + (MH)^2 \Rightarrow$
$\Rightarrow (AM)^2 = (AB)^2 - (BH)^2 + (BH - BM)^2 \Rightarrow$
$\Rightarrow (AM)^2 = (AB)^2 - 2(BH)(BM) + (BM)^2 \Rightarrow$
$\Rightarrow (AM)^2 = (AB)^2 - (BM)(2BH - (BM)) \Rightarrow$
$\Rightarrow (AM)^2 = (AB)^2 - BM((BC) - (BM)) \Rightarrow$
$\Rightarrow (AM)^2 = (AB)^2 - (BM)(MC) \Rightarrow$
$\Rightarrow (AB)^2 - (AM)^2 = (MB)(MC)$



590. Na figura, temos:
$\triangle PER$ é retângulo, $\overline{EF}$ é altura relativa à hipotenusa $\Rightarrow$
$\Rightarrow (PE)^2 = (PR) \cdot (PF) \Rightarrow$
$\Rightarrow h^2 = \ell \cdot a \Rightarrow \ell = \dfrac{h^2}{a}$
$\triangle PER \Rightarrow h^2 + b^2 = \ell^2 \Rightarrow$
$\Rightarrow h^2 + b^2 = \dfrac{h^4}{a^2} \Rightarrow$
$b = \dfrac{h}{a}\sqrt{h^2 - a^2}$
$2p = 2\ell + 2b \Rightarrow$
$\Rightarrow 2p = 2h^2 + \dfrac{2h}{a}\sqrt{h^2 - a^2} \Rightarrow$
$$\Rightarrow 2p = \frac{2h\left(h + \sqrt{h^2 - a^2}\right)}{a}$$



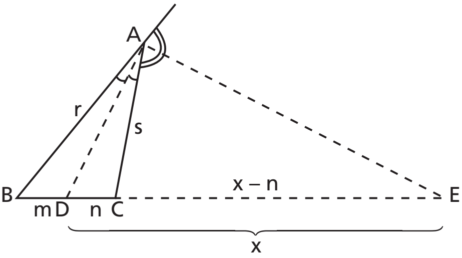
591.
$$\left.\begin{array}{l} \triangle AOE \xRightarrow{\text{Pitágoras}} OE = \dfrac{a\sqrt{5}}{4} \\[2ex] \triangle BOC \xRightarrow{\text{Pitágoras}} OC = \dfrac{a\sqrt{5}}{2} \\[2ex] \triangle CDE \xRightarrow{\text{Pitágoras}} EC = \dfrac{5a}{4} \end{array}\right\} \Rightarrow$$



Pela recíproca do teorema de Pitágoras, a igualdade obtida acima nos garante que $\triangle COE$ é retângulo em O. Como $\overline{OP}$ é altura relativa à hipotenusa, temos:

$$(OP)^2 = (EP) \cdot (CP)$$

592.



Hipótese | Tese

AD é bissetriz interna
AE é bissetriz externa $\Rightarrow \dfrac{\sqrt{AD^2 + AE^2}}{CD} - \dfrac{\sqrt{AD^2 + AE^2}}{BD} = 2$

Demonstração

$2 \cdot C\hat{A}D + 2 \cdot C\hat{A}E = 180° \Rightarrow C\hat{A}D + C\hat{A}E = 90° \Rightarrow \triangle ADE$ retângulo em A $\Rightarrow DE = \sqrt{AD^2 + AE^2} = x$

Utilizando as medidas indicadas, devemos provar que $\dfrac{x}{n} - \dfrac{x}{m} = 2$.

$$\left.\begin{array}{l} \text{Teor. biss. interna} \Rightarrow \dfrac{m}{r} = \dfrac{n}{s} \Rightarrow \dfrac{m}{n} = \dfrac{r}{s} \\ \text{Teor. biss. externa} \Rightarrow \dfrac{x+m}{r} = \dfrac{x-n}{s} \Rightarrow \dfrac{x+m}{x-n} = \dfrac{r}{s} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \frac{x+m}{x-n} = \frac{m}{n} \Rightarrow mn = \frac{(m-n)}{2} \cdot x$$

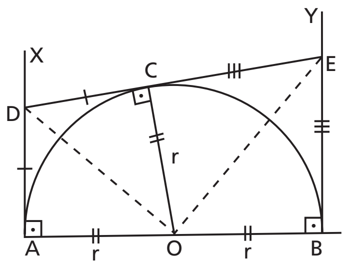
$$\text{Daí: } \frac{x}{n} - \frac{x}{m} = \frac{m-n}{mn} \cdot x = \frac{m-n}{\dfrac{(m-n)}{2}x} \cdot x = 2.$$

593.

$$\left.\begin{array}{l} (\overline{DC} \equiv \overline{DA}, \overline{OC} \equiv \overline{OA}, \overline{OD} \text{ comum}) \overset{LLL}{\Longrightarrow} \triangle CDO \equiv \triangle ADO \Rightarrow C\hat{O}D \equiv A\hat{O}D \\ (\overline{CE} \equiv \overline{BE}, \overline{OC} \equiv \overline{OB}, \overline{OE} \text{ comum}) \overset{LLL}{\Longrightarrow} \triangle COE \equiv \triangle BOE \Rightarrow C\hat{O}E \equiv B\hat{O}E \end{array}\right\} \Rightarrow$$

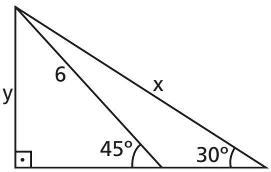
$\Rightarrow 2 \cdot C\hat{O}D + 2 \cdot C\hat{O}E = 180° \Rightarrow C\hat{O}D + C\hat{O}E = 90° \Rightarrow \triangle DEO$ é retângulo em O.

$\overline{OC}$ é altura relativa à hipotenusa $\Rightarrow OC^2 = CD \cdot CE \Rightarrow r^2 = CD \cdot CE$.



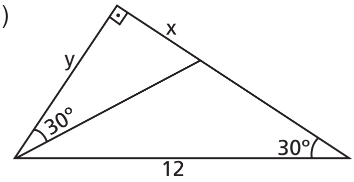
Aplicações do teorema de Pitágoras

598. a)



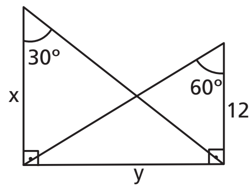
$$\text{sen } 45° = \frac{y}{6} \Rightarrow \frac{\sqrt{2}}{2} = \frac{y}{6} \Rightarrow$$
$$\Rightarrow y = 3\sqrt{2}$$
$$\text{sen } 30° = \frac{y}{x} \Rightarrow \frac{1}{2} = \frac{3\sqrt{2}}{x} \Rightarrow$$
$$\Rightarrow x = 6\sqrt{2}$$

b)



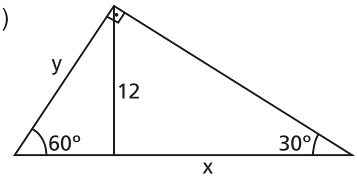
$$\text{sen } 30° = \frac{y}{12} \Rightarrow \frac{1}{2} = \frac{y}{12} \Rightarrow$$
$$\Rightarrow y = 6$$
$$\text{tg } 30° = \frac{x}{y} \Rightarrow \frac{\sqrt{3}}{3} = \frac{x}{6} \Rightarrow$$
$$\Rightarrow x = 2\sqrt{3}$$

c)



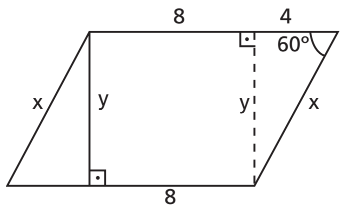
$$\text{tg } 60° = \frac{y}{2} \Rightarrow \sqrt{3} = \frac{y}{12} \Rightarrow$$
$$\Rightarrow y = 12\sqrt{3}$$
$$\text{tg } 30° = \frac{y}{x} \Rightarrow \frac{\sqrt{3}}{3} = \frac{12\sqrt{3}}{x} \Rightarrow$$
$$\Rightarrow x = 36$$

d)



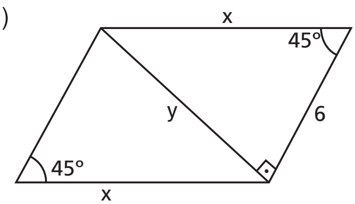
$$\text{sen } 60° = \frac{12}{y} \Rightarrow \frac{\sqrt{3}}{2} = \frac{12}{y} \Rightarrow$$
$$\Rightarrow y = 8\sqrt{3}$$
$$\text{sen } 30° = \frac{y}{x} \Rightarrow \frac{1}{2} = \frac{8\sqrt{3}}{x} \Rightarrow$$
$$\Rightarrow x = 16\sqrt{3}$$

599. b)



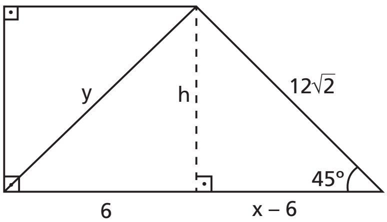
$$\cos 60° = \frac{4}{x} \Rightarrow \frac{1}{2} = \frac{4}{x} \Rightarrow$$
$$\Rightarrow x = 8$$
$$4^2 + y^2 = x^2 \Rightarrow 16 + y^2 = 64 \Rightarrow$$
$$\Rightarrow y = 4\sqrt{3}$$

c)



$$\cos 45° = \frac{6}{x} \Rightarrow \frac{\sqrt{2}}{2} = \frac{6}{x} \Rightarrow$$
$$\Rightarrow x = 6\sqrt{2}$$
$$y^2 + 6^2 = x^2 \Rightarrow y^2 + 36 = 72 \Rightarrow$$
$$\Rightarrow y = 6$$

d)



$\cos 45° = \dfrac{x-6}{12\sqrt{2}} \Rightarrow$

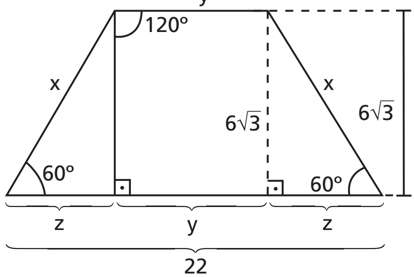
$\Rightarrow \dfrac{\sqrt{2}}{2} = \dfrac{x-6}{12\sqrt{2}} \Rightarrow x = 18$

$\text{sen } 45° = \dfrac{h}{12\sqrt{2}} \Rightarrow$

$\Rightarrow \dfrac{\sqrt{2}}{2} = \dfrac{h}{12\sqrt{2}} \Rightarrow h = 12$

$y^2 = 6^2 + 12^2 \Rightarrow y = 6\sqrt{5}$

e)



$\text{sen } 60° = \dfrac{6\sqrt{3}}{x} \Rightarrow$

$\Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{6\sqrt{3}}{x} \Rightarrow x = 12$

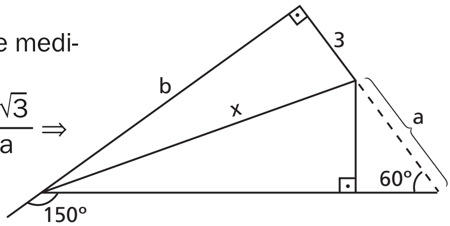
$\cos 60° = \dfrac{z}{x} \Rightarrow \dfrac{1}{2} = \dfrac{z}{12} \Rightarrow$

$\Rightarrow z = 6$

$2z + y = 22 \Rightarrow 12 + y = 22 \Rightarrow$

$\Rightarrow y = 10$

600. a) Prolongando o segmento de medida 3, temos:



$\text{sen } 60° = \dfrac{3\sqrt{3}}{a} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{3\sqrt{3}}{a} \Rightarrow$

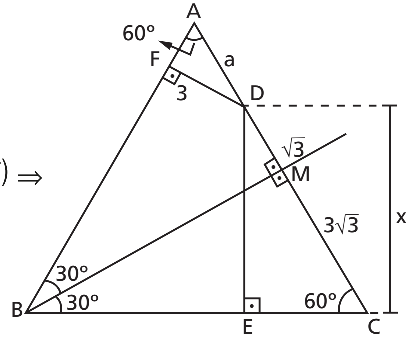
$\Rightarrow a = 6$

$\text{tg } 30° = \dfrac{9}{b} \Rightarrow \dfrac{\sqrt{3}}{3} = \dfrac{9}{b} \Rightarrow$

$\Rightarrow b = 9\sqrt{3}$

$x^2 = b^2 + 3^2 \Rightarrow x^2 = (9\sqrt{3})^2 + 9 \Rightarrow x = 6\sqrt{7}$

b) Prolongando o segmento de medida $\sqrt{3}$, temos:



$\triangle ADF \Rightarrow \text{sen } 60° = \dfrac{3}{a} \Rightarrow$

$\Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{3}{a} \Rightarrow a = 2\sqrt{3} \Rightarrow$

$\Rightarrow AM = 3\sqrt{3}$

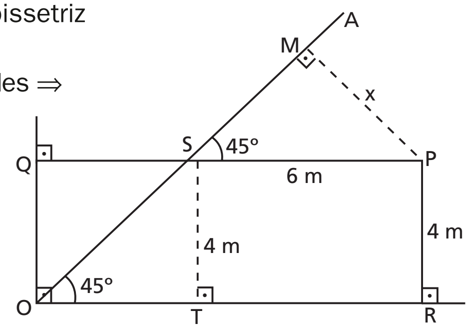
$(\triangle ABC$ é equilátero, $\overline{BM} \perp \overline{AC}) \Rightarrow$

$\Rightarrow AM = MC$

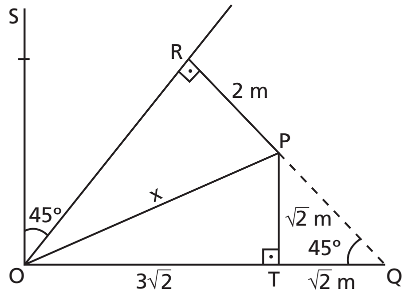
$\triangle CDE \Rightarrow \text{sen } 60° = \dfrac{DE}{CD} \Rightarrow$

$\Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{x}{4\sqrt{3}} \Rightarrow x = 6$

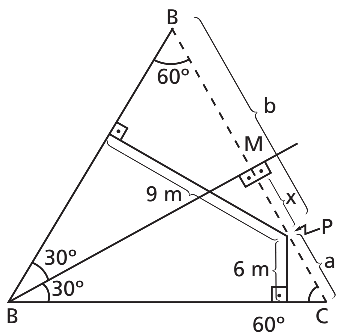
604. Na figura temos:
$PQ = 10$ m, $PR = 4$ m, $\overrightarrow{OA}$ bissetriz de $Q\hat{O}R$.
$S\hat{O}T = 45° \Rightarrow \triangle SOT$ é isósceles $\Rightarrow$
$\Rightarrow ST = TO = QS = 4$ m $\Rightarrow$
$\Rightarrow QS = 4$m $\Rightarrow PS = 6$ m
$\triangle MPS$: $\text{sen } 45° = \dfrac{x}{6} \Rightarrow$
$\Rightarrow \dfrac{\sqrt{2}}{2} = \dfrac{x}{6} \Rightarrow$
$\Rightarrow x = 3\sqrt{2}$ m



605. Na figura, $\overrightarrow{OR}$ é bissetriz de $S\hat{O}Q$.
Prolongamos o segmento de medida 2 m, formando o triângulo PQT.
Note que $O\hat{Q}R - 45° \Rightarrow \triangle PQT$ isósceles $\Rightarrow (QT = \sqrt{2}$ m, $PQ = 2$ m$)$
$PQ = 2$ m $\Rightarrow RQ = 4$ m.
$\triangle RQO$: $\text{sen } 45° = \dfrac{4}{OQ} \Rightarrow$
$\Rightarrow \dfrac{\sqrt{2}}{2} = \dfrac{4}{OQ} \Rightarrow OQ = 4\sqrt{2} \Rightarrow$
$\Rightarrow OT = 3\sqrt{2}$ m
$\triangle POT$: $x^2 = (3\sqrt{2})^2 + (\sqrt{2})^2 \Rightarrow x = 2\sqrt{5}$ m



606. Prolongando o segmento, cuja medida é procurada, até interceptar os lados do ângulo, obtemos um triângulo equilátero e os segmentos *a* e *b*.
Temos:
$\text{sen } 60° = \dfrac{6}{a} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{6}{a} \Rightarrow$
$\Rightarrow a = 4\sqrt{3}$ m
$\text{sen } 60° = \dfrac{9}{b} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{9}{b} \Rightarrow$
$\Rightarrow b = 6\sqrt{3}$ m
O lado do triângulo equilátero é igual a $a + b = 10\sqrt{3}$ m. Temos:
$MC = \dfrac{a + b}{2} \Rightarrow MC = 5\sqrt{3}$ m
$x = MC - a \Rightarrow x = 5\sqrt{3} - 4\sqrt{3} \Rightarrow x = \sqrt{3}$m



607. Prolongando o segmento de medida 6 m, obtemos o triângulo BCD com $C\hat{B}D = 30°$. Daí:

$\text{sen } 30° = \dfrac{CD}{BD} \Rightarrow \dfrac{1}{2} = \dfrac{3}{BD} \Rightarrow$

$\Rightarrow BD = 6 \text{ m}$

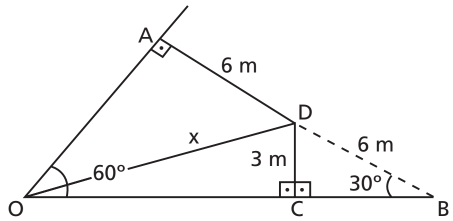
$BD = 6 \text{ m} \Rightarrow AB = 12 \text{ m}$

$\triangle AOB\text{: tg } 60° = \dfrac{AB}{AO} \Rightarrow$

$\Rightarrow \sqrt{3} = \dfrac{12}{AO} \Rightarrow AO = 4\sqrt{3}$

$\triangle AOD\text{: } x^2 = AO^2 + AD^2 \Rightarrow$

$\Rightarrow x^2 = 48 + 36 \Rightarrow x = 2\sqrt{21} \text{ m}$



608. Prolongamos o segmento cuja medida vamos determinar e obtemos o triângulo ABC. Temos:

$y^2 + 3^2 = (3\sqrt{13})^2 \Rightarrow y = 6\sqrt{3} \text{ m}$

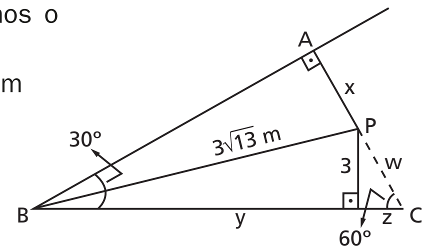
$\text{tg } 60° = \dfrac{3}{z} \Rightarrow \sqrt{3} = \dfrac{3}{z} \Rightarrow$

$\Rightarrow z = \sqrt{3} \text{ m}$

$\text{sen } 60° = \dfrac{3}{w} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{3}{w} \Rightarrow$

$\Rightarrow w = 2\sqrt{3}$

$\triangle ABC\text{: sen } 30° = \dfrac{x + w}{y + z} \Rightarrow \dfrac{1}{2} = \dfrac{x + 2\sqrt{3}}{7\sqrt{3}} \Rightarrow x = \dfrac{3\sqrt{3}}{2} \text{ m}$



609. Na figura ao lado precisamos determinar a distância PT. Temos:

$\triangle PQW \Rightarrow \text{sen } 60° = \dfrac{3\sqrt{3}}{y} \Rightarrow$

$\Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{3\sqrt{3}}{y} \Rightarrow y = 6 \text{ m}$

$\triangle PRS\text{: sen } 60° = \dfrac{9\sqrt{3}}{PS} \Rightarrow$

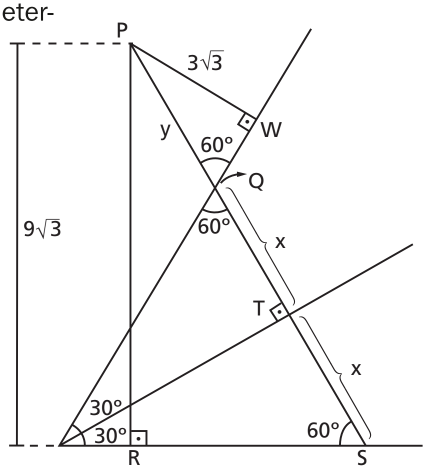
$\Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{9\sqrt{3}}{PS} \Rightarrow PS = 18 \text{ m}$

$PS = 18 \Rightarrow y + 2x = 18 \Rightarrow$

$\Rightarrow 6 + 2x = 18 \Rightarrow x = 6 \text{ m}$

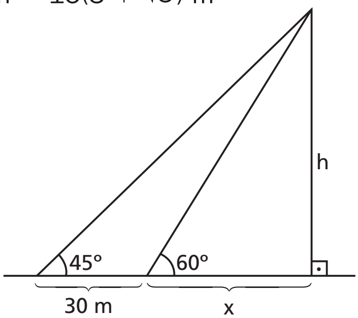
$PT = x + y \Rightarrow PT = 6 + 6 \Rightarrow$

$\Rightarrow PT = 12 \text{ m}$



612. $\left.\begin{array}{l} \text{tg } 60° = \dfrac{h}{x} \Rightarrow \sqrt{3} = \dfrac{h}{x} \Rightarrow x = \dfrac{h}{\sqrt{3}} \\ \text{tg } 45° = \dfrac{h}{30 + x} \Rightarrow 1 = \dfrac{h}{30 + x} \Rightarrow x = h - 30 \end{array}\right\} \Rightarrow$

$\Rightarrow \dfrac{h}{\sqrt{3}} = h - 30 \Rightarrow h = 15\left(3 + \sqrt{3}\right) \text{ m}$



613. $(B\hat{A}D = 45°, \hat{D} = 90°) \Rightarrow$

$\Rightarrow A\hat{B}D = 45° \Rightarrow AD = BD$

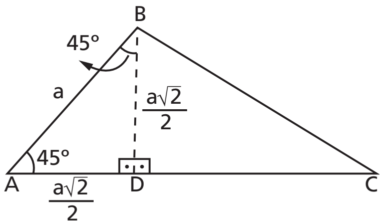
$\triangle ABD \Rightarrow a^2 = BD^2 + BD^2 \Rightarrow$

$\Rightarrow BD = \dfrac{a\sqrt{2}}{2} = AD$

$\left(AC = 2a,\ AD = \dfrac{a\sqrt{2}}{2}\right) \Rightarrow$

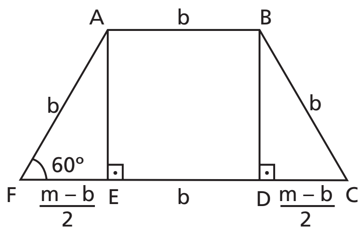
$\Rightarrow CD = 2a - \dfrac{a\sqrt{2}}{2} \Rightarrow$

$\Rightarrow CD = \dfrac{(4 - \sqrt{2})}{2} \cdot a$



$\triangle BCD: BC^2 = \left(\dfrac{a\sqrt{2}}{2}\right)^2 + \left(\dfrac{(4 - \sqrt{2})}{2} \cdot a\right)^2 \Rightarrow BC = \sqrt{5 - 2\sqrt{2}} \cdot a$

615. Seja *b* a base menor. Daí
$AF = BC = b$.
Traçando as alturas $\overline{AE}$ e $\overline{BD}$, temos
$DE = b$ e $CD = EF = \dfrac{m - b}{2}$.



$\triangle AEF: \cos 60° = \dfrac{\dfrac{(m - b)}{2}}{b} \Rightarrow \dfrac{1}{2} = \dfrac{\dfrac{(m - b)}{2}}{b} \Rightarrow b = \dfrac{m}{2}$

$2p = 3 \cdot b + m \Rightarrow 2p = \dfrac{3m}{2} + m \Rightarrow 2p = \dfrac{5m}{2}$

616. Sejam B e *b* as bases maior e menor, respectivamente.

Traçando as alturas $\overline{PR}$ e $\overline{TS}$, temos:

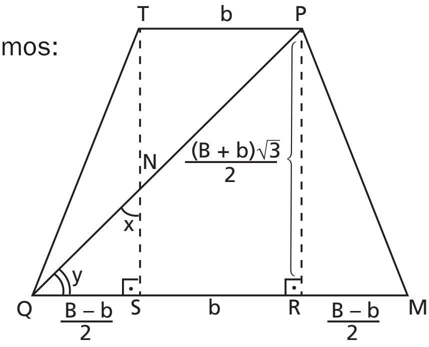
$QS = RM = \dfrac{B - b}{2}$ e

$QR = \dfrac{B + b}{2}$

$\triangle PQR$: $\text{tg } y = \dfrac{\left(\dfrac{b + b}{2}\right)\sqrt{3}}{\dfrac{(B + b)}{2}} \Rightarrow$

$\Rightarrow \text{tg } y = \sqrt{3} \Rightarrow y = 60°$

$\triangle QSN \Rightarrow x = 30°$



618. Seja ℓ a medida do lado do quadrado. Traçamos $\overline{EF}$ tal que $\overline{EF} \parallel \overline{AB}$.

Temos:

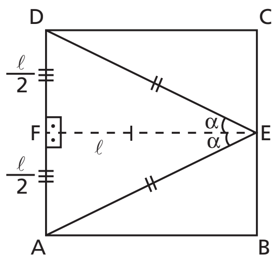
$\triangle CDE \equiv \triangle BAE$ (LAL) $\Rightarrow$

$\Rightarrow \overline{DE} \equiv \overline{AE}$

$\triangle DEF \equiv AEF$ (LLL) $\Rightarrow$

$\Rightarrow D\hat{E}F = A\hat{E}F = \alpha$

$\triangle DEF$: $\text{tg } \alpha = \dfrac{\frac{\ell}{2}}{\ell} \Rightarrow \text{tg } \alpha = \dfrac{1}{2}$



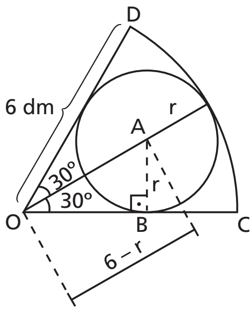
619. No $\triangle ABC \Rightarrow$

$\Rightarrow (A\hat{O}B = 30°, AB = r,$

$OA = 6 - r)$

$\text{sen } 30° = \dfrac{AB}{OA} \Rightarrow \dfrac{1}{2} = \dfrac{r}{6 - r} \Rightarrow$

$\Rightarrow r = 2$ dm



620. Na figura ao lado, precisamos determinar CD. Temos:

1) $(D\hat{B}C = \alpha, D\hat{C}B = \beta) \Rightarrow$

$\Rightarrow \alpha + \beta = 90°$

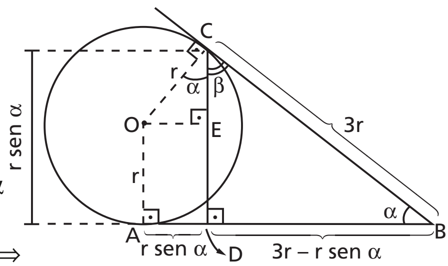
2) $E\hat{C}B = \beta \Rightarrow E\hat{C}O = \alpha$

3) $\triangle COE$: $OE = AD = r \text{ sen } \alpha$

4) $AB = 3r \Rightarrow BD = 3r - r \text{ sen } \alpha$

5) $\triangle BCD$: $CD = 3r \text{ sen } \alpha$

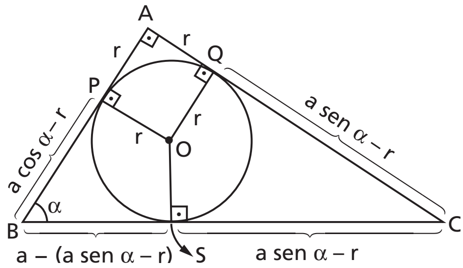
6) $\triangle BCD$: $BD^2 + CD^2 = BC^2 \Rightarrow$



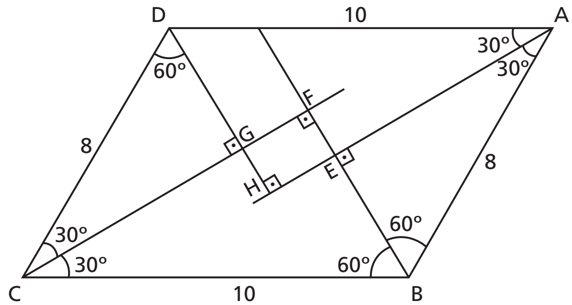
$\Rightarrow (3r - r \text{ sen } \alpha)^2 + (r \text{ sen } \alpha)^2 = 9r^2 \Rightarrow \text{sen } \alpha = \dfrac{3}{5}$

mas: $\text{sen } \alpha = \dfrac{CD}{3r} \Rightarrow \dfrac{CD}{3r} = \dfrac{3}{5} \Rightarrow CD = \dfrac{9}{5}r$

621. 1) POQA é quadrado $\Rightarrow$
$\Rightarrow PA = AQ = r$
2) $\triangle ABC$: $(AB = a \cos \alpha,$
$AC = a \text{ sen } \alpha)$
1), 2) $\Rightarrow (PB = a \cos \alpha - r,$
$QC = a \text{ sen } \alpha - r)$
3) $QC = CS = a \text{ sen } \alpha - r$
4) $BC = a \Rightarrow BS = a - (a \text{ sen } \alpha - r)$
5) $BS = BP \Rightarrow$
$\Rightarrow a - a \text{ sen } \alpha - r = a \cos \alpha - r \Rightarrow$
$\Rightarrow r = \dfrac{a}{2}(\text{sen } \alpha + \cos \alpha - 1)$



622.



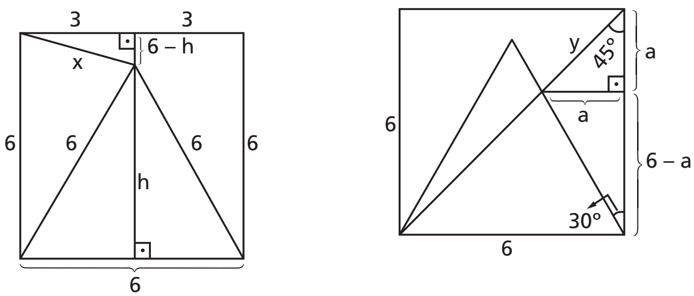
$\triangle ABE$: $(\hat{A} = 30°, \hat{B} = 60°) \Rightarrow \hat{E} = 90°$. Analogamente,
$\hat{F} = \hat{G} = \hat{H} = 90° \Rightarrow$ EFGH é retângulo.

$$\left.\begin{array}{l} \triangle BCF: \text{sen } 30° = \dfrac{BF}{BC} \Rightarrow \dfrac{1}{2} = \dfrac{BF}{10} \Rightarrow BF = 5 \\ \triangle ABE: \text{sen } 30° = \dfrac{BE}{AB} \Rightarrow \dfrac{1}{2} = \dfrac{BE}{8} \Rightarrow BE = 4 \end{array}\right\} \Rightarrow FE = GH = 1 \text{ cm}$$

$$\left.\begin{array}{l} \triangle BCF: \cos 30° = \dfrac{CF}{BC} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{CF}{10} \Rightarrow CF = 5\sqrt{3} \text{ cm} \\ \triangle CDG: \cos 30° = \dfrac{CG}{CD} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{CG}{8} \Rightarrow CG = 4\sqrt{3} \text{ cm} \end{array}\right\} \Rightarrow FG = EH = \sqrt{3} \text{ cm}$$

Sendo 2p o perímetro de EFGH, temos $2p = 2(\sqrt{3} + 1)$ cm.

623. a)



Considerando as medidas indicadas na figura, temos:

$h = \dfrac{6\sqrt{3}}{2} \Rightarrow h = 3\sqrt{3}$

$x^2 = 3^2 + (6 - h)^2 \Rightarrow$

$\Rightarrow x^2 = 9 + (6 - 3\sqrt{3})^2 \Rightarrow$

$\Rightarrow x = 6\sqrt{2 - \sqrt{3}} \Rightarrow$

$\Rightarrow x = 3(\sqrt{6} - \sqrt{2})$

$\text{tg } 30° = \dfrac{a}{6 - a} \Rightarrow$

$\Rightarrow \dfrac{\sqrt{3}}{3} = \dfrac{a}{6 - a} \Rightarrow$

$\Rightarrow a = 3(\sqrt{3} - 1)$

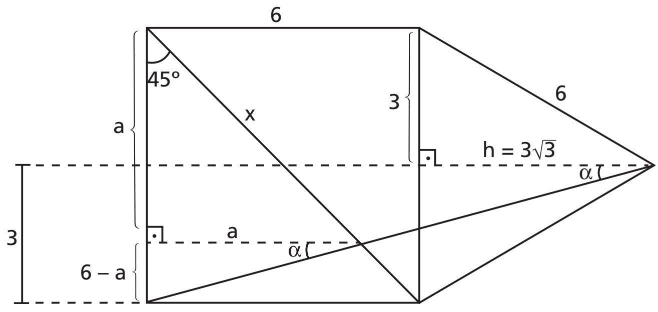
$\text{sen } 45° = \dfrac{a}{y} \Rightarrow$

$\Rightarrow \dfrac{\sqrt{2}}{2} = \dfrac{3(\sqrt{3} - 1)}{y} \Rightarrow$

$\Rightarrow y = \dfrac{6(\sqrt{3} - 1)}{\sqrt{2}} \Rightarrow$

$\Rightarrow y = 3(\sqrt{6} - \sqrt{2})$

b)



Considerando as medidas indicadas na figura, temos:

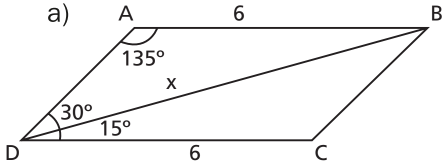
$$\left.\begin{array}{l} \text{tg } \alpha = \dfrac{6 - a}{a} \\ \text{tg } \alpha = \dfrac{3}{6 + 3\sqrt{3}} \end{array}\right\} \Rightarrow \dfrac{6 - a}{a} = \dfrac{1}{3 + \sqrt{3}} \Rightarrow a = 3 + \sqrt{3}$$

$$\text{sen } 45° = \dfrac{a}{x} \Rightarrow \dfrac{\sqrt{2}}{2} = \dfrac{3 + \sqrt{3}}{x} \Rightarrow x = 3\sqrt{2} + \sqrt{6}$$

CAPÍTULO XV — Triângulos quaisquer

Teorema dos senos

627. a)

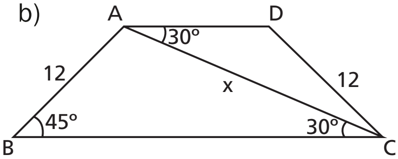


Paralelogramo ABCD $\Rightarrow$
$\Rightarrow$ ($\hat{A} = 135°$, AB $= 6$)

$\triangle ABD: \dfrac{x}{\text{sen } 135°} = \dfrac{6}{\text{sen } 30°} \Rightarrow$

$\Rightarrow \dfrac{x}{\frac{\sqrt{2}}{2}} = \dfrac{6}{\frac{1}{2}} \Rightarrow x = 6\sqrt{2}$

b)

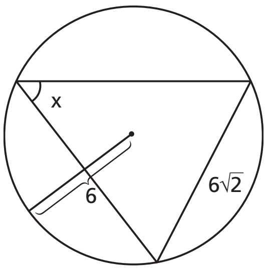


ABCD trapézio $\Rightarrow$
$\Rightarrow D\hat{A}C = B\hat{C}A$ (alternos)

$\triangle ABC: \dfrac{x}{\text{sen } 45°} = \dfrac{12}{\text{sen } 30°} \Rightarrow$

$\Rightarrow \dfrac{x}{\frac{\sqrt{2}}{2}} = \dfrac{12}{\frac{1}{2}} \Rightarrow x = 12\sqrt{2}$

628. b)



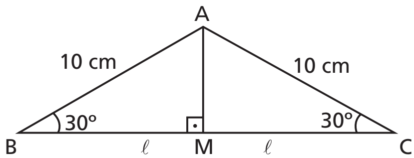
$\dfrac{6\sqrt{2}}{\text{sen } x} = 2R \Rightarrow \dfrac{6\sqrt{2}}{\text{sen } x} = 12 \Rightarrow$

$\Rightarrow \text{sen } x = \dfrac{\sqrt{2}}{2} \Rightarrow x = 45°$

630. $\triangle ABM \Rightarrow \ell = 10 \cdot \cos 30° \Rightarrow$

$\Rightarrow \ell = 10 \cdot \dfrac{\sqrt{3}}{2} \Rightarrow \ell = 5\sqrt{3}$

Logo: BC $= 10\sqrt{3}$ cm.



632. Devemos provar que $\dfrac{a + b}{b} = \dfrac{\text{sen A} + \text{sen B}}{\text{sen B}}$.

Da lei dos senos, temos:

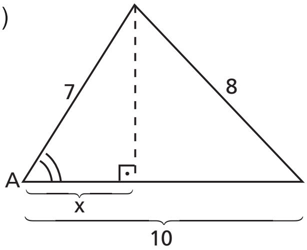
$\dfrac{a}{\text{sen A}} = \dfrac{b}{\text{sen B}} = \dfrac{c}{\text{sen C}} = 2R$

Daí: $a = 2R \text{ sen A}$, $b = 2R \text{ sen B}$, $c = 2R \text{ sen C}$.

Logo: $\dfrac{a + b}{b} = \dfrac{2R \text{ sen A} + 2R \text{ sen B}}{2R \text{ sen B}} = \dfrac{\text{sen A} + \text{sen B}}{\text{sen B}}$

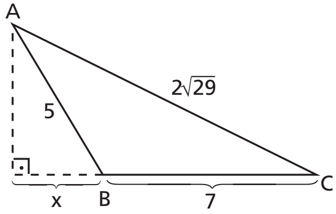
Relações métricas — Teorema dos cossenos

635. a)



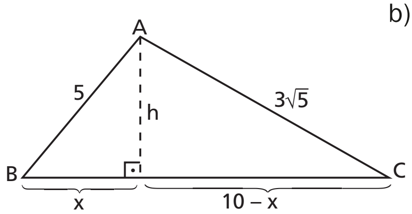
$\hat{A}$ é agudo $\Rightarrow$
$\Rightarrow 8^2 = 7^2 + 10^2 - 2 \cdot 10x \Rightarrow$
$\Rightarrow x = \dfrac{17}{4}$

b)



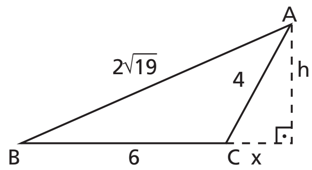
$A\hat{B}C$ é obtuso $\Rightarrow$
$\Rightarrow (2\sqrt{29})^2 = 5^2 + 7^2 +$
$+ 2 \cdot 7 \cdot x \Rightarrow x = 3$

637. a)



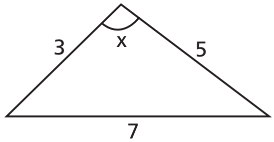
$\hat{B}$ é agudo $\Rightarrow$
$\Rightarrow (3\sqrt{5})^2 = 10^2 + 5^2 -$
$- 2 \cdot 10 \cdot x \Rightarrow x = 4$
$h^2 + 4^2 = 5^2 \Rightarrow h = 3$

b)

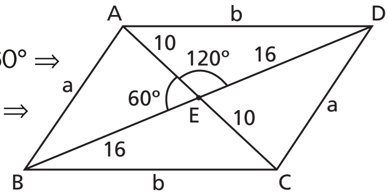


$\triangle ABC$, $\hat{C}$ é obtuso $\Rightarrow$
$\Rightarrow (2\sqrt{19})^2 = 4^2 + 6^2 +$
$+ 2 \cdot 6 \cdot x \Rightarrow x = 2$
$h^2 + 2^2 = 4^2 \Rightarrow h = 2\sqrt{3}$

639. b) Pela lei dos cossenos, temos:
$7^2 = 3^2 + 5^2 - 2 \cdot 3 \cdot 5 \cdot \cos x \Rightarrow$
$\Rightarrow \cos x = -\dfrac{1}{2} \Rightarrow x = 120°$



640. $\triangle ABE \Rightarrow$
$\Rightarrow a^2 = 10^2 + 16^2 - 2 \cdot 10 \cdot 16 \cdot \cos 60° \Rightarrow$
$\Rightarrow a^2 = 100 + 256 - 2 \cdot 10 \cdot 16 \cdot \dfrac{1}{2} \Rightarrow$
$\Rightarrow a = 14$ cm

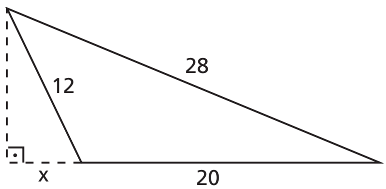


$\triangle ADE \Rightarrow$
$\Rightarrow b^2 = 10^2 + 16^2 - 2 \cdot 10 \cdot 16 \cdot \cos 120° \Rightarrow$
$\Rightarrow b^2 = 100 + 256 - 2 \cdot 10 \cdot 16 \cdot \left(-\dfrac{1}{2}\right) \Rightarrow b = 2\sqrt{129}$ cm

641. d) Os lados são da forma 3k, 4k e 4,5k ou 6k, 8k e 9k.
Temos: $(9k)^2 = 81k^2 < (6k)^2 + (8k)^2 = 100k^2 \Rightarrow$ o triângulo é acutângulo.

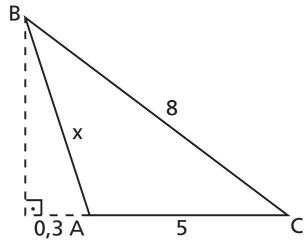
e) Os lados são da forma $\frac{k}{3}, \frac{k}{4}, \frac{k}{6}$ ou 4k, 3k, 2k.
Temos: $(4k)^2 = 16k^2 > (3k)^2 + (2k)^2 = 13k^2 \Rightarrow$ o triângulo é obtusângulo.

644. $(28^2 = 784,\ 12^2 + 20^2 = 544) \Rightarrow$
$\Rightarrow 28^2 > 12^2 + 20^2 \Rightarrow$
$\Rightarrow$ o triângulo é obtusângulo.
Aplicando relações métricas:
$28^2 = 12^2 + 20^2 + 2 \cdot 20 \cdot x \Rightarrow$
$\Rightarrow x = 6$ m

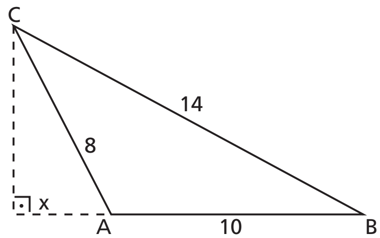


646. $\triangle ABC$ é acutângulo $\Rightarrow x^2 = 7^2 + 5^2 - 2 \cdot 5 \cdot 1 \Rightarrow x^2 = 64 \Rightarrow x = 8$ cm.

647. Usando as medidas em cm, temos a projeção de $\overline{AB}$ sobre $\overline{AC}$ igual a 0,3 cm.
Daí:
$8^2 = 5^2 + x^2 + 2 \cdot 5 \cdot 0{,}3 \Rightarrow$
$\Rightarrow x^2 = 64 - 25 - 3 \Rightarrow x = 6$ cm.



649. $(14^2 = 196;\ 8^2 + 10^2 = 164) \Rightarrow$
$\Rightarrow 14^2 > 8^2 + 10^2 \Rightarrow$
$\Rightarrow$ o triângulo é obtusângulo e temos da figura ao lado:
$14^2 = 8^2 + 10^2 + 2 \cdot 10 \cdot x \Rightarrow$
$\Rightarrow x = \frac{8}{5}$ cm.

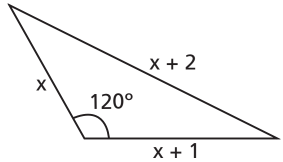


A projeção de AC sobre a base AB mede $\frac{8}{5}$ cm.
A projeção de BC sobre a base AB mede $10 + \frac{8}{5} = \frac{58}{5}$ cm.

655. Aplicamos a lei dos cossenos:
$(x + 2)^2 = x^2 + (x + 1)^2 - 2 \cdot x(x + 1) \cdot \cos 120° \Rightarrow$
$\Rightarrow (x + 2)^2 = x^2 + (x + 1)^2 - 2 \cdot x(x + 1) \cdot \left(-\frac{1}{2}\right) \Rightarrow$
$\Rightarrow 2x^2 - x - 3 = 0 \Rightarrow x = -1$ (não serve) ou $x = \frac{3}{2}$.

Temos:

$p = x + 2 + x + 1 + x \Rightarrow p = \frac{3}{2} + 2 + \frac{3}{2} + 1 + \frac{3}{2} \Rightarrow p = \frac{15}{2} \Rightarrow p = 7{,}5.$

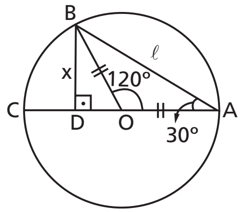


657. Na figura, $\triangle AOB$ é isósceles $\Rightarrow$

$\Rightarrow O\hat{A}B = 30°$

$\triangle ABD \Rightarrow \text{sen } 30° = \frac{x}{\ell} \Rightarrow$

$\Rightarrow \frac{1}{2} = \frac{x}{\ell} \Rightarrow x = \frac{\ell}{2}$



658. Seja R o raio do círculo. Temos:

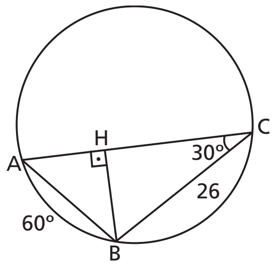
$AB = R \Rightarrow \triangle AOB$ equilátero $\Rightarrow$

$\Rightarrow \overset{\frown}{AB} = 60°$

$A\hat{C}B = \frac{\overset{\frown}{AB}}{2} \Rightarrow A\hat{C}B = 30°$

$\triangle BHC$: $\cos 30° = \frac{HC}{BC} \Rightarrow$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{HC}{26} \Rightarrow HC = 13\sqrt{3}$



660. $(6^2 = 36,\ 4^2 + 5^2 = 41) \Rightarrow$

$\Rightarrow 6^2 < 4^2 + 5^2 \Rightarrow$

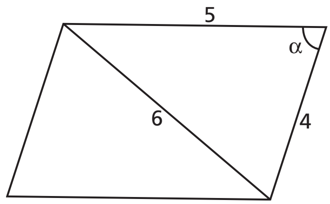
$\Rightarrow$ o triângulo de lados 4, 5 e 6 é acutângulo $\Rightarrow$

$\Rightarrow$ a diagonal de medida 6 é oposta ao ângulo agudo do paralelogramo.

Pela lei dos cossenos:

$6^2 = 4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cdot \cos \alpha \Rightarrow$

$\Rightarrow \cos \alpha = \frac{1}{8}.$

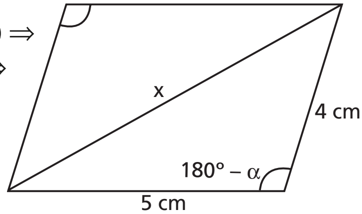


Agora,

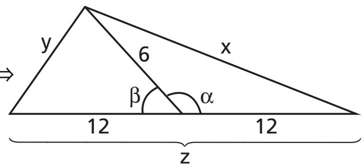
$x^2 = 4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cdot \cos (180° - \alpha) \Rightarrow$

$\Rightarrow x^2 = 16 + 25 - 2 \cdot 4 \cdot 5 \cdot (-\cos \alpha) \Rightarrow$

$\Rightarrow x^2 = 41 + 40 \cdot \frac{1}{8} \Rightarrow x = \sqrt{46}$ cm.



662. $\left.\begin{array}{l}\alpha + \beta = 180° \\ \alpha - \beta = 60°\end{array}\right\} \Rightarrow \alpha = 120°,\ \beta = 60°$



$x^2 = 6^2 + 12^2 - 2 \cdot 6 \cdot 12 \cdot \cos 120° \Rightarrow$

$\Rightarrow x = 6\sqrt{7}$ cm

$y^2 = 6^2 + 12^2 - 2 \cdot 6 \cdot 12 \cdot \cos 60° \Rightarrow$

$\Rightarrow x = 6\sqrt{3}$ cm

$z = 12 + 12 \Rightarrow z = 24$ cm.

663. Pela lei dos cossenos, temos:

$AC^2 = AB^2 + BC^2 - 2(AB)(BC) \cdot \cos \beta \Rightarrow$

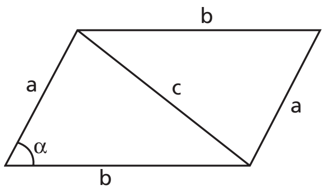
$\Rightarrow 1 = AB^2 + 100 - 2 \cdot AB \cdot 10 \cdot \dfrac{\sqrt{3}}{2} \Rightarrow$

$\Rightarrow AB^2 - 10\sqrt{3}AB + 99 = 0 \Rightarrow$ não possui solução real.

Resposta: não existe o triângulo com as medidas indicadas.

664. $c^2 = a^2 + b^2 - 2ab \cos \alpha \Rightarrow$

$\Rightarrow \cos \alpha = \dfrac{a^2 + b^2 - c^2}{2ab}$

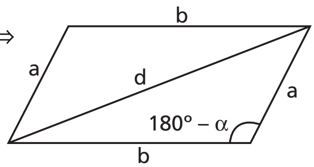


Seja a medida da outra diagonal igual a *d*.

$d^2 = a^2 + b^2 - 2ab \cos (180° - \alpha) \Rightarrow$

$\Rightarrow d^2 = a^2 + b^2 - 2ab\,(-\cos \alpha) \Rightarrow$

$\Rightarrow d^2 = a^2 + b^2 + \cancel{2ab} \cdot \dfrac{a^2 + b^2 - c^2}{\cancel{2ab}} \Rightarrow$



$\Rightarrow d^2 = 2a^2 + 2b^2 - c^2 \Rightarrow$

$\Rightarrow d = \sqrt{2a^2 + 2b^2 - c^2}$

668. Sejam P, Q e R os centros dos quadrados.

PA e PB medem metade da diagonal do quadrado de lado 6 cm.

Então, $PA = PB = 3\sqrt{2}$ cm. Analogamente, $QA = QC = 3\sqrt{6}$ cm.

Observando os ângulos formados no vértice, pode-se concluir que os pontos P, A e Q estão alinhados; logo, $PQ = 3\left(\sqrt{2} + \sqrt{6}\right)$ cm.

Agora, no triângulo ABC temos:

$\text{sen } \hat{B} = \dfrac{6\sqrt{3}}{12} \Rightarrow \text{sen } \hat{B} = \dfrac{\sqrt{3}}{2} \Rightarrow \hat{B} = 60° \Rightarrow \hat{C} = 30°.$

Aplicando a lei dos cossenos ao triângulo PBR:

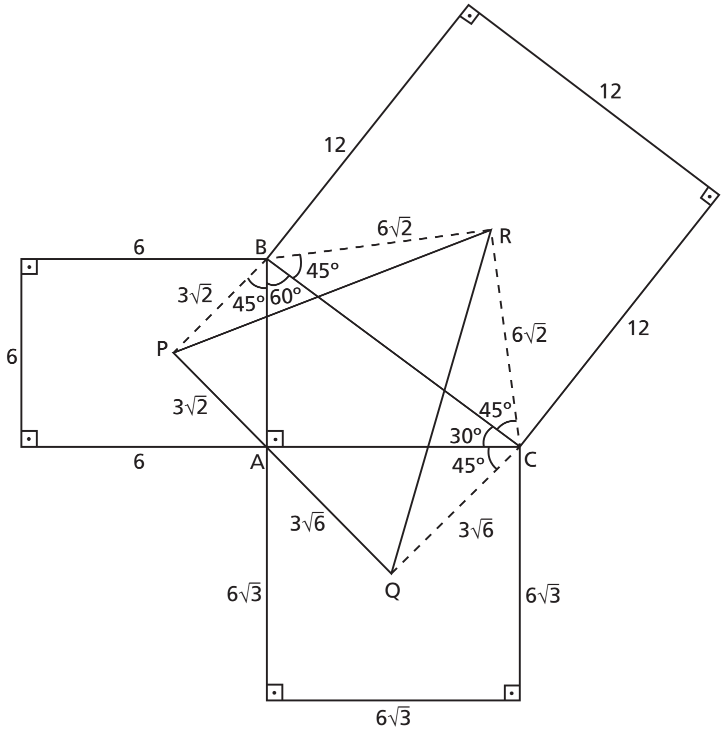
$PR^2 = PB^2 + BR^2 - 2 \cdot (PB)(BR) \cdot \cos \hat{B} \Rightarrow$

$\Rightarrow PR^2 = \left(3\sqrt{2}\right)^2 + \left(6\sqrt{2}\right)^2 - 2 \cdot 3\sqrt{2} \cdot 6\sqrt{2} \cdot \cos 150° \Rightarrow$

$\Rightarrow PR^2 = 18 + 72 - 72 \cdot (-\cos 30°) \Rightarrow PR = 3\sqrt{10 + 4\sqrt{3}}$ cm.

Aplicando a lei dos cossenos ao triângulo QCR:

$QR^2 = QC^2 + CR^2 - 2(QC)(CR)\cos\hat{C} \Rightarrow$
$\Rightarrow QR^2 = \left(3\sqrt{6}\right)^2 + \left(6\sqrt{2}\right)^2 - 2 \cdot 3\sqrt{6} \cdot 6\sqrt{2} \cdot \cos 120° \Rightarrow$
$\Rightarrow QR^2 = 54 + 72 - 72 \cdot \sqrt{3}(-\cos 60°) \Rightarrow QR = 3\sqrt{14 + 4\sqrt{3}}$ cm.



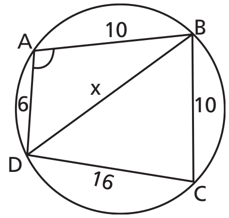
669. Quadrilátero ABCD é inscrito $\Rightarrow \hat{A} + \hat{C} = 180°$

$$\left.\begin{array}{l} \triangle ABD \Rightarrow x^2 = 6^2 + 10^2 - 2 \cdot 6 \cdot 10 \cdot \cos\hat{A}\ (1) \\ \triangle BCD \Rightarrow x^2 = 10^2 + 16^2 - 2 \cdot 10 \cdot 16 \cdot \cos(180° - A) \end{array}\right\} \Rightarrow$$

$\Rightarrow 6^2 + 10^2 - 120\cos\hat{A} = 10^2 + 16^2 - 320(-\cos\hat{A}) \Rightarrow$

$\Rightarrow \cos\hat{A} = \frac{1}{2}$. Substituindo em (1):

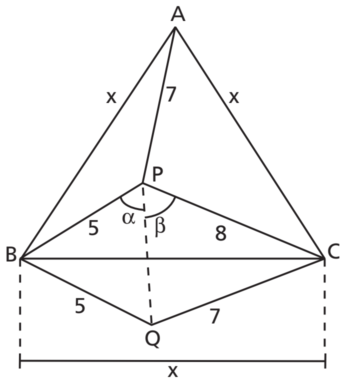
$x^2 = 6^2 + 10^2 - 120 \cdot \left(-\frac{1}{2}\right) \Rightarrow x = 14$ m.



670. 1) Considere um ponto Q externo ao triângulo, tal que BQ = 5 e CQ = = 7. Note que $\triangle PBA \equiv \triangle QBC$ (LLL), donde se obtém: $P\hat{B}Q = 60°$. Então, $\triangle PBQ$ é equilátero. Logo, $B\hat{P}Q = 60°$ ($\alpha = 60°$).

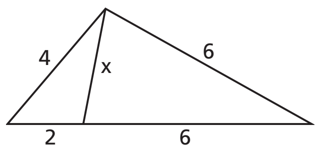
2) Aplicando a lei dos cossenos no $\triangle PQC$, temos:
$\beta = 60°$.

3) $(\alpha = 60°, \beta = 60°) \Rightarrow$
$\Rightarrow B\hat{P}C = 120°$.
Aplicando a lei dos cossenos no $\triangle BPC$, temos:
$BC = \sqrt{129} \Rightarrow x = \sqrt{129}$ cm.



Linhas notáveis — Relações de Stewart

674. a) Usando a relação de Stewart:
$4^2 \cdot 6 + 6^2 \cdot 2 - x^2 \cdot 8 = 8 \cdot 2 \cdot 6 \Rightarrow$
$\Rightarrow 96 + 72 - 8x^2 = 96 \Rightarrow x = 3.$



675. $m_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$; $m_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$;

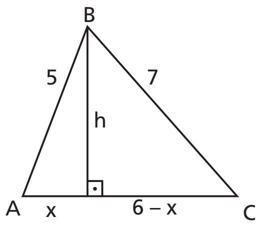
$m_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$$m_a^2 + m_b^2 + m_c^2 = \frac{1}{4}[2(b^2 + c^2) - a^2] + \frac{1}{4}[2(a^2 + c^2) - b^2] +$$
$$+ \frac{1}{4}[2(a^2 + b^2) - c^2] \Rightarrow$$
$$\Rightarrow m_a^2 + m_b^2 + m_c^2 = \frac{1}{4}[2b^2 + 2c^2 - a^2 + 2a^2 + 2c^2 - b^2 +$$
$$+ 2a^2 + 2b^2 - c^2] \Rightarrow$$
$$\Rightarrow m_a^2 + m_b^2 + m_c^2 = \frac{1}{4}[3a^2 + 3b^2 + 3c^2] = \frac{3}{4}(a^2 + b^2 + c^2)$$

Logo, $\frac{m_a^2 + m_b^2 + m_c^2}{a^2 + b^2 + c^2} = \frac{3}{4}$.

677. 1) $\begin{cases} x^2 + h^2 = 25 \\ (6 - x)^2 + h^2 = 49 \end{cases} \Rightarrow \begin{cases} h^2 = 25 - x^2 \\ h^2 = 49 - (6 - x)^2 \end{cases} \Rightarrow$

$\Rightarrow 25 - x^2 = 49 - (6 - x)^2 \Rightarrow$
$\Rightarrow x = 1$
$h^2 + x^2 = 25 \Rightarrow h^2 + 1 = 25 \Rightarrow$
$\Rightarrow h = 2\sqrt{6}$ cm

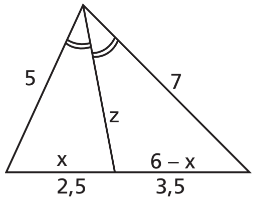


2) Teorema da bissetriz interna:

$\dfrac{x}{5} = \dfrac{6 - x}{7} \Rightarrow x = 2{,}5$ cm

Relação de Stewart:

$5^2 \cdot 3{,}5 + 7^2 \cdot 2{,}5 - z^2 \cdot 6 =$
$= 6 \cdot 2{,}5 \cdot 3{,}5 \Rightarrow$
$\Rightarrow z^2 = \dfrac{1575}{60} \Rightarrow z = \dfrac{\sqrt{105}}{2}$ cm

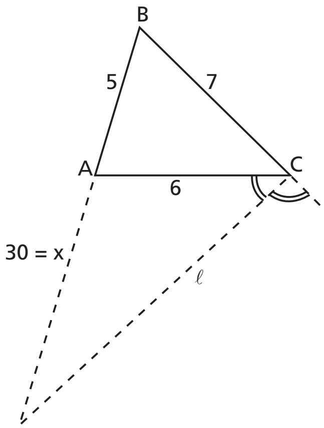


3) Teorema da bissetriz externa:

$\dfrac{5 + x}{7} = \dfrac{x}{6} \Rightarrow x = 30$

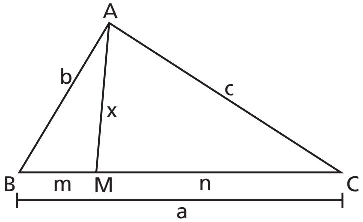
Relação de Stewart:

$\ell^2 \cdot 5 + 7^2 \cdot 30 - 6^2 \cdot 35 =$
$= 35 \cdot 5 \cdot 30 \Rightarrow$
$\Rightarrow 5\ell^2 = 5\,040 \Rightarrow \ell = 12\sqrt{7}$ cm



678. Aplicando a relação de Stewart:

$b^2n + c^2m - x^2(m + n) = \underbrace{(m + n)}_{a} \cdot m \cdot n \Rightarrow$



$\Rightarrow x^2(m + n) = b^2n + c^2m - amn.$

Multiplicando ambos os membros por $(m + n)$:

$x^2(m + n)^2 = b^2n(m + n) + c^2m(m + n) - amn\underbrace{(m + n)}_{a} \Rightarrow$

$\Rightarrow x^2(m + n)^2 = b^2n^2 + c^2m^2 + b^2mn + c^2mn - a^2mn \Rightarrow$

$$\Rightarrow x^2 = \frac{\sqrt{b^2n^2 + c^2m^2 + mn(b^2 + c^2 - a^2)}}{m + n}$$

679. Teor. biss. interna $\Rightarrow \frac{3}{x} = \frac{4}{y} \Rightarrow$

$\Rightarrow 4x = 3y \Rightarrow x^2 = \frac{9}{16}y^2$

Aplicando a relação de Stewart, vem:

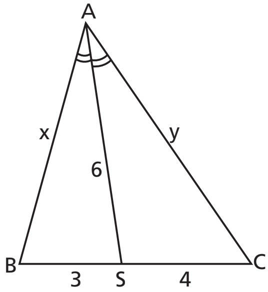
$x^2 \cdot 4 + y^2 \cdot 3 - 6^2 \cdot 7 = 7 \cdot 3 \cdot 4 \Rightarrow$

$\Rightarrow \frac{9}{16}y^2 \cdot 4 + 3y^2 - 252 = 84 \Rightarrow$

$\Rightarrow y = 8$

$x^2 = \frac{9}{16}y^2 \Rightarrow x^2 = \frac{9}{16} \cdot 64 \Rightarrow$

$\Rightarrow x = 6$



680. Teor. biss. externa $\Rightarrow \frac{36}{x} = \frac{18}{y} \Rightarrow$

$\Rightarrow x = 2y \Rightarrow x^2 = 4y^2$

Com a relação de Stewart, temos:

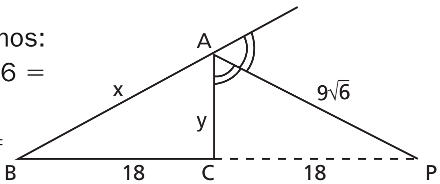
$(9\sqrt{6})^2 \cdot 18 + x^2 \cdot 18 - y^2 \cdot 36 =$

$= 36 \cdot 18 \cdot 18 \Rightarrow$

$\Rightarrow 8\,748 + 4y^2 \cdot 18 - 36y^2 =$

$= 11\,664 \Rightarrow$

$\Rightarrow 36y^2 = 2\,916 \Rightarrow y = 9 \Rightarrow x = 18$



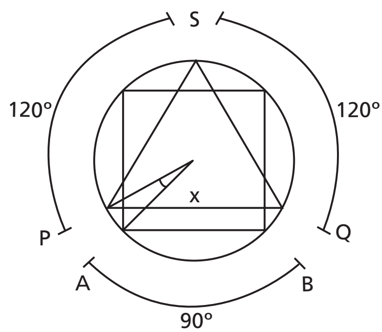
CAPÍTULO XVI — Polígonos regulares

686. Note que $\widehat{PS} = \widehat{QS} = 120°$ e

$\widehat{AB} = 90° \Rightarrow \widehat{PA} + \widehat{QB} = 60°$

$\left.\begin{array}{l} \overline{PQ} \parallel \overline{AB} \Rightarrow \widehat{PA} = \widehat{PB} \\ \widehat{PA} + \widehat{QB} = 60° \end{array}\right\} \Rightarrow$

$\Rightarrow \widehat{PA} = 30° \Rightarrow x = 15°$



687. $\overline{AB}$ é lado do pentadecágono regular $\Rightarrow$

$\Rightarrow \widehat{AB} = \dfrac{360°}{15} = \widehat{AB} = 24°$

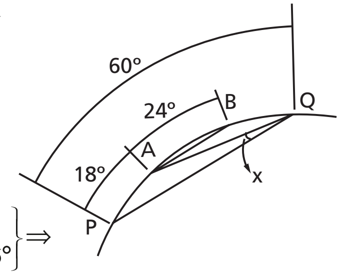
$\overline{PQ}$ é lado do hexágono regular $\Rightarrow$

$\Rightarrow \widehat{PQ} = \dfrac{360°}{6} \Rightarrow \widehat{PQ} = 60°$

$\left.\begin{array}{l} \overline{AB} \,//\, \overline{PQ} \Rightarrow \widehat{AP} = \widehat{BQ} \\ \widehat{AP} + \widehat{AB} + \widehat{BQ} = 60° \Rightarrow \widehat{AP} = \widehat{BQ} = 36° \end{array}\right\} \Rightarrow$

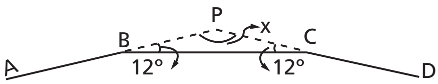
$\Rightarrow \widehat{AP} = 18°$

$A\hat{Q}P$ é inscrito e subtende $\widehat{AP} \Rightarrow A\hat{Q}P = 9°$.



689. $a_e = \dfrac{360°}{30°} \Rightarrow a_e = 12$

$\triangle BCP \Rightarrow \hat{P} = 156°$

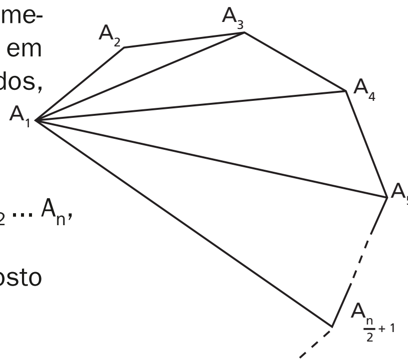


691. g) O número de diagonais com medidas duas a duas diferentes em um polígono regular de *n* lados, *n* par, é dado por:

$$\frac{n-2}{2}$$

Dedução: Seja o polígono $A_1A_2 \ldots A_n$, *n* par.

O vértice diametralmente oposto a A_1 é $A_{\frac{n}{2}+1}$.



De A_1 partem $\dfrac{n}{2} + 1 - 2$ diagonais com medidas duas a duas diferentes.

Então, $\dfrac{n}{2} + 1 - 2 = \dfrac{n-2}{2}$.

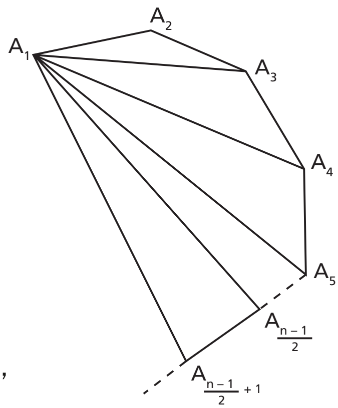
A união de A_1 com os demais vértices fornece diagonais com medidas iguais às já obtidas.

h) O número de diagonais com medidas duas a duas diferentes em um polígono regular de *n* lados, *n* ímpar, é dado por:

$$\frac{n-3}{2}$$

Dedução: Seja o polígono $A_1A_2 \ldots A_n$, *n* ímpar.

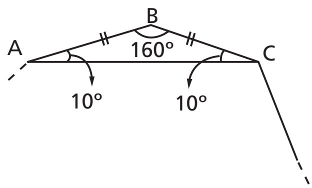
O vértice que unido a A_1 fornece a maior medida de diagonal possível é $A_{\frac{n-1}{2}+1}$.



De A_1 partem $\frac{n-1}{2} + 1 - 2$ diagonais com medidas duas a duas diferentes.
Então, $\frac{n-1}{2} + 1 - 2 = \frac{n-3}{2}$.
A união de A_1 com os demais vértices resulta em diagonais com medidas iguais às já obtidas.

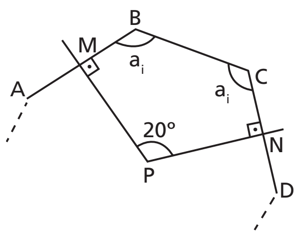
692. Temos:
$\frac{n-2}{2} = 6$ ou $\frac{n-3}{2} = 6 \Rightarrow n = 14$ ou $n = 15 \Rightarrow S_i = 2160°$ ou $S_i = 2340°$.

693. Na figura, temos:
$\overline{AB} \equiv \overline{BC} \Rightarrow \triangle ABC$ isósceles $\Rightarrow$
$\Rightarrow \hat{A} = 10°, \hat{B} = 160°$
$a_i = 160° \Rightarrow a_e = 20° \Rightarrow$
$\Rightarrow \frac{360°}{n} = 20° \Rightarrow n = 18$
$d = \frac{n(n-3)}{2} \Rightarrow$
$\Rightarrow d = \frac{18 \cdot (18-3)}{2} \Rightarrow d = 135$

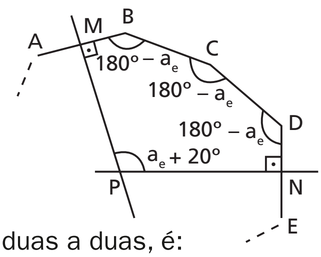


$n = 18 \Rightarrow 9$ diagonais passam pelo centro.
Logo, não passam pelo centro $135 - 9 = 126$ diagonais.

695. Polígono MBCNP: $S_i = 540° \Rightarrow$
$\Rightarrow 2a_i + 200° = 540° \Rightarrow a_i = 170°$
$a_i = 170° \Rightarrow a_e = 10° \Rightarrow$
$\Rightarrow \frac{360°}{n} = 10° \Rightarrow n = 36$
Logo, passam pelo centro $\frac{n}{2} = 18$ diagonais.



697. Soma dos ângulos internos do polígono MBCDNP é igual a 720°. Então: $3 \cdot (180° - a_e) + 180° + a_e + 20° = 720° \Rightarrow a_e = 10° \Rightarrow \frac{360°}{n} = 10° \Rightarrow n = 36$.



Logo, o número de diagonais diferentes, duas a duas, é:
$\frac{n-2}{2} = \frac{36-2}{2} = 17$.

698. Na figura, prolongamos também os lados $\overline{BC}$ e $\overline{DE}$, formando o triângulo ZCD.

Temos:

$E\hat{Y}Z$ externo ao $\triangle BXY \Rightarrow E\hat{Y}Z = 3 \cdot a_e$.

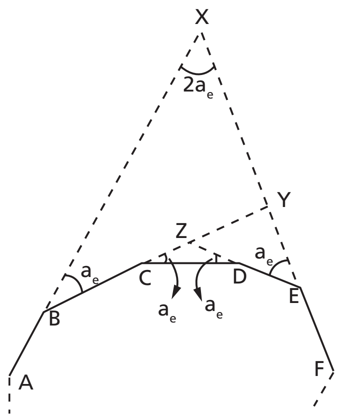
$C\hat{Z}D$ externo ao $\triangle EYZ \Rightarrow C\hat{Z}D = 4 \cdot a_e$.

$\triangle CZD \Rightarrow 6a_e = 180° \Rightarrow a_e = 30° \Rightarrow$

$\Rightarrow \dfrac{360°}{n} = 30° \Rightarrow n = 12$

$d = \dfrac{n(n-3)}{2} \Rightarrow$

$\Rightarrow d = \dfrac{12(12-3)}{2} \Rightarrow d = 54.$



702. Seja ℓ_6 o lado do hexágono.

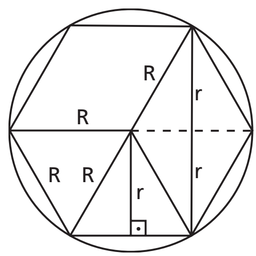
a) Se os triângulos são equiláteros, temos $R = \ell_6$.
Assim, diagonal maior $= 2R = 2\ell_6 = 12$ m.

b) R é lado do triângulo equilátero $\Rightarrow$
$\Rightarrow R = \ell_6 = 6$ m.

c) *r* é altura do triângulo equilátero $\Rightarrow$
$\Rightarrow r = \dfrac{\ell_6\sqrt{3}}{2} \Rightarrow r = \dfrac{6\sqrt{3}}{2} = r = 3\sqrt{3}$ m.

d) diagonal menor $\Rightarrow 2r = 2 \cdot 3\sqrt{3} = 6\sqrt{3}$ m.

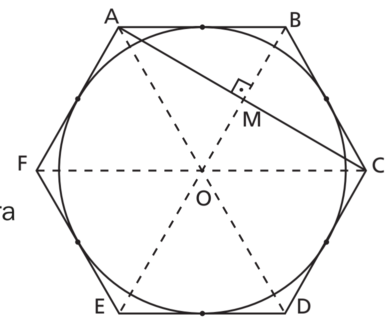
e) apótema $= r \Rightarrow a_6 = 3\sqrt{3}$ m.



710. a) $\ell_6 = R \Rightarrow R = 5 \text{ cm} \Rightarrow$
$\Rightarrow a_6 = \dfrac{R\sqrt{3}}{2} \Rightarrow a_6 = \dfrac{5\sqrt{3}}{2}$ cm

b) $r = a_6 \Rightarrow r = \dfrac{5\sqrt{3}}{2}$ cm

c) Note que $\overline{AM} \perp \overline{BC} \Rightarrow \overline{AM}$ é altura do triângulo equilátero OAB $\Rightarrow$
$\Rightarrow AC = 2\ AM \Rightarrow AC = 2 \cdot r \Rightarrow$
$\Rightarrow AC = 5\sqrt{3}$ cm.



712. Sejam R_1 e R_2 os raios dos círculos onde estão inscritos o quadrado e o triângulo equilátero, respectivamente. Temos:

$\ell_4 = R_1\sqrt{2} \Rightarrow 2p_1 = 4R_1\sqrt{2}$

$\ell_3 = R_2\sqrt{3} \Rightarrow 2p_2 = 3R_2\sqrt{3}$

$2p_1 = 2p_2 \Rightarrow 4R_1\sqrt{2} = 3R_2\sqrt{3} \Rightarrow \dfrac{R_1}{R_2} = \dfrac{4\sqrt{6}}{9}.$

714. ACEG quadrado $\Rightarrow AC = R\sqrt{2} \Rightarrow$

$\Rightarrow OM = \dfrac{R\sqrt{2}}{2} \Rightarrow$

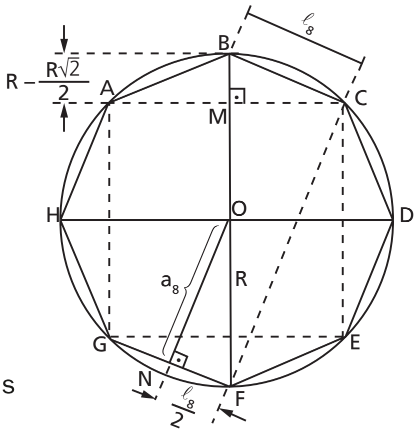
$\Rightarrow BM = R - \dfrac{R\sqrt{2}}{2}$

Aplicando relações métricas no $\triangle BCF$, retângulo em C, vem:

$(BC)^2 = (BF) \cdot (BM)$

$\ell_8^2 = 2R\left(R - \dfrac{R\sqrt{2}}{2}\right)$

$\ell_8 = R\sqrt{2 - \sqrt{2}}$.



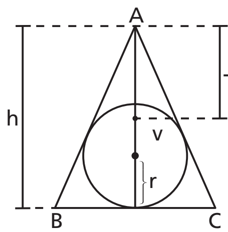
Aplicando o teorema de Pitágoras no $\triangle FNO$:

$a_8^2 = R^2 - \left(\dfrac{\ell_8}{2}\right)^2 \Rightarrow$

$\Rightarrow a_8^2 = R^2 - \dfrac{R^2\left(2 - \sqrt{2}\right)}{4} \Rightarrow a_8 = \dfrac{R\sqrt{2 + \sqrt{2}}}{2}$

716. $\left.\begin{array}{l} r = 1 \\ 2r = \dfrac{\sqrt{5} - 1}{2} \cdot h \end{array}\right\} \Rightarrow h = \sqrt{5} + 1$

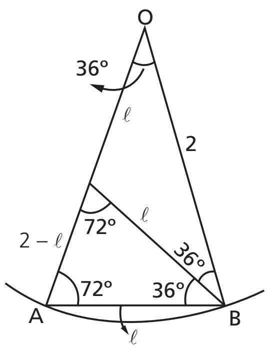
$AV = \dfrac{h}{2} \Rightarrow AV = \dfrac{\sqrt{5} + 1}{2}$



717. Aplicando o teorema da bissetriz interna no $\triangle OAB$:

$\dfrac{\ell}{2} = \dfrac{2 - \ell}{\ell} \Rightarrow \ell^2 + 2\ell - 4 = 0 \Rightarrow$

$\Rightarrow \ell = \left(\sqrt{5} - 1\right)$ m.



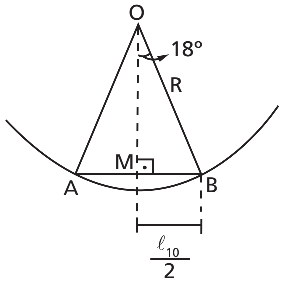
719. No $\triangle OAB$ temos:

$\overline{OM}$ bissetriz $\Rightarrow A\hat{O}B = 36° \Rightarrow$

$$\Rightarrow \left(AB = \ell_{10},\ OB = R,\ MB = \frac{\ell_{10}}{2}\right)$$

$$\text{sen } 18° = \frac{MB}{OB} = \frac{\frac{\ell_{10}}{2}}{R} = \frac{\frac{\sqrt{5}-1}{2} \cdot R}{2R} \Rightarrow$$

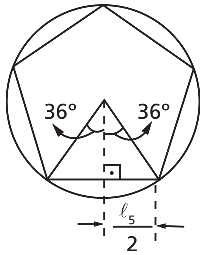
$$\Rightarrow \text{sen } 18° = \frac{\sqrt{5}-1}{4}$$



720. $$\ell_5^2 = \ell_6^2 + \ell_{10}^2 \Rightarrow \ell_5^2 = R^2 + \left(\frac{\sqrt{5}-1}{2} \cdot R\right)^2 \Rightarrow \ell_5 = \frac{R}{2}\sqrt{10 - 2\sqrt{5}}$$

721. $$\text{sen } 36° = \frac{\frac{\ell_5}{2}}{R} = \frac{\cancel{R}\sqrt{10-2\sqrt{5}}}{4\cancel{R}} \Rightarrow$$

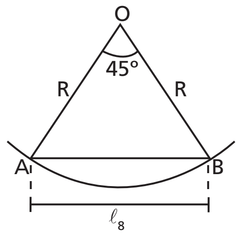
$$\Rightarrow \text{sen } 36° = \frac{\sqrt{10-2\sqrt{5}}}{4}$$



725. $$\ell_8^2 = R^2 + R^2 - 2 \cdot R \cdot R \cdot \cos 45°$$

$$\ell_8^2 = 2R^2 - 2R^2 \cdot \frac{\sqrt{2}}{2} \Rightarrow$$

$$\Rightarrow \ell_8 = R\sqrt{2 - \sqrt{2}}$$



726. $$\ell = R\sqrt{2-\sqrt{2}} \Rightarrow R = \frac{\ell}{\sqrt{2-\sqrt{2}}} \Rightarrow R = \frac{\ell \cdot \sqrt{2+\sqrt{2}} \cdot \sqrt{2}}{\sqrt{2-\sqrt{2}} \cdot \sqrt{2+\sqrt{2}} \cdot \sqrt{2}} \Rightarrow$$

$$\Rightarrow R = \frac{\ell}{2}\sqrt{4 + 2\sqrt{2}}$$

727. Temos: $a_i = 135°$.

Lei dos cossenos no $\triangle ABC$:

$$AC^2 = \ell^2 + \ell^2 - 2 \cdot \ell \cdot \ell \cdot (-\cos 45°) \Rightarrow$$

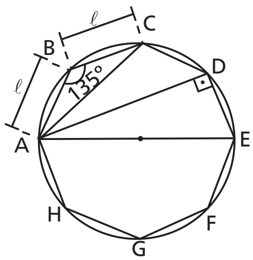
$$\Rightarrow AC^2 = 2\ell^2 + \ell^2\sqrt{2} \Rightarrow$$

$$\Rightarrow AC = \ell\sqrt{2+\sqrt{2}}$$

$$AE = 2R \xrightarrow[726]{\text{exercício}}$$

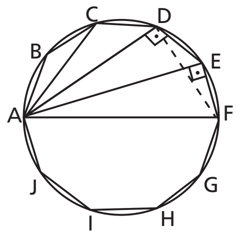
$$\Rightarrow AE = 2 \cdot \frac{\ell}{2}\sqrt{4+2\sqrt{2}} \Rightarrow$$

$$\Rightarrow AE = \ell\sqrt{4+2\sqrt{2}}$$



AE é diâmetro $\Rightarrow$
$\Rightarrow \triangle ADE$ é retângulo em D.
Aplicando Pitágoras ao $\triangle ADE$:
$AD^2 = AE^2 - DE^2 \Rightarrow$
$\Rightarrow AD^2 = \ell^2(4 + 2\sqrt{2}) - \ell^2 \Rightarrow$
$\Rightarrow AD = \ell\sqrt{3 + 2\sqrt{2}} \Rightarrow$
$\Rightarrow AD = \ell\sqrt{(2 + 2\sqrt{2} + 1)} \Rightarrow AD = \ell \cdot \sqrt{(\sqrt{2} + 1)^2} \Rightarrow AD = \ell \cdot (\sqrt{2} + 1)$

728. a) $\ell = \dfrac{\sqrt{5} - 1}{2} \cdot R \Rightarrow R = \dfrac{\ell(\sqrt{5} + 1)}{2}$

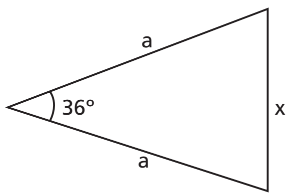


b) $\triangle AEF$ é retângulo $\Rightarrow$
$\Rightarrow AE^2 = AF^2 - EF^2 \Rightarrow$
$\Rightarrow AE^2 = (2R)^2 - \ell^2 \Rightarrow$
$\Rightarrow AE^2 = \ell^2(\sqrt{5} + 1)^2 - \ell^2 \Rightarrow$
$\Rightarrow AE^2 = \ell^2(5 + 2\sqrt{5}) \Rightarrow$
$\Rightarrow AE = \ell\sqrt{5 + 2\sqrt{5}}$

c) $\overset{\frown}{AB} + \overset{\frown}{BC} = 72° \Rightarrow AC = \ell_5 = \dfrac{\ell}{2}\sqrt{10 + 2\sqrt{5}}$

d) $\triangle ADF$ é retângulo em D, $DF = \ell_5$, $AF = (\sqrt{5} + 1)\ell$
Daí:
$AD^2 = AF^2 - DF^2 \Rightarrow AD^2 = (6 + 2\sqrt{5})\ell^2 - \dfrac{10 + 2\sqrt{5}}{4} \cdot \ell^2 \Rightarrow$
$\Rightarrow AD^2 = \dfrac{7 + 3\sqrt{5}}{2} \cdot \ell^2 \Rightarrow AD = \dfrac{\ell}{2}\sqrt{14 + 6\sqrt{5}}$

729. $x^2 = a^2 + a^2 - 2 \cdot a \cdot a \cdot \cos 36° \Rightarrow$
$\Rightarrow x^2 = 2a^2 - 2a^2 \cdot \dfrac{\sqrt{5} + 1}{4} \Rightarrow$
$\Rightarrow x^2 = \dfrac{(6 - 2\sqrt{5})}{4} \cdot a^2 \Rightarrow$
$\Rightarrow x^2 = \dfrac{(3 - \sqrt{5})}{2} \cdot a^2 \Rightarrow$
$\Rightarrow x = \dfrac{\sqrt{6 - 2\sqrt{5}}}{2} \cdot a \Rightarrow$
$\Rightarrow x = \dfrac{\sqrt{(5 - 2\sqrt{5} + 1)}}{2} \cdot a \Rightarrow$
$\Rightarrow x = \dfrac{\sqrt{(\sqrt{5} - 1)^2}}{2} \cdot a \Rightarrow x = \dfrac{\sqrt{5} - 1}{2} \cdot a$

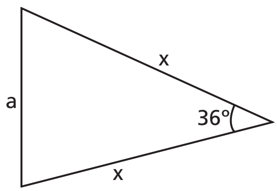


730. Usando o resultado do exercício 729:

$a = \dfrac{\sqrt{5} - 1}{2} x \Rightarrow$

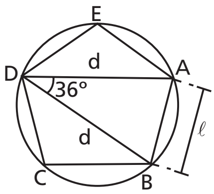
$\Rightarrow x = \dfrac{2}{\sqrt{5} - 1} \cdot a \Rightarrow$

$\Rightarrow x = \dfrac{\sqrt{5} + 1}{2} \cdot a.$



731. Usando o resultado do exercício 730 no $\triangle ABD$:

$d = \dfrac{\sqrt{5} + 1}{2} \cdot \ell.$



732. a) Os triângulos A'AB, B'BC, C'CD, D'DE e E'EA são congruentes e isósceles de bases $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$ e $\overline{EA}$. Daí:

$\hat{A}' = \hat{B}' = \hat{C}' = \hat{D}' = \hat{E}'$ (1)

e, por diferença, obtemos:

$A'B' = B'C' = C'D' = D'E' =$
$= E'A'$ (2)

(1) e (2) $\Rightarrow$ A'B'C'D'E' é pentágono regular.

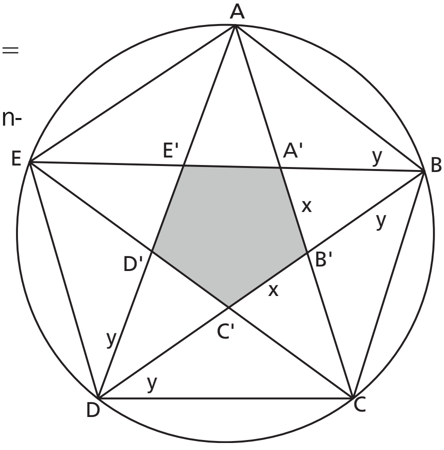
b) $\triangle A'B'B \xrightarrow[729]{\text{exercício}}$

$\Rightarrow x = \dfrac{y}{2}\left(\sqrt{5} - 1\right)$ (1)

$\overline{BD}$ é diagonal $\xrightarrow[731]{\text{exercício}}$

$\Rightarrow 2x + y = \dfrac{\ell}{2}\left(\sqrt{5} + 1\right)$ (2)

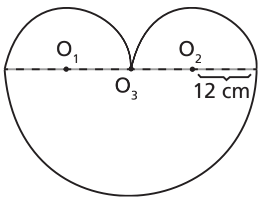
(2) $\Rightarrow y = \dfrac{\ell\left(\sqrt{5} + 1\right)}{2} - 2x.$



Substituindo em (1), obtemos $x = \dfrac{3 - \sqrt{5}}{2} \ell.$

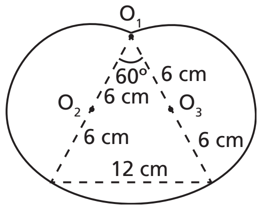
CAPÍTULO XVII — Comprimento da circunferência

735. a)



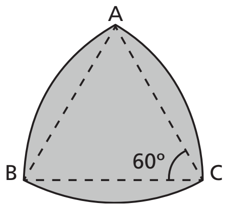
$R_1 = R_2 = 12 \Rightarrow$
$\Rightarrow C_1 = C_2 = \pi R_1 = 12\pi$
$R_3 = 24 \Rightarrow C_3 = \pi R_3 \Rightarrow$
$\Rightarrow C_3 = 24\pi$
$C_1 + C_2 + C_3 = 12\pi +$
$+ 12\pi + 24\pi \Rightarrow C_1 +$
$+ C_2 + C_3 = 48\pi$ cm

b)



$R_1 = 12$ cm
$R_2 = R_3 = 6 \Rightarrow$
$\Rightarrow C_2 = C_3 = \pi R_2 = 6\pi$
$C_1 = \dfrac{1}{6} \cdot 2\pi R_1 \Rightarrow$
$\Rightarrow C_1 = \dfrac{1}{6} \cdot 2 \cdot \pi \cdot 12 \Rightarrow$
$\Rightarrow C_1 = 4\pi$ cm
$C_1 + C_2 + C_3 = 4\pi + 6\pi +$
$+ 6\pi = 16\pi$ cm

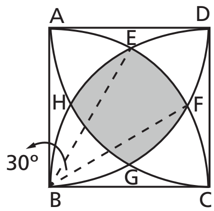
736. a)



$\triangle ABC$ é equilátero, pois seus lados são os raios dos arcos.

$A\hat{C}B = 60° \Rightarrow \overset{\frown}{AB} = \dfrac{60°}{360°} \cdot 2\pi R \Rightarrow$
$\Rightarrow \overset{\frown}{AB} = \dfrac{1}{3}\pi \cdot 12 \Rightarrow$
$\left.\begin{array}{l}\Rightarrow \overset{\frown}{AB} = 4\pi \text{ m} \\ \overset{\frown}{AB} = \overset{\frown}{AC} = \overset{\frown}{BC}\end{array}\right\} \Rightarrow$
$\Rightarrow \overset{\frown}{AB} + \overset{\frown}{AC} + \overset{\frown}{BC} = 12\pi$ m.

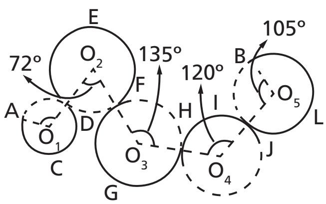
b)



Note que $E\hat{B}F = 30°$. Daí:

$\overset{\frown}{EF} = \dfrac{30°}{360°} \cdot 2\pi \cdot R \Rightarrow$
$\Rightarrow \overset{\frown}{EF} = \dfrac{1}{6} \cdot \pi \cdot 48 \Rightarrow$
$\Rightarrow \overset{\frown}{EF} = 8\pi$ m.

Logo, como $\overset{\frown}{EF} = \overset{\frown}{FG} = \overset{\frown}{GH} =$
$= \overset{\frown}{HE}$, temos:
$\overset{\frown}{EF} + \overset{\frown}{FG} + \overset{\frown}{GH} + \overset{\frown}{HE} = 32\pi$ m.

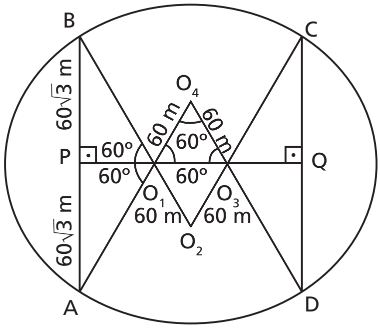


737.

$$\left.\begin{array}{l}
\overset{\frown}{ACD} = \dfrac{270°}{360°} \cdot 2\pi \cdot 18 \Rightarrow \overset{\frown}{ACD} = 27\pi \text{ cm} \\
\overset{\frown}{DEF} = \dfrac{288°}{360°} \cdot 2\pi \cdot 35 \Rightarrow \overset{\frown}{DEF} = 56\pi \text{ cm} \\
\overset{\frown}{FGH} = \dfrac{225°}{360°} \cdot 2\pi \cdot 24 \Rightarrow \overset{\frown}{FGH} = 30\pi \text{ cm} \\
\overset{\frown}{HIJ} = \dfrac{120°}{360°} \cdot 2\pi \cdot 36 \Rightarrow \overset{\frown}{HIJ} = 24\pi \text{ cm} \\
\overset{\frown}{JLB} = \dfrac{255°}{360°} \cdot 2\pi \cdot 48 \Rightarrow \overset{\frown}{JLB} = 68\pi \text{ cm}
\end{array}\right\} \Rightarrow$$

$$\Rightarrow \overset{\frown}{ACD} + \overset{\frown}{DEF} + \overset{\frown}{FGH} + \overset{\frown}{HIJ} + \overset{\frown}{JLB} = 205\pi \text{ cm}$$

738.



$$\triangle PAO_1 \Rightarrow \text{sen } 60° = \frac{AP}{O_1A} \Rightarrow \frac{\sqrt{3}}{2} = \frac{60\sqrt{3}}{O_1A} \Rightarrow O_1A = 120 \text{ m} \Rightarrow$$
$$\Rightarrow O_4A = 180 \text{ m}$$

$$\overset{\frown}{AB} = \frac{120°}{360°} \cdot 2\pi(O_1A) \Rightarrow \overset{\frown}{AB} = \frac{1}{3} \cdot 2\pi \cdot (120) \Rightarrow \overset{\frown}{AB} = 80\pi \text{ m}$$

$$\overset{\frown}{AD} = \frac{60°}{360°} \cdot 2\pi(O_4A) \Rightarrow \overset{\frown}{AD} = \frac{1}{6} \cdot 2\pi \cdot 180 \Rightarrow \overset{\frown}{AD} = 60\pi \text{ m}$$

Seja 2p o comprimento total da pista. Temos:

$$2p = \overset{\frown}{AB} + \overset{\frown}{CD} + \overset{\frown}{AD} + \overset{\frown}{BC} \Rightarrow 2p = 80\pi + 80\pi + 60\pi + 60\pi \Rightarrow$$
$$\Rightarrow 2p = 280\pi \text{ m.}$$

745. Seja C o comprimento da circunferência e C_1, C_2 e C_3 os comprimentos quando o raio é aumentado em 2 m, em 3 m e em *a* metros, respectivamente. Temos:

$C = 2\pi R$

$C_1 = 2\pi(R + 2) \Rightarrow C_1 = 2\pi R + 4\pi \Rightarrow C_1 = C + 4\pi$

$C_2 = 2\pi(R + 3) \Rightarrow C_2 = 2\pi R + 6\pi \Rightarrow C_2 = C + 6\pi$

$C_3 = 2\pi(R + a) \Rightarrow C_3 = 2\pi R + 2a\pi \Rightarrow C_3 = C + 2a\pi$

Portanto o comprimento aumenta em 4π m, 6π m e $2a\pi$ m, respectivamente.

746. $\left.\begin{array}{l} p_2 = 2\pi R_2 = 1 + 10^3 \\ p_1 = 2\pi R_1 = 10^3 \end{array}\right\} \Rightarrow 2\pi R_2 - 2\pi R_1 = 1 + 10^3 - 10^3 \Rightarrow$

$\Rightarrow R_2 - R_1 = \dfrac{1}{2\pi}$

747. Sejam o comprimento normal e o comprimento com o raio duplicado iguais a C e C_1, respectivamente. Temos:

$C = 2\pi R$

$C_1 = 2\pi(2R) \Rightarrow C_1 = 2 \cdot 2\pi R \Rightarrow C_1 = 2\pi C$

Logo, o comprimento também duplica.

748. $(\ell = 2\pi R,\ r = 2R,\ \ell = r \cdot \alpha) \Rightarrow 2\pi R = 2R \cdot \alpha \Rightarrow \alpha = \pi \Rightarrow \alpha = 180°$

750. $C \rightarrow$ comprimento normal da circunferência.

$C_1 \rightarrow$ comprimento da circunferência cujo raio aumentou 50%.

Temos:

$C = 2\pi R$

$C_1 = 2\pi(R + 0{,}5R) \Rightarrow C_1 = 2\pi R + \pi R \Rightarrow C_1 = C + \dfrac{C}{2} \Rightarrow$

$\Rightarrow C_1 = C + 0{,}5C.$

Resposta: o comprimento aumenta 50%.

755. $C_1 = 2\pi R_1 \Rightarrow 1\,500 = 2\pi R_1 \Rightarrow$

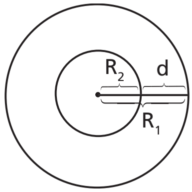
$\Rightarrow R_1 = \dfrac{750}{\pi}$ m

$C_2 = 2\pi R_2 \Rightarrow 1\,200 = 2\pi R_2 \Rightarrow$

$\Rightarrow R_2 = \dfrac{600}{\pi}$ m

$d = R_1 - R_2 \Rightarrow d = \dfrac{750}{\pi} - \dfrac{600}{\pi} \Rightarrow$

$\Rightarrow d = \dfrac{150}{\pi}$ m



756. $C = 2\pi R \Rightarrow C = 2\pi \cdot 40 \Rightarrow C = 80\pi$ cm

n: nº de voltas, 26 km $= 26 \times 10^5$ cm

$d = n \cdot C \Rightarrow n = \dfrac{d}{C} \Rightarrow n = \dfrac{26 \times 10^5}{80\pi} \Rightarrow n \cong 10\,350$ voltas

1 h 50 min = 110 min

nº de voltas/min $= \dfrac{10\,350}{110} \cong 94$

757. Sejam R_F: raio da roda dianteira; R_T: raio da roda traseira; *d*: distância percorrida. Distância percorrida quando R_F dá 25 voltas:

$d = 25 \cdot 2\pi R_F \Rightarrow d = 25 \cdot 2\pi \cdot 1 \Rightarrow d = 50\pi$ m.

Nessa distância, R_T dá 20 voltas. Então:

$d = 20 \cdot 2\pi R_T \Rightarrow 50\pi = 20 \cdot 2 \cdot \pi \cdot R_T \Rightarrow R_T = \dfrac{5}{4}$ m.

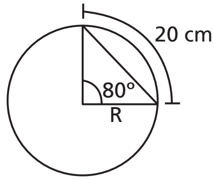
Distância percorrida depois que R_F deu 100 voltas:

$d = 100 \cdot 2\pi R_F \Rightarrow d = 100 \cdot 2\pi \cdot 1 \Rightarrow d = 200\pi$ m.

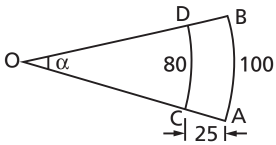
758. $C_1 = 2\pi R_1 \Rightarrow C_1 = 2\pi \cdot 1{,}5 \Rightarrow C_1 = 3\pi$ cm
$C_2 = 2\pi R_2 \Rightarrow C_2 = 2\pi \cdot 1 \Rightarrow C_2 = 2\pi$ cm
$C_1 - C_2 = 3\pi - 2\pi \Rightarrow C_1 - C_2 = \pi$ cm

765. $\left(a = 80°, \ell = 20 \text{ cm}, \ell = \dfrac{\pi\alpha R}{180°}\right) \Rightarrow$

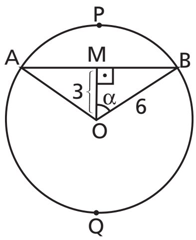
$\Rightarrow 20 = \dfrac{\pi \cdot 80° \cdot R}{180°} \Rightarrow R = \dfrac{45}{\pi}$ cm



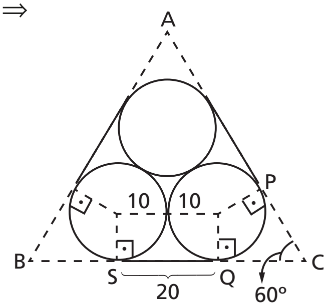
767. $(\widehat{AB} = \alpha \cdot OB;\ \widehat{CD} = \alpha \cdot OD) \Rightarrow$
$\Rightarrow \widehat{AB} - \widehat{CD} = \alpha(OB - OD) \Rightarrow$
$\Rightarrow 100 - 80 = \alpha \cdot 25 \Rightarrow$
$\Rightarrow \alpha = \dfrac{4}{5}$ rad



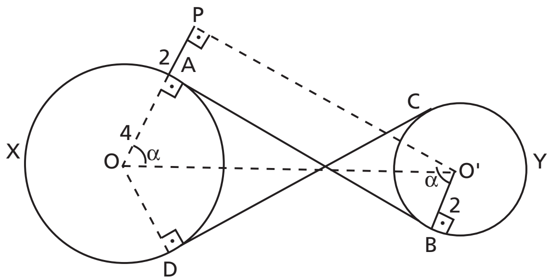
770. Na figura, temos:
$\triangle OMB \Rightarrow \cos\alpha = \dfrac{OM}{OB} \Rightarrow$
$\Rightarrow \cos\alpha = \dfrac{3}{6} \Rightarrow \cos\alpha = \dfrac{1}{2} \Rightarrow$
$\Rightarrow \alpha = 60°$
$\alpha = 60° = A\hat{O}B = 120° \Rightarrow$
$\Rightarrow (\widehat{APB} = 120°, \widehat{AQB} = 240°)$
Como os comprimentos dos arcos são proporcionais aos ângulos centrais determinados, temos:
$\dfrac{\widehat{AQB}}{\widehat{APB}} = \dfrac{2}{1} = 2.$



773. Note o $\triangle ABC$, equilátero. Temos $\hat{C} = 60° \Rightarrow$
$\Rightarrow P\hat{O}Q = 120° \Rightarrow$
$\Rightarrow \widehat{PQ} = \dfrac{1}{3} \cdot 2\pi \cdot R \Rightarrow$
$\Rightarrow \widehat{PQ} = \dfrac{1}{3} \cdot 2\pi \cdot 10 \Rightarrow$
$\Rightarrow \widehat{PQ} = \dfrac{20}{3}\pi$ cm
Também temos:
QS = 2R = QS = 20 cm.
Logo, o comprimento da correia será dado por:
$3\left(20 + \dfrac{20}{3}\pi\right) = 60 + 20\pi = 20(3 + \pi)$ cm.



774.



$\triangle OPO' \Rightarrow (OP = 6, OO' = 12) \xrightarrow{\text{Pitágoras}} O'P = 6\sqrt{3}$ cm $\Rightarrow$
$\Rightarrow \left(AB = 6\sqrt{3} \text{ cm}, CD = 6\sqrt{3} \text{ cm}\right)$
Seja $P\hat{O}O' = \alpha$. Temos:
$\cos\alpha = \dfrac{OP}{OO'} \Rightarrow \cos\alpha = \dfrac{6}{12} \Rightarrow \cos\alpha\ \dfrac{1}{2} \Rightarrow \alpha = 60° \Rightarrow A\hat{O}D = 120°$
AOO' e $BO'O$ são alternos $\Rightarrow BO'O = \alpha = 60° \Rightarrow BO'C = 120°$

$A\hat{O}D = 120° \Rightarrow \overset{\frown}{AXD} = 240° \Rightarrow \overset{\frown}{AXD} = \dfrac{240°}{360°} \cdot 2\pi \cdot 4 \Rightarrow \overset{\frown}{AXD} = \dfrac{16}{3}\pi$ cm

$BO'C = 120° \Rightarrow \overset{\frown}{BYC} = 240° \Rightarrow \overset{\frown}{BYC} = \dfrac{240°}{360°} \cdot 2\pi \cdot 2 \Rightarrow \overset{\frown}{BYC} = \dfrac{8\pi}{3}$ cm

Logo, o comprimento da correia será dado por:
$AB + CD + \overset{\frown}{AXD} + \overset{\frown}{BYC} = 6\sqrt{3} + 6\sqrt{3} + \dfrac{16}{3}\pi + \dfrac{8}{3}\pi = 4\left(3\sqrt{3} + 2\pi\right)$ cm.

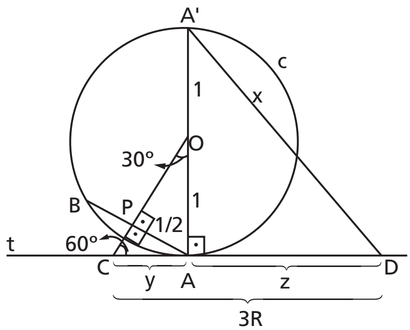
775. $\triangle A'AD \Rightarrow x^2 = 4 + z^2$
Cálculo de z:
Note que $y = 3 - z$.
$\triangle OPA \Rightarrow AP = \dfrac{1}{2}$
$\triangle APC \Rightarrow C = 60° \Rightarrow$
$\Rightarrow \text{sen } 60° = \dfrac{AP}{AC} \Rightarrow$

$\Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{\frac{1}{2}}{3 - z} \Rightarrow$

$\Rightarrow z = \dfrac{9 - \sqrt{3}}{3}$. Daí:

$x^2 = 4 + \left(\dfrac{9 - \sqrt{3}}{3}\right)^2 \Rightarrow$

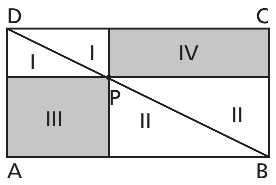
$\Rightarrow x \cong 3{,}1415333...$



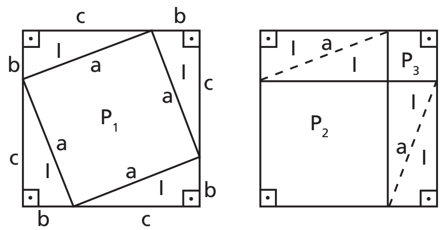
CAPÍTULO XVIII — Equivalência plana

783. Pelos itens 235 e 236 da teoria podemos concluir que todo triângulo é equivalente a um retângulo de base congruente à base do triângulo e altura igual à metade da altura do triângulo.
Se reduzirmos à metade a base de um triângulo, o retângulo equivalente também terá sua base reduzida à metade. Para manter a equivalência, a altura deverá dobrar.

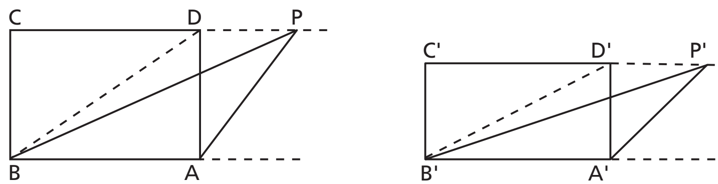
784. $\triangle ABD \approx \triangle BCD \Rightarrow$
$\Rightarrow \text{I} + \text{II} + \text{III} \equiv \text{I} + \text{II} + \text{IV} \Rightarrow$
$\Rightarrow \text{III} \equiv \text{IV}$



789. $4 \cdot (\text{I}) + P_1 \approx 4 \cdot (\text{I}) + P_2 + P_3 \Rightarrow$
$\Rightarrow P_1 \approx P_2 + P_3$



790.



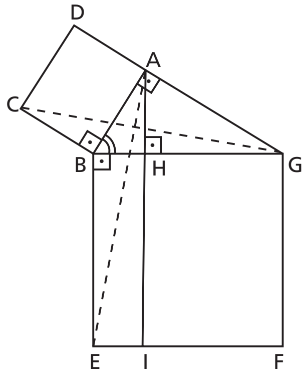
$\left.\begin{array}{l}\triangle PAB \approx \triangle DAB \text{ (mesma base e mesma altura)} \\ \triangle P'A'B' \approx \triangle D'A'B' \text{ (mesma base e mesma altura)}\end{array}\right\} \Rightarrow \triangle DAB \approx \triangle D'A'B'$

Como a diagonal do retângulo o divide em triângulos equivalentes, concluímos que os retângulos são equivalentes.

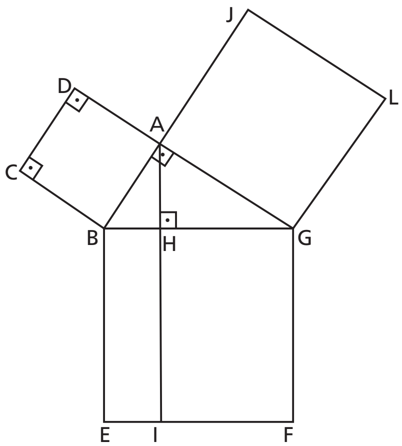
791. $\triangle CBG \equiv \triangle ABE$ (LAL)

Utilizando a dedução feita no exercício anterior, temos:

$ABCD \approx BEIH$.



792.



$$\left.\begin{array}{l}\text{Exercício 791} \Rightarrow ABCD \approx BEIH \\ \text{Exercício 791} \Rightarrow AJLG \approx GHIF\end{array}\right\} \Rightarrow ABCD + AJLG \approx BEIH + GHIF$$

$$\Rightarrow ABCD + AJLG \approx BGFE$$

CAPÍTULO XIX — Áreas de superfícies planas

798. g) $\text{sen } 60° = \dfrac{h}{6} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{h}{6} \Rightarrow$

$\Rightarrow h = 3\sqrt{3}$ m

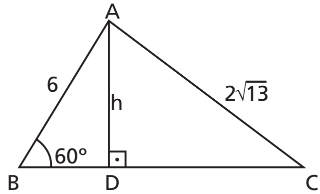
$\cos 60° = \dfrac{BD}{6} \Rightarrow \dfrac{1}{2} = \dfrac{BD}{6} \Rightarrow$

$\Rightarrow BD = 3$ m

$\triangle ACD \Rightarrow CD = 5$ m

$S_{ABC} = \dfrac{BC \cdot h}{2} \Rightarrow$

$\Rightarrow S_{ABC} = \dfrac{8 \cdot 3\sqrt{3}}{2} \Rightarrow S_{ABC} = 12\sqrt{3}$ m^2



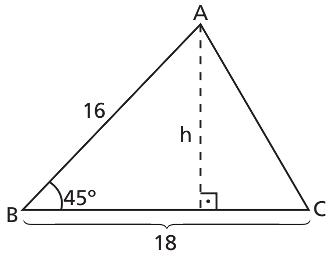
h) $\text{sen } 45° = \dfrac{h}{16} \Rightarrow \dfrac{\sqrt{2}}{2} = \dfrac{h}{16} \Rightarrow$

$\Rightarrow h = 8\sqrt{2}\text{ m}$

$S_{ABC} = \dfrac{BC \cdot h}{2} \Rightarrow$

$\Rightarrow S_{ABC} = \dfrac{18 \cdot 8\sqrt{2}}{2} \Rightarrow$

$\Rightarrow S_{ABC} = 72\sqrt{2}\text{ m}^2$

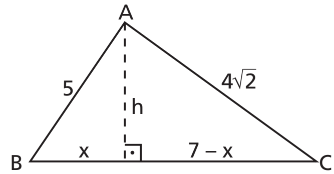


i) $\left.\begin{array}{l} x^2 + h^2 = 5^2 \\ (7 - x)^2 + h^2 = (4\sqrt{2})^2 \end{array}\right\} \Rightarrow$

$\Rightarrow x = 3\text{ m} \Rightarrow h = 4\text{ m}$

$S_{ABC} = \dfrac{BC \cdot h}{2} \Rightarrow$

$\Rightarrow S_{ABC} = \dfrac{7 \cdot 4}{2} \Rightarrow S_{ABC} = 14\text{ m}^2$

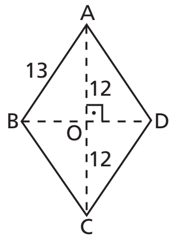


799. a) $\triangle ABO \xRightarrow{\text{Pitágoras}} BO = 5\text{ m} \Rightarrow$

$\Rightarrow BD = 10\text{ m}$

$S = \dfrac{BD \cdot AC}{2} \Rightarrow$

$\Rightarrow S = \dfrac{10 \cdot 24}{2} \Rightarrow S = 120\text{ m}^2$

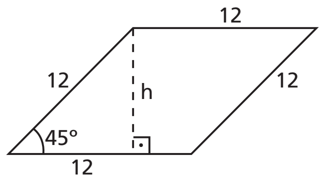


b) $\text{sen } 45° = \dfrac{h}{12} \Rightarrow \dfrac{\sqrt{2}}{2} = \dfrac{h}{12} \Rightarrow$

$\Rightarrow h = 6\sqrt{2}\text{ m}$

$S = 12 \cdot h \Rightarrow S = 12 \cdot 6\sqrt{2} \Rightarrow$

$\Rightarrow S = 72\sqrt{2}\text{ m}^2$



c) $\text{tg } 60° = \dfrac{OD}{AO} \Rightarrow \sqrt{3} = \dfrac{12}{AO} \Rightarrow$

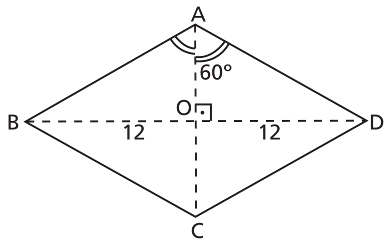
$\Rightarrow AO = 4\sqrt{3}\text{ m}$

$AO = 4\sqrt{3} \Rightarrow AC = B\sqrt{3}\text{ m}$

$S = \dfrac{AC \cdot BD}{2} \Rightarrow$

$\Rightarrow S = \dfrac{8\sqrt{3} \cdot 24}{2} \Rightarrow$

$\Rightarrow S = 96\sqrt{3}\text{ m}^2$



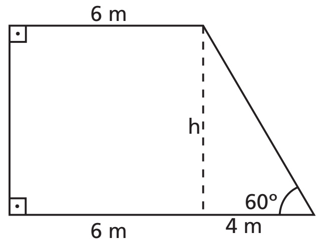
800. d) $\text{tg } 60° = \dfrac{h}{4} \Rightarrow \sqrt{3} = \dfrac{h}{4} \Rightarrow$

$\Rightarrow h = 4\sqrt{3}$ m

$S = \dfrac{(B + b) \cdot h}{2} \Rightarrow$

$\Rightarrow S = \dfrac{(10 + 6) \cdot 4\sqrt{3}}{2} \Rightarrow$

$\Rightarrow S = 32\sqrt{3}$ m^2



e) $\text{sen } 30° = \dfrac{h}{6} \Rightarrow \dfrac{1}{2} = \dfrac{h}{6} \Rightarrow$

$\Rightarrow h = 3$ m

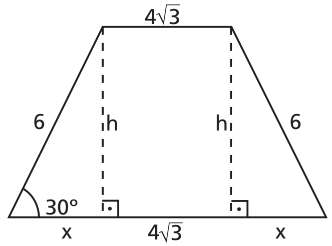
$\cos 30° = \dfrac{x}{6} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{x}{6} \Rightarrow$

$\Rightarrow x = 3\sqrt{3}$ m

$S = \dfrac{(B + b) \cdot h}{2} \Rightarrow$

$\Rightarrow S = \dfrac{\left(10\sqrt{3} + 4\sqrt{3}\right) \cdot 3}{2} \Rightarrow$

$\Rightarrow S = 21\sqrt{3}$ m^2



f) $\text{sen } 60° = \dfrac{h}{6} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{h}{6} \Rightarrow$

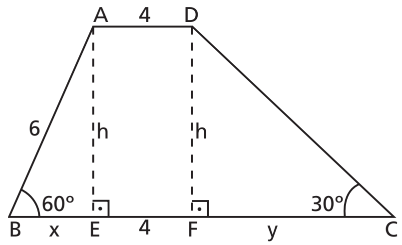
$\Rightarrow h = 3\sqrt{3}$ m

$\cos 60° = \dfrac{x}{6} \Rightarrow \dfrac{1}{2} = \dfrac{x}{6} \Rightarrow$

$\Rightarrow x = 3$ m

$\text{tg } 30° = \dfrac{h}{y} \Rightarrow \dfrac{\sqrt{3}}{3} = \dfrac{3\sqrt{3}}{y} \Rightarrow$

$\Rightarrow y = 9$ m



$S = \dfrac{(B + b) \cdot h}{2} \Rightarrow S = \dfrac{(16 + 4) \cdot 3\sqrt{3}}{2} \Rightarrow S = 30\sqrt{3}$ m^2

803. b) $\text{sen } 30° = \dfrac{x}{12} \Rightarrow \dfrac{1}{2} = \dfrac{x}{12} \Rightarrow$

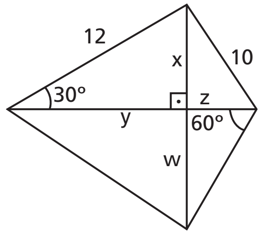
$\Rightarrow x = 6$ m

$\cos 30° = \dfrac{y}{12} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{y}{12} \Rightarrow$

$\Rightarrow y = 6\sqrt{3}$ m

$6^2 + z^2 = 10^2 \Rightarrow z = 8$ m

$\text{tg } 60° = \dfrac{w}{z} \Rightarrow \sqrt{3} = \dfrac{w}{8} \Rightarrow w = 8\sqrt{3}$ m



$S = \dfrac{(x + w)(y + z)}{2} \Rightarrow$

$\Rightarrow S = \dfrac{(6 + 8\sqrt{3})(6\sqrt{3} + 8)}{2} \Rightarrow S = 2\left(25\sqrt{3} + 48\right)$ m^2

809. Sejam D e *d* as diagonais maior e menor, respectivamente, do losango, e ℓ o lado do quadrado. Temos:

$\ell^2 = 81 \Rightarrow \ell = 9$ cm $\Rightarrow 2p = 36$ cm.

$$\left.\begin{array}{l} \dfrac{d}{D} = \dfrac{2}{7} \\ d + D = 36 \end{array}\right\} \Rightarrow D = 28 \text{ cm}, d = 8 \text{ cm} \Rightarrow S = \frac{D \cdot d}{2} \Rightarrow S = \frac{28 \cdot 8}{2} \Rightarrow$$

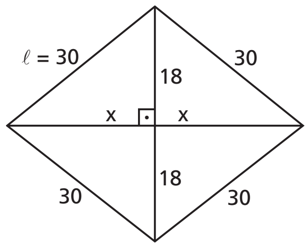
$\Rightarrow S = 112$ cm^2

811. $4\ell = 120 \Rightarrow \ell = 30$ cm

$x^2 + 18^2 = 30^2 \Rightarrow x = 24$ cm

$$S = \frac{D \cdot d}{2} \Rightarrow S = \frac{48 \cdot 36}{2} \Rightarrow$$

$\Rightarrow S = 864$ cm^2



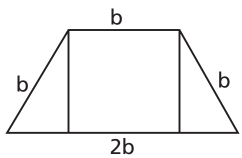
812. Perímetro do quadrado $= 40 \Rightarrow 4\ell = 40 \Rightarrow$

$\Rightarrow \ell = 10$ m $\Rightarrow S_Q = 100$ m^2

No trapézio da figura:

$2p = 40 \Rightarrow 5b = 40 \Rightarrow b = 8$ m

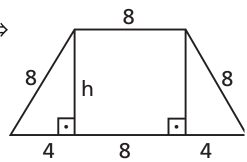
$h^2 + 4^2 = 8^2 \Rightarrow h = 4\sqrt{3}$ m



$$S_{Tra} = \frac{(B + b)h}{2} \Rightarrow S_{Tra} = \frac{(16 + 8) \cdot 4\sqrt{3}}{2} \Rightarrow$$

$\Rightarrow S_{Tra} = 48\sqrt{3}$ m^2

$$\frac{S_Q}{S_{Tra}} = \frac{100}{48\sqrt{3}} \Rightarrow \frac{S_Q}{S_{Tra}} = \frac{25\sqrt{3}}{36}$$



814. Sendo *b* e *h* a base e a altura do retângulo, temos:

$$\left.\begin{array}{l} b = h + 3 \\ 2b + 3h = 66 \end{array}\right\} \Rightarrow (b = 15 \text{ cm}, h = 12 \text{ cm}) \Rightarrow 2p = 54 \text{ cm}.$$

Sendo ℓ o lado do quadrado, temos:

$$4\ell = 54 \Rightarrow \ell = \frac{27}{2} \text{ cm} \Rightarrow S = \ell^2 \Rightarrow S = \frac{729}{4} \text{ cm}^2.$$

815. Sejam D e *d* as diagonais do losango. Temos:

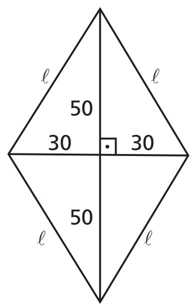
$$\left.\begin{array}{l} \dfrac{d}{D} = \dfrac{3}{5} \\ D - d = 40 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 3D - 5d = 0 \\ D - d = 40 \end{array}\right\} \Rightarrow$$

$\Rightarrow (D = 100$ cm, $d = 60$ cm$)$

Na figura ao lado:
$\ell^2 = 50^2 + 30^2 \Rightarrow \ell = 10\sqrt{34}$ cm
Sendo o perímetro do quadrado igual ao do losango, o lado do quadrado também mede $10\sqrt{34}$ cm.

$$\frac{A_{qua}}{A_{los}} = \frac{\ell^2}{\frac{D \cdot d}{2}} \Rightarrow$$

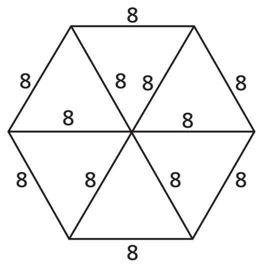
$$\Rightarrow \frac{A_{qua}}{A_{los}} = \frac{3\,400}{\frac{100 \cdot 60}{2}} \Rightarrow \frac{A_{qua}}{A_{los}} = \frac{17}{15}$$



819. a) $\ell = 8$

$$S = \frac{3\sqrt{3}\ell^2}{2} \Rightarrow S = \frac{3\sqrt{3} \cdot 8^2}{2} \Rightarrow$$

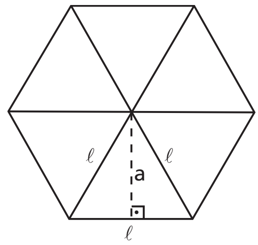
$$\Rightarrow S = 96\sqrt{3} \text{ m}^2$$



b) $a = \frac{\ell\sqrt{3}}{2} \Rightarrow 2\sqrt{3} = \frac{\ell\sqrt{3}}{2} \Rightarrow \ell = 4$ m

$$S = \frac{3\sqrt{3}\ell^2}{2} \Rightarrow S = \frac{3\sqrt{3} \cdot 4^2}{2} \Rightarrow$$

$$\Rightarrow S = 24\sqrt{3} \text{ m}^2$$



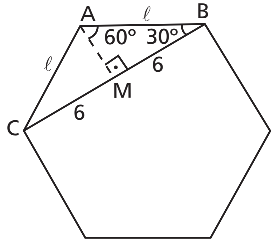
c) $BC = 12 \Rightarrow MB = 6$
$B\hat{A}C = 120° \Rightarrow M\hat{A}B = 60°$
$M\hat{A}B = 60° \Rightarrow M\hat{B}A = 30°$

$$\cos 30° = \frac{6}{\ell} \Rightarrow \frac{\sqrt{3}}{2} = \frac{6}{\ell} \Rightarrow$$

$$\Rightarrow \ell = 4\sqrt{3} \text{ m}$$

$$S = \frac{3\sqrt{3}\ell^2}{2} \Rightarrow S = \frac{3\sqrt{3}(4\sqrt{3})^2}{2} \Rightarrow$$

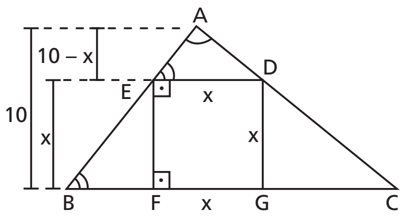
$$\Rightarrow S = 72\sqrt{3} \text{ m}^2$$



824. $\triangle AED \sim \triangle ABC \Rightarrow$

$$\Rightarrow \frac{10 - x}{10} = \frac{x}{15} \Rightarrow x = 6 \text{ m} \Rightarrow$$

$$\Rightarrow S_{DEFG} = x^2 \Rightarrow S_{DEFG} = 36 \text{ m}^2$$



825. $\triangle ADE \sim \triangle ACB \Rightarrow$

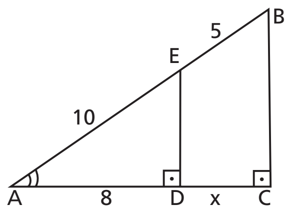
$$\Rightarrow \frac{AE}{AB} = \frac{AD}{AC} \Rightarrow$$

$$\Rightarrow \frac{10}{15} = \frac{8}{8 + x} \Rightarrow x = 4 \text{ m}$$

$\triangle ABC \Rightarrow 12^2 + BC^2 = 15^2 \Rightarrow$

$\Rightarrow BC = 9 \text{ m}$

$$S_{ABC} = \frac{(AC) \cdot (BC)}{2} \Rightarrow S_{ABC} = \frac{12 \cdot 9}{2} \Rightarrow S_{ABC} = 54 \text{ m}^2$$



826.

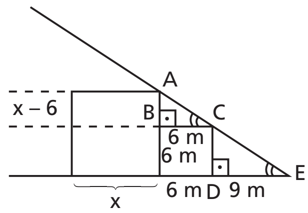
$$\left.\begin{array}{l} A\hat{C}B \equiv C\hat{E}D \text{ (correspondentes)} \\ A\hat{B}C \equiv C\hat{D}E \text{ (retos)} \end{array}\right\} \Rightarrow$$

$\Rightarrow \triangle ABC \sim \triangle CDE \Rightarrow$

$$\Rightarrow \frac{AB}{CD} = \frac{BC}{DE} \Rightarrow$$

$$\Rightarrow \frac{x - 6}{6} = \frac{6}{9} \Rightarrow x = 10 \text{ m}$$

$S = x^2 \Rightarrow S = 10^2 \Rightarrow S = 100 \text{ m}^2$



827. De acordo com a figura ao lado:

$$\begin{cases} 2p = 36 \\ a^2 = b^2 + 12^2 \end{cases} \Rightarrow \begin{cases} 2a + 2b = 36 \\ a^2 - b^2 = 144 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} a + b = 18 \text{ (1)} \\ a^2 - b^2 = 144 \text{ (2)} \end{cases}$$

$(2) \Rightarrow (a + b)(a - b) = 144 \overset{(1)}{\Rightarrow}$

$\Rightarrow 18(a - b) = 144 \Rightarrow$

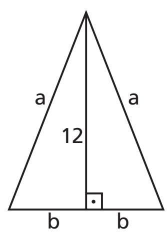
$\Rightarrow a - b = 8 \text{ (3)}$

(1) e (3) $\Rightarrow a = 13, b = 5$

Seja S a área procurada. Então:

$$S = \frac{2b \cdot h}{2} \Rightarrow S = \frac{2 \cdot 5 \cdot 12}{2} \Rightarrow$$

$\Rightarrow S = 60 \text{ m}^2$

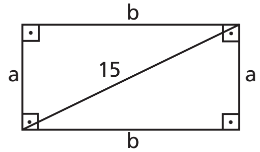


828. Considerando as medidas indicadas na figura:

$$\left.\begin{array}{l} a^2 + b^2 = 15^2 \\ 2(a + b) = 42 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a^2 + b^2 = 225 \\ a + b = 21 \end{array}\right\} \Rightarrow$$

$$\Rightarrow a \cdot b = 108 \Rightarrow S = 108\ m^2$$



830. Na figura ao lado temos um trapézio isósceles de altura $3\sqrt{3}$ m, base maior 14 m e perímetro 34 m.

Para facilitar os cálculos fizemos a base menor igual a 2a. Daí:

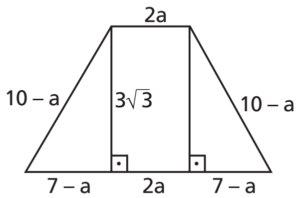
$$(7 - a)^2 + \left(3\sqrt{3}\right)^2 = (10 - a)^2 \Rightarrow$$

$$\Rightarrow a = 4\ m$$

$$S = \frac{(B + b)h}{2} \Rightarrow$$

$$\Rightarrow S = \frac{(14 + 8)}{2} \cdot 3\sqrt{3} \Rightarrow$$

$$\Rightarrow S = 33\sqrt{3}\ m^2$$



831. Considerando as medidas indicadas na figura, temos:

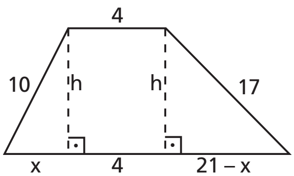
$$\begin{cases} h^2 + (21 - x)^2 = 17^2 \\ h^2 + x^2 = 10^2 \end{cases} \Rightarrow$$

$$\Rightarrow (x = 6\ m,\ h = 8\ m)$$

$$S = \frac{(B + b)h}{2} \Rightarrow$$

$$\Rightarrow S = \frac{(25 + 4) \cdot 8}{2} \Rightarrow$$

$$\Rightarrow S = 116\ m^2$$



832. Para simplificar os cálculos, seja 2x a medida de uma diagonal. A outra medirá 2x + 4 e o lado medirá 2x − 2.

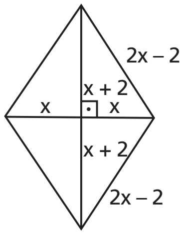
Considerando as medidas indicadas na figura:

$$x^2 + (x + 2)^2 = (2x - 2)^2 \Rightarrow$$

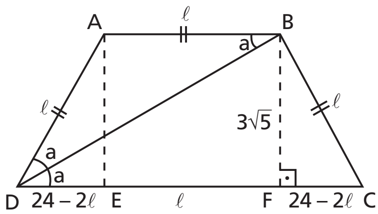
$$\Rightarrow x^2 - 6x = 0 \Rightarrow x = 0 \text{ ou } x = 6\ m$$

$$x = 6m \Rightarrow (d = 12\ m,\ D = 16\ m).$$

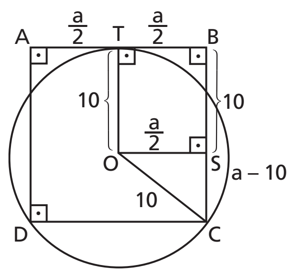
$$S = \frac{D \cdot d}{2} \Rightarrow S = \frac{16 \cdot 12}{2} \Rightarrow S = 96\ m^2.$$



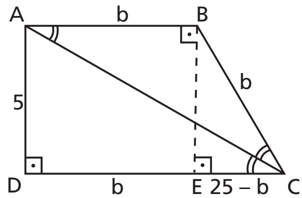
833. $A\hat{B}D = C\hat{D}B$ (alternos) $\Rightarrow \triangle ABD$ isósceles $\Rightarrow AB = AD = \ell$.
O trapézio é isósceles $\Rightarrow AD = BC = \ell$.
$AB = \ell \Rightarrow EF = \ell$
$2p = 48 \Rightarrow 4\ell + DE + FC = 48 \Rightarrow$
$\Rightarrow DE + FC = 48 - 4\ell$
$(DE = FC;\ DE + FC = 48 - \ell) \Rightarrow$
$\Rightarrow DE = FC = 24 - 2\ell$
$\triangle BCF$: $\ell^2 = \left(3\sqrt{5}\right)^2 + (24 - 2\ell)^2 \Rightarrow$
$\Rightarrow \ell = 23$ ou $\ell = 9$
Sendo B a base maior, temos:
$B = 48 - 3\ell$.
$\ell = 23 \Rightarrow B = 48 - 3 \cdot 23 \Rightarrow$
$\Rightarrow B = -21$ (não serve)
$\ell = 9 \Rightarrow B = 48 - 3 \cdot 9 \Rightarrow$
$\Rightarrow B = 21$
$S = \dfrac{(B + b)h}{2} \Rightarrow$
$\Rightarrow S = \dfrac{21 + 9}{2} \cdot 3\sqrt{5} \Rightarrow$
$\Rightarrow S = 45\sqrt{5}\ m^2$.



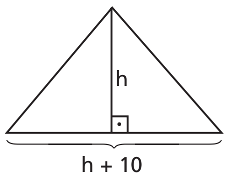
834. Seja $\overline{OT}$ o raio perpendicular ao lado $\overline{AB}$ e $\overline{OS}$ o segmento paralelo a $\overline{AB}$, com S em BC.
Sendo *a* o lado do quadrado e considerando as medidas indicadas na figura, temos:
$\triangle OCS$: $(a - 10)^2 + \left(\dfrac{a}{2}\right)^2 = 10^2 \Rightarrow$
$\Rightarrow a = 16$ m.
$S = a^2 \Rightarrow S = 16^2 \Rightarrow S = 256\ m^2$.

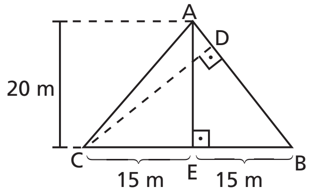


835. $A\hat{C}D \equiv B\hat{A}C$ (alternos) $\Rightarrow$
$\Rightarrow \triangle ABC$ isósceles $\Rightarrow AB = BC = b$.
$AB = b \Rightarrow DE = b \Rightarrow CE = 25 - b$
$\triangle BEC \Rightarrow b^2 = 5^2 + (25 - b)^2 \Rightarrow$
$\Rightarrow b = 13$
$S = \dfrac{(B + b)h}{2} \Rightarrow$
$\Rightarrow S = \dfrac{(25 + 13) \cdot 5}{2} \Rightarrow$
$\Rightarrow S = 95\ m^2$.



836.

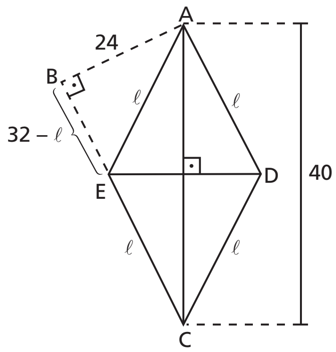




1) $S = 300 \Rightarrow \dfrac{(h + 10) \cdot h}{2} =$
$= 300 \Rightarrow h^2 + 10h - 600 =$
$= 0 \Rightarrow h = -30$ (não serve)
ou $h = 20$ m

2) $\triangle ABE \Rightarrow AB^2 = 20^2 + 15^2 \Rightarrow$
$\Rightarrow AB = 25$ m
$S = 300 \Rightarrow \dfrac{(AB)\cdot(CD)}{2} = 300 \Rightarrow$
$\Rightarrow \dfrac{25 \cdot CD}{2} = 300 \Rightarrow CD = 24$ m

837. $\triangle ABC \Rightarrow BC^2 + 24^2 = 40^2 \Rightarrow$
$\Rightarrow BC = 32$ m
Sendo a medida do lado do losango igual a ℓ:
$\triangle ABE \Rightarrow (32 - \ell)^2 + 24^2 = \ell^2 \Rightarrow$
$\Rightarrow \ell = 25$ m
$S = CE \cdot AB \Rightarrow S = 25 \cdot 24 \Rightarrow$
$\Rightarrow S = 600\ m^2$.



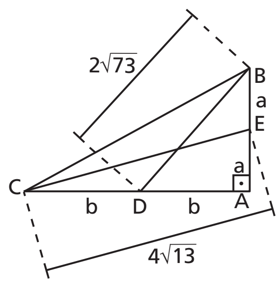
838. Na figura, o $\triangle ABC$ é retângulo em A; $\overline{BD}$ mediana relativa a $\overline{AC}$; $\overline{CE}$ mediana relativa a $\overline{AB}$; $\overline{BD} = 2\sqrt{73}$ m, $CE = 4\sqrt{13}$ m. Temos:
$\triangle ACE \Rightarrow a^2 + (2b)^2 = \left(2\sqrt{73}\right)^2$
$\triangle ABD \Rightarrow (2a)^2 + b^2 = \left(4\sqrt{13}\right)^2 \Rightarrow$

$$\Rightarrow \begin{cases} a^2 + 4b^2 = 292 \\ 4a^2 + b^2 = 208 \end{cases} \Rightarrow$$

$\Rightarrow a = 6$ m, $b = 8$ m $\Rightarrow$

$$\Rightarrow S = \frac{2a \cdot 2b}{2} \Rightarrow$$

$$\Rightarrow S = \frac{12 \cdot 16}{2} \Rightarrow S = 96\ m^2.$$



839. A menor altura é relativa ao maior lado.

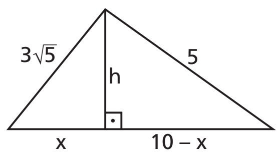
De acordo com a figura:

$$\left.\begin{aligned} h^2 + x^2 &= 45 \\ h^2 + (10 - x)^2 &= 25 \end{aligned}\right\} \Rightarrow$$

$\Rightarrow (x = 6 \text{ m}, h = 3 \text{ m})$

$$S = \frac{10 \cdot h}{2} \Rightarrow S = \frac{10 \cdot 3}{2} \Rightarrow$$

$\Rightarrow S = 15 \text{ m}^2.$



840. $\triangle ABC \Rightarrow (BF = BD = 4 \text{ m}, CF = CE = 6 \text{ m})$

$AB^2 + AC^2 = BC^2 \Rightarrow$

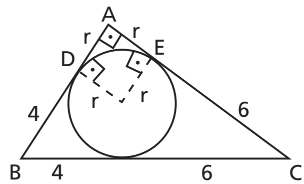
$\Rightarrow (4 + r)^2 + (6 + r)^2 = 10^2 \Rightarrow$

$\Rightarrow r = -12$ (não serve) ou $r = 2$ m

$r = 2 \text{ m} \Rightarrow (AB = 6 \text{ m}, AC = 8 \text{ m})$

$$S = \frac{(AB) \cdot (AC)}{2} \Rightarrow$$

$$\Rightarrow S = \frac{6 \cdot 8}{2} \Rightarrow S = 24 \text{ m}^2.$$



841. (1) $\triangle ABC \approx \triangle ACD$ (mesma base $AB = AD$ e mesma altura)

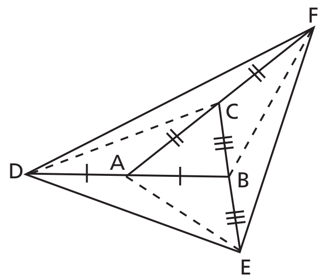
(2) $\triangle CDF \approx \triangle ACD$ (mesma base $AC = CF$ e mesma altura)

(1) e (2) $\Rightarrow S_{ADF} = 2 \cdot S_{ABC}$

Analogamente,

$S_{CEF} = 2 \cdot S_{ABC}$; $S_{ADE} = 2 \cdot S_{ABC}$.

Portanto, $S_{DEF} = 7 \cdot S_{ABC}$.



843. Na figura, os triângulos que têm áreas iguais assim foram marcados por possuírem mesma base e mesma altura relativa a essas bases.

Agora, temos:

$\triangle ABP \approx \triangle ACP$ (mesma base e mesma altura) $\Rightarrow$

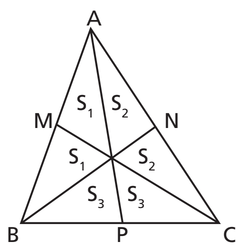
$\Rightarrow 2S_1 + S_3 = 2S_2 + S_3 \Rightarrow$

$\Rightarrow S_1 = S_2$ (1)

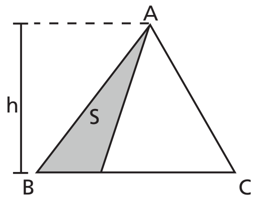
$\triangle ABN \approx \triangle CBN \Rightarrow$

$\Rightarrow 2S_1 + S_2 = 2S_3 + S_2 \Rightarrow S_1 = S_3$ (2)

(1) e (2) $\Rightarrow S_1 = S_2 = S_3$.



844. a)

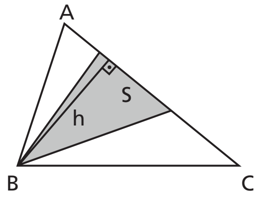


$k = \frac{BC \cdot h}{2}$

$S = \frac{\frac{1}{3} BC \cdot h}{2} \Rightarrow$

$\Rightarrow S = \frac{1}{3}k$

b)

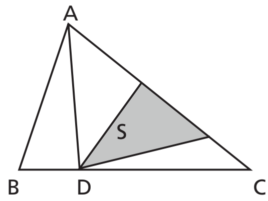


$k = \frac{AC \cdot h}{2}$

$S = \frac{\frac{2}{5} AC \cdot h}{2} \Rightarrow$

$\Rightarrow S = \frac{2}{5}k$

c)



$S_{ABD} = \frac{k}{4} \Rightarrow$

$\Rightarrow S_{ACD} = \frac{3k}{4}$

$S = \frac{3}{6} \cdot S_{ACD} \Rightarrow$

$\Rightarrow S = \frac{3}{6} \cdot \frac{3}{4}k \Rightarrow$

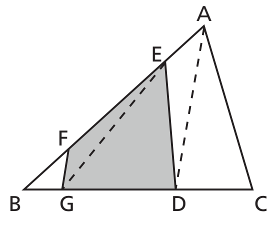
$\Rightarrow S = \frac{3}{8}k$

d) $S_{ACD} = \frac{2}{6}k \Rightarrow S_{ACD} = \frac{k}{3}$

$S_{ABD} = \frac{2}{3}k \Rightarrow S_{BDE} = \frac{3}{4}S_{ABD} \Rightarrow$

$\Rightarrow S_{BDE} = \frac{3}{4} \cdot \frac{2}{3}k \Rightarrow$

$\Rightarrow S_{BDE} = \frac{k}{2}$



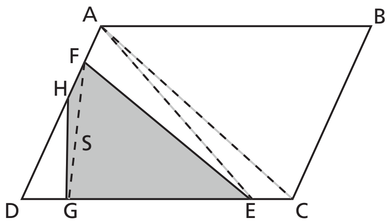
$S_{BGE} = \frac{1}{4} \cdot S_{BDE} \Rightarrow$

$\Rightarrow S_{BGE} = \frac{1}{4} \cdot \frac{k}{2} \Rightarrow S_{BGE} = \frac{k}{8}$

$S_{BFG} = \frac{1}{3} \cdot S_{BGE} \Rightarrow S_{BFG} = \frac{1}{3} \cdot \frac{k}{8} \Rightarrow S_{BFG} = \frac{k}{24}$

$S_{DEFG} = S_{BDE} - S_{BFG} \Rightarrow S_{DEFG} = \frac{k}{2} - \frac{k}{24} \Rightarrow S_{DEFG} = \frac{11}{24}k$

845. a)



$$S_{ACD} = \frac{k}{2} \Rightarrow S_{ACE} = \frac{1}{6} \cdot \frac{k}{2} \Rightarrow$$

$$\Rightarrow S_{ACE} = \frac{k}{12} \Rightarrow S_{ADE} = \frac{5k}{12}$$

$$S_{AFE} = \frac{1}{5} \cdot \frac{5k}{12} \Rightarrow S_{AFE} = \frac{k}{12}$$

$$S_{FDE} = S_{ACD} - S_{ACEF} =$$

$$= \frac{k}{2} - 2 \cdot \frac{k}{12} = \frac{k}{3}$$

$$S_{FGE} = \frac{4}{5} S_{FDE} = \frac{4}{5} \cdot \frac{k}{3} = \frac{4k}{15}$$

$$S_{FGD} = \frac{1}{5} \cdot S_{FDE} =$$

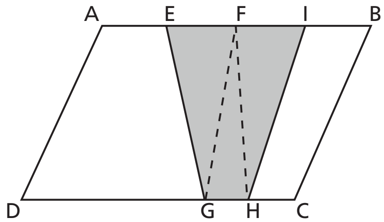
$$= \frac{1}{5} \cdot \frac{k}{3} = \frac{k}{15} \Rightarrow$$

$$\Rightarrow S_{FGH} = \frac{1}{4} \cdot \frac{k}{15} = \frac{k}{60}$$

$$S = S_{FGE} + S_{FGH} = \frac{4k}{15} + \frac{k}{60} \Rightarrow$$

$$\Rightarrow S = \frac{17k}{60}$$

b)



$$S_{EGHI} = S_{EFG} + S_{FGH} + S_{FHI} \Rightarrow$$

$$\Rightarrow S_{EGHI} = \frac{1}{4} \cdot \frac{k}{2} + \frac{1}{6} \cdot \frac{k}{2} +$$

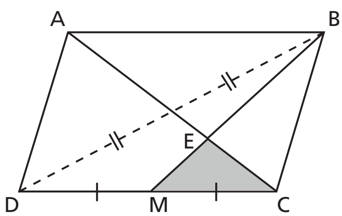
$$+ \frac{1}{4} \cdot \frac{k}{2} \Rightarrow$$

$$\Rightarrow S_{EGHI} = \frac{k}{3}$$

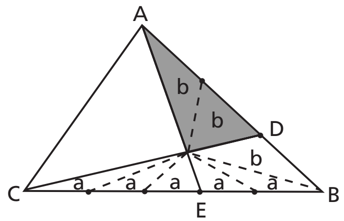
846. E é baricentro do $\triangle BCD \Rightarrow$

$$\Rightarrow S_{EMC} = \frac{1}{6} \cdot \frac{S}{2} \Rightarrow$$

$$\Rightarrow S_{EMC} = \frac{S}{12}$$



847. Observando as áreas indicadas na figura, temos:



$$\left.\begin{array}{l} \triangle ABE \Rightarrow \\ \\ \triangle BCD \Rightarrow \end{array}\left\{\begin{array}{l} 2a + 3b = \dfrac{2}{5}k \\ \\ 5a + b = \dfrac{k}{3} \end{array}\right.\right. \Rightarrow$$

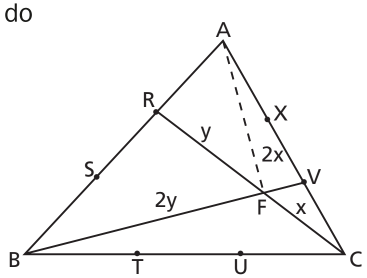
$$\Rightarrow b = \frac{4k}{39} \Rightarrow S = \frac{8k}{39}$$

848. Unindo os pontos A e F.

Sendo *x* a área do $\triangle FVC$, a área do $\triangle FVA$ será 2x.

Sendo *y* a área do $\triangle FAR$, a área do $\triangle FBR$ será 2y.

Temos:



$$\left.\begin{array}{l} 3x + y = \dfrac{k}{3} \\ \\ 2x + 3y = \dfrac{2}{3}k \end{array}\right\} \Rightarrow$$

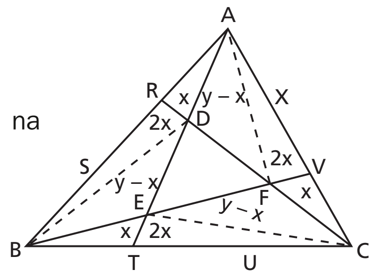
$$\Rightarrow \left(x = \frac{k}{21};\ y = \frac{4k}{21}\right)$$

De modo análogo, obtemos:

$$S_{ARD} = S_{BTE} = S_{FVC} = \frac{k}{21} \text{ e}$$

$$S_{BTD} = S_{CVD} = S_{FAR} = \frac{4k}{21}.$$

Observando as áreas indicadas na figura, temos:



$$S_{DEF} = k - 6x - 3y =$$

$$= k - 6 \cdot \frac{k}{21} - 3 \cdot \frac{4k}{21} \Rightarrow$$

$$\Rightarrow S_{DEF} = \frac{k}{7}.$$

849. Exercício 714 $\Rightarrow \left(a_8 = \dfrac{R\sqrt{2+\sqrt{2}}}{2};\ \ell_8 = R\sqrt{2-\sqrt{2}}\right) \Rightarrow$

$$\left.\begin{array}{l} \Rightarrow a_8 = \dfrac{(\sqrt{2}+1)\ell_8}{2} \\ \\ 2p = 8\ell \Rightarrow p = 4\ell_8 \end{array}\right\} \Rightarrow S = p \cdot a \Rightarrow S = \frac{4(\sqrt{2}+1)\ell_8^2}{2} \Rightarrow$$

$$\Rightarrow S = 2(\sqrt{2}+1)\ell_8^2$$

850. $\left(a_{10}=\frac{R}{4}\sqrt{10+2\sqrt{5}};\ \ell_{10}=\frac{\sqrt{5}-1}{2}\cdot R\right)\Rightarrow a_{10}=\frac{\sqrt{5}+1}{8}\cdot\sqrt{10+2\sqrt{5}}\,\ell_{10}$

$2p = 10\ell \Rightarrow p = 5\ell$

$S = p\cdot a \Rightarrow S = 5\cdot\frac{\sqrt{5}+1}{8}\cdot\sqrt{10+2\sqrt{5}}\,\ell^2 =$

$= \frac{5}{8}\sqrt{(\sqrt{5}+1)^2(10+2\sqrt{5})}\cdot\ell^2 \Rightarrow$

$\Rightarrow S = \frac{5}{8}\sqrt{16(5+2\sqrt{5})}\,\ell^2 \Rightarrow S = \frac{5}{2}\sqrt{5+2\sqrt{5}}\,\ell^2$

851. $\left(a_5 = \frac{R}{4}(\sqrt{5}+1);\ \ell_5 = \frac{R}{2}\sqrt{10-2\sqrt{5}}\right)\Rightarrow a_5 = \frac{\sqrt{25+10\sqrt{5}}}{10}\ell_5$

$2p = 5\ell \Rightarrow p = \frac{5}{2}\ell$

$S = p\cdot a \Rightarrow S = \frac{5}{2}\cdot\frac{\sqrt{25+10\sqrt{5}}}{10}\cdot\ell^2 \Rightarrow S = \frac{\sqrt{25+10\sqrt{5}}}{4}\cdot\ell^2$

852. $b = \ell_5 \Rightarrow b = \frac{r}{2}\sqrt{10-2\sqrt{5}};\ h = a_5 \Rightarrow h = \frac{r}{4}(\sqrt{5}+1)$

$S = bh \Rightarrow S = \frac{\sqrt{10-2\sqrt{5}}\,(\sqrt{5}+1)}{8}\cdot r^2 \Rightarrow S = \frac{\sqrt{10+2\sqrt{5}}}{4}r^2$

853. Exercício 714 $\Rightarrow \ell_8 = r\sqrt{2-\sqrt{2}}$

$S = \ell_8^2 \Rightarrow S = \left(r\sqrt{2-\sqrt{2}}\right)^2 \Rightarrow S = (2-\sqrt{2})r^2$

854. De acordo com a figura, temos:

$\left.\begin{array}{r}\frac{84+x+40}{y+35+30} = \frac{a}{b}\\ \frac{40}{30} = \frac{a}{b}\end{array}\right\}\Rightarrow$

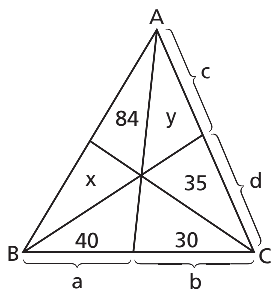
$\Rightarrow \frac{84+x+40}{y+35+30} = \frac{4}{3}\ (1)$

$\left.\begin{array}{r}\frac{x+84+y}{40+30+35} = \frac{c}{d}\\ \frac{y}{35} = \frac{c}{d}\end{array}\right\}\Rightarrow$

$\Rightarrow \frac{x+84+y}{40+30+35} = \frac{y}{35}\ (2)$

$\left.\begin{array}{l}(1) \Rightarrow 4y - 3x = 112\\ (2) \Rightarrow 70y - 35x = 2\,940\end{array}\right\}\Rightarrow$

$\Rightarrow (x = 56,\ y = 70) \Rightarrow S_{ABC} = 315$



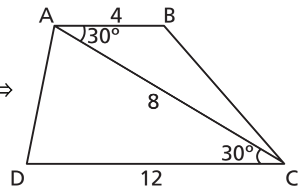
Expressões da área do triângulo

858. $A\hat{C}D \equiv B\hat{A}C$ (alternos)

$S = S_{ACD} + S_{ABC} \Rightarrow$

$\Rightarrow S = \dfrac{8 \cdot 12 \cdot \text{sen } 30°}{2} + \dfrac{4 \cdot 8 \cdot \text{sen } 30°}{2} \Rightarrow$

$\Rightarrow S = 32 \text{ m}^2.$



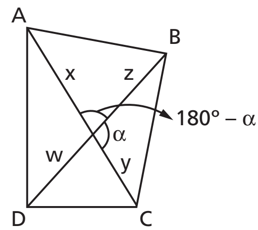
860. Seja o quadrilátero ABCD, onde AC = a, BD = b.

Lembrando que sen $(180° - \alpha) = \text{sen } \alpha$, temos:

$S_{Tra} = P + Q + R + S \Rightarrow$

$\Rightarrow S_{Tra} = \dfrac{xz \text{ sen}}{2} + \dfrac{xw \text{ sen } \alpha}{2} +$

$+ \dfrac{wy \text{ sen } \alpha}{2} + \dfrac{zy \text{ sen } \alpha}{2} \Rightarrow$



$\Rightarrow S_{Tra} = \dfrac{\text{sen } \alpha}{2}[x(z + w) + y(z + w)] \Rightarrow S_{Tra} = \dfrac{\text{sen } \alpha}{2}[(x + y)(z + w)] \Rightarrow$

$\Rightarrow S_{Tra} = \dfrac{1}{2} ab \text{ sen } \alpha$

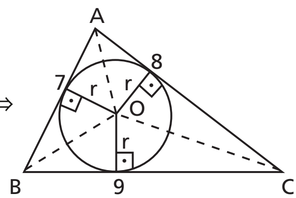
862. a) $p = \dfrac{7 + 8 + 9}{2} \Rightarrow p = 12$

$S = \sqrt{p(p - a)(p - b)(p - c)} \Rightarrow$

$\Rightarrow S = \sqrt{12(12 - 7)(12 - 8)(12 - 9)} \Rightarrow$

$\Rightarrow S = 12\sqrt{5}$

$S = p \cdot r \Rightarrow 12\sqrt{5} = 12 \cdot r \Rightarrow r = \sqrt{5}$



b) $p = \dfrac{16 + 20 + 18}{2} \Rightarrow p = 27$

$S = \sqrt{p(p - a)(p - b)(p - c)} \Rightarrow$

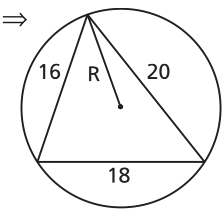
$\Rightarrow S = \sqrt{27(27 - 16)(27 - 20)(27 - 18)} \text{ m} \Rightarrow$

$\Rightarrow S = 9\sqrt{231}$

$S = \dfrac{abc}{4R} \Rightarrow$

$\Rightarrow 9\sqrt{231} = \dfrac{16 \cdot 20 \cdot 18}{4R} \Rightarrow$

$\Rightarrow R = \dfrac{160\sqrt{231}}{231}$



863. a) $p = \dfrac{6 + 10 + 12}{2} \Rightarrow p = 14$ m

$S = \sqrt{p(p - a)(p - b)(p - c)} \Rightarrow S = \sqrt{14(14 - 6)(14 - 10)(14 - 12)} \Rightarrow$
$\Rightarrow S = 8\sqrt{14}$ m^2

b) A menor altura é relativa ao maior lado; no caso, 12 m.

$S = \dfrac{12 \cdot h}{2} \Rightarrow 8\sqrt{14} = \dfrac{12h}{2} \Rightarrow h = \dfrac{4\sqrt{14}}{3}$ m

c) A maior altura é relativa ao menor lado; no caso, 6 m.

$S = \dfrac{6 \cdot H}{2} \Rightarrow 8\sqrt{14} = \dfrac{6H}{2} \Rightarrow H = \dfrac{8\sqrt{14}}{3}$

d) $S = p \cdot r \Rightarrow 8\sqrt{14} = 14 \cdot r \Rightarrow r = \dfrac{4\sqrt{14}}{7}$ m

e) $S = \dfrac{abc}{4R} \Rightarrow 8\sqrt{14} = \dfrac{6 \cdot 10 \cdot 12}{4R} \Rightarrow R = \dfrac{45\sqrt{14}}{28}$

864. $p = \dfrac{14 + 10 + 16}{2} \Rightarrow p = 20$ m

$S = \sqrt{p(p - a)(p - b)(p - c)} \Rightarrow S = \sqrt{20(20 - 14)(20 - 10)(20 - 16)} \Rightarrow$
$\Rightarrow S = 40\sqrt{3}$ m^2

$S = (p - 10) \cdot r \Rightarrow 40\sqrt{3} = (20 - 10) \cdot r \Rightarrow r = 4\sqrt{3}$ m

868. Para facilitar os cálculos, seja ℓ a medida de cada um dos lados congruentes. Considerando as medidas indicadas na figura 1, temos:
$2p = 32 \Rightarrow 3\ell + 2 = 32 \Rightarrow$
$\Rightarrow \ell = 10$ cm.
Substituindo $\ell = 10$ cm na figura 1, obtemos a figura 2, onde, pelo teorema de Pitágoras, h = 8 cm. Daí:

$S = \dfrac{b \cdot h}{2} \Rightarrow S = \dfrac{12 \cdot 8}{2} \Rightarrow$
$\Rightarrow S = 48$ cm^2

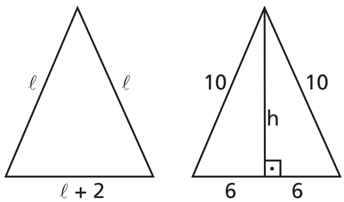


fig. 1 fig. 2

871. Os lados são da forma 5k, 12k e 13k. Temos:
$2p = 90 \Rightarrow (5 + 12 + 13)k = 90 \Rightarrow k = 3$. Logo, os catetos medem 15 cm e 36 cm.

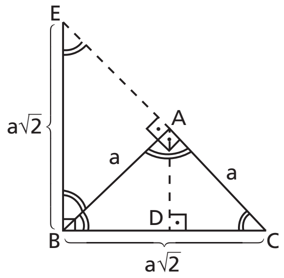
$S = \dfrac{15 \cdot 36}{2} \Rightarrow S = 270$ cm^2

874. $\left.\begin{array}{l}\overline{AD} \text{ é bissetriz} \\ \overline{BE} \,//\, \overline{AD}\end{array}\right\} \Rightarrow$

$\Rightarrow A\hat{B}C \equiv A\hat{B}E \equiv A\hat{C}B \equiv B\hat{E}A$

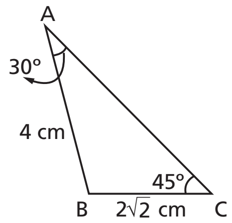
$\triangle BEC$ isósceles $\Rightarrow BE = BC = a\sqrt{2}$

$$S_{CBE} = \frac{a\sqrt{2} \cdot a\sqrt{2}}{2} \Rightarrow S_{CBE} = a^2$$



875. $$\frac{4}{\text{sen } 45°} = \frac{BC}{\text{sen } 30°} \Rightarrow$$

$$\Rightarrow \frac{4}{\frac{\sqrt{2}}{2}} = \frac{BC}{\frac{1}{2}} \Rightarrow BC = 2\sqrt{2} \text{ cm}$$



$$AB^2 = AC^2 + BC^2 - 2(AB)(AC) \cos 45° \Rightarrow$$

$$\Rightarrow 16 = AC^2 + 8 - 2 \cdot 4 \cdot AC \cdot \frac{\sqrt{2}}{2} \Rightarrow$$

$\Rightarrow AC^2 - 4AC - 8 = 0 \Rightarrow$

$\Rightarrow AC = 2 - 2\sqrt{3}$ (não serve) ou

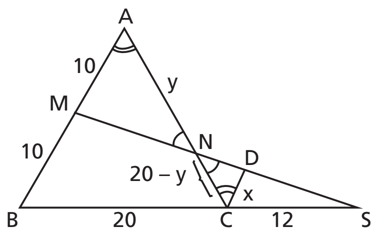
$AC = \left(2 + 2\sqrt{3}\right)$ cm

$$S = \frac{(AB)(AC) \text{ sen } 30°}{2} \Rightarrow S = \frac{4 \cdot \left(2 + 2\sqrt{3}\right)}{4} \Rightarrow S = 2\left(\sqrt{3} + 1\right) \text{ cm}^2$$

876. Traçamos $\overline{CD} \,//\, \overline{AB}$.

$\overline{CD} \,//\, \overline{AB} \Rightarrow \triangle BMS \sim \triangle CDS \Rightarrow$

$$\Rightarrow \frac{10}{x} = \frac{32}{12} \Rightarrow x = \frac{15}{4} \text{ m}$$



$\triangle AMN \sim \triangle CDN \Rightarrow$

$$\Rightarrow \frac{y}{20 - y} = \frac{10}{x} \Rightarrow y = \frac{160}{11}$$

$S_{BCMN} = S_{ABC} - S_{AMN} \Rightarrow$

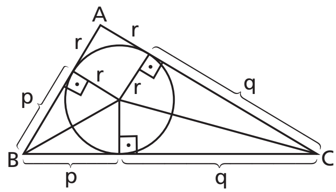
$$\Rightarrow S_{BCMN} = \frac{20^2\sqrt{3}}{4} - \frac{10 \cdot y \cdot \text{sen } 60°}{2} \Rightarrow S_{BCMN} = \frac{700\sqrt{3}}{11} \text{ m}^2$$

879. Seja S a área do $\triangle ABC$.

$$S = \frac{2 \cdot p \cdot r}{2} + \frac{2 \cdot q \cdot r}{2} + r^2 \Rightarrow$$

$\Rightarrow S = pr + qr + r^2$

Aplicando o teorema de Pitágoras ao $\triangle ABC$:



$(p + q)^2 = (p + r)^2 + (q + r)^2 \Rightarrow 2pq = 2pr + 2qr + 2r^2 \Rightarrow$
$\Rightarrow pq = pr + qr + r^2 \Rightarrow pq = S.$

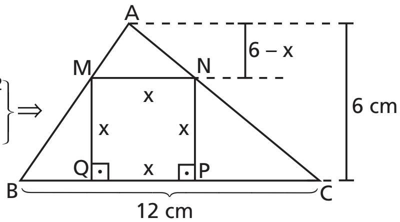
880. $\overline{MN} \mathbin{/\!/} \overline{BC} \Rightarrow \triangle AMN \sim \triangle ABC \Rightarrow$

$\Rightarrow \dfrac{BC}{MN} = \dfrac{H}{h} \Rightarrow \dfrac{12}{x} = \dfrac{6}{6 - x} \Rightarrow$

$\Rightarrow x = 4$ cm

$$\left.\begin{array}{l} S_{ABC} = \dfrac{12 \cdot 6}{2} \Rightarrow S_{ABC} = 36 \text{ cm}^2 \\ S_{MNPQ} = 4^2 \Rightarrow S_{MNPQ} = 16 \text{ cm}^2 \end{array}\right\} \Rightarrow$$

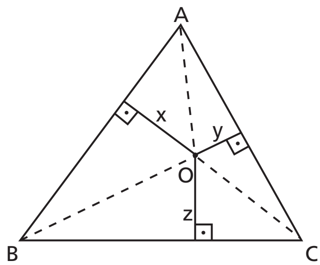
$\Rightarrow \dfrac{S_{ABC}}{S_{MNPQ}} = \dfrac{9}{4}$



886. $S_{ABC} = S_{ABO} + S_{ACO} + S_{BCO} \Rightarrow$

$\Rightarrow \dfrac{ah}{2} = \dfrac{ax}{2} + \dfrac{ay}{2} + \dfrac{az}{2} \Rightarrow$

$\Rightarrow h = x + y + z$



888. a) $(\overline{DA} \equiv \overline{DC}) \Rightarrow \overline{BD}$ é bissetriz de $A\hat{B}C$.

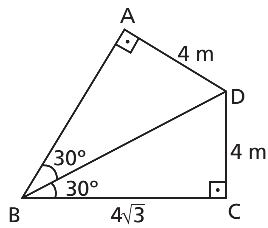
$\triangle BCD \Rightarrow \text{tg } 30° = \dfrac{4}{BC} \Rightarrow$

$\Rightarrow \dfrac{\sqrt{3}}{3} = \dfrac{4}{BC} \Rightarrow BC = 4\sqrt{3}$

$S_{ABCD} = 2 \cdot S_{BCD} \Rightarrow$

$\Rightarrow S_{ABCD} = \dfrac{2 \cdot 4\sqrt{3} \cdot 4}{2} \Rightarrow$

$\Rightarrow S_{ABCD} = 16\sqrt{3} \text{ m}^2$



b) $\text{tg } 60° = \dfrac{CE}{BC} \Rightarrow \sqrt{3} = \dfrac{CE}{6\sqrt{3}} \Rightarrow$

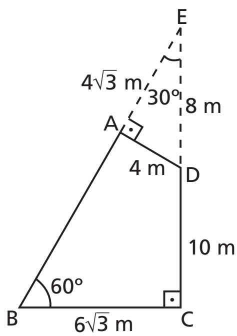
$\Rightarrow (CE = 18 \text{ m}, ED = 8 \text{ m})$

Note $A\hat{E}D = 30°$. No triângulo AED, obtemos: $AD = 4$ m, $AE = 4\sqrt{3}$ m.

$S_{ABCD} = S_{BCE} - S_{ADE} \Rightarrow$

$\Rightarrow S_{ABCD} = \dfrac{6\sqrt{3} \cdot 18}{2} - \dfrac{4 \cdot 4\sqrt{3}}{2} \Rightarrow$

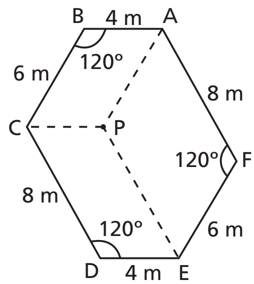
$\Rightarrow S_{ABCD} = 46\sqrt{3} \text{ m}^2.$



889. Na figura ao lado construímos $\overline{AP}$ // $\overline{BC}$, // $\overline{CP}$ // $\overline{DE}$ e $\overline{EP}$ // $\overline{AF}$, de modo que obtivemos os paralelogramos ABCP, CDEP e EFAP.

A área do hexágono será a soma das áreas destes. Assim:

$S_{hex} = S_{ABCP} + S_{CDEP} + S_{EFAP} \Rightarrow$
$\Rightarrow S_{hex} = 4 \cdot 6 \cdot \text{sen } 120° +$
$+ 4 \cdot 8 \cdot \text{sen } 120° +$
$+ 6 \cdot 8 \cdot \text{sen } 120° \Rightarrow$
$\Rightarrow S_{hex} = 52\sqrt{3}\ m^2$.

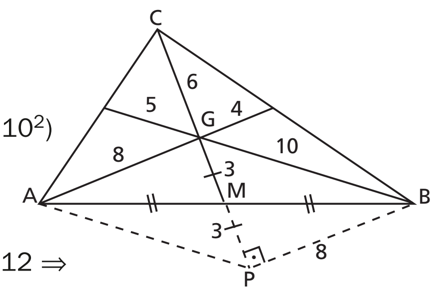


890. Prolongamos $\overline{CM}$, tomando P em $\overline{CM}$, tal que MP = GM.

AM = MB $\Rightarrow$
$\Rightarrow$ APBG é paralelogramo $\Rightarrow$
$\Rightarrow$ BP = AG = 8 m
$\triangle$BGP é retângulo ($8^2 + 6^2 = 10^2$)

$S_{BPM} = \dfrac{8 \cdot 3}{2} \Rightarrow$

$\Rightarrow S_{BPM} = 12\ m^2 = S_{BMG}$
$S_{ABC} = 6 \cdot S_{BMG} \Rightarrow S_{ABC} = 6 \cdot 12 \Rightarrow$
$\Rightarrow S_{ABC} = 72\ m^2$

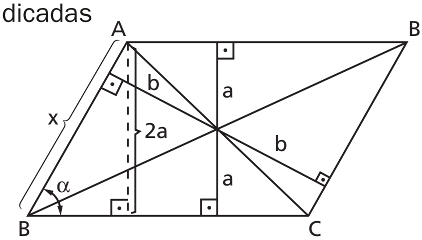


891. Considerando as medidas indicadas na figura, temos:

$\text{sen } \alpha = \dfrac{2a}{x} \Rightarrow x = \dfrac{2a}{\text{sen } \alpha}$

$S_{ABCD} = x \cdot 2b \Rightarrow$

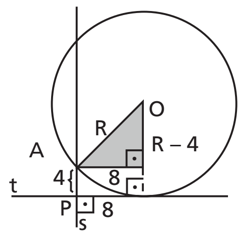
$\Rightarrow S_{ABCD} = \dfrac{4ab}{\text{sen } \alpha}$.



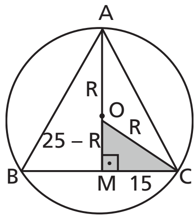
Área do círculo e de suas partes

893. a) No triângulo sombreado:

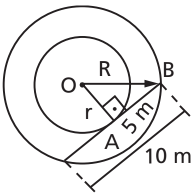
$R^2 = (R - 4)^2 + 8^2 \Rightarrow R = 10$ m
$S = \pi R^2 \Rightarrow S = \pi \cdot 10^2 \Rightarrow$
$\Rightarrow S = 100\pi\ m^2$



b) $(AM = 25,\ OA = R) \Rightarrow$
$\Rightarrow OM = 25 - R$
$\left.\begin{array}{l}\overline{AM} \perp \overline{BC} \Rightarrow \overline{BM} = \overline{MC} \\ BM = 15\end{array}\right\} \Rightarrow$
$\Rightarrow MC = 15$
No triângulo sombreado:
$R^2 = (25 - R)^2 + 15^2 \Rightarrow R = 17\text{ m}$
$S = \pi R^2 \Rightarrow S = 17^2 \cdot \pi \Rightarrow$
$\Rightarrow S = 289\pi\text{ m}^2$

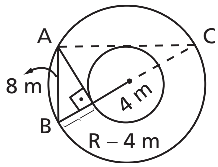


894. b)



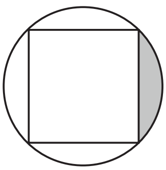
$\triangle OAB \Rightarrow R^2 = r^2 + 5^2 \Rightarrow$
$\Rightarrow R^2 - r^2 = 25\text{ m}^2$
$S_{coroa} = \pi(R^2 - r^2) \Rightarrow$
$\Rightarrow S_{coroa} = 25\pi\text{ m}^2$

c)



Relações métricas no $\triangle ABC$:
$8^2 = 2R \cdot (R - 4) \Rightarrow$
$\Rightarrow R = -4$ (não serve) ou $R = 8\text{ m}$
$S_{coroa} = \pi(R^2 - r^2) \Rightarrow$
$\Rightarrow S_{coroa} = \pi(64 - 16) \Rightarrow$
$\Rightarrow S_{coroa} = 48\pi\text{ m}^2$

903. a)



$\ell = 8 \Rightarrow R\sqrt{2} = 8 \Rightarrow R = 4\sqrt{2}\text{ m}$

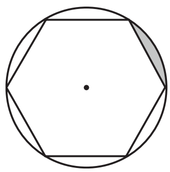
$$S = -\frac{1}{4}(S_{círc} - S_{qua}) \Rightarrow$$

$$\Rightarrow S = \frac{1}{4}(\pi R^2 - \ell^2) \Rightarrow$$

$$\Rightarrow S = \frac{1}{4}(32\pi - 64) \Rightarrow$$

$$\Rightarrow S = 8(\pi - 2)\text{ m}^2$$

b)



$\ell = 6 \Rightarrow R = 6\text{ m}$

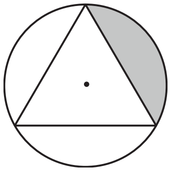
$$S = \frac{1}{6}(S_{círc} - S_{hex}) \Rightarrow$$

$$\Rightarrow S = \frac{1}{6} \cdot \left(\pi R^2 - \frac{3\sqrt{3}\ \ell^2}{2}\right) \Rightarrow$$

$$\Rightarrow S = \frac{1}{6} \cdot \left(36\pi - \frac{3\sqrt{3}}{2} \cdot 36\right) \Rightarrow$$

$$\Rightarrow S = 3\left(2\pi - 3\sqrt{3}\right)\text{ m}^2$$

c)



$\ell = 12 \Rightarrow R\sqrt{3} = 12 \Rightarrow R = 4\sqrt{3}\,\text{m}$

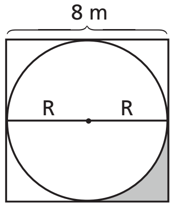
$S = \frac{1}{3}(S_{\text{círc}} - S_{\text{Tri}}) \Rightarrow$

$\Rightarrow S = \frac{1}{3}\left(\pi R^2 - \frac{\ell^2\sqrt{3}}{4}\right) \Rightarrow$

$\Rightarrow S = \frac{1}{3}\left(48\pi - \frac{144\sqrt{3}}{4}\right) \Rightarrow$

$\Rightarrow S = 4(4\pi - 3\sqrt{3})\ \text{m}^2$

d)



$\ell = 8\ \text{m} \Rightarrow r = 4\ \text{m}$

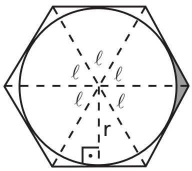
$S = \frac{1}{4}(S_{\text{qua}} - S_{\text{círc}}) \Rightarrow$

$\Rightarrow S = \frac{1}{4}(\ell^2 - \pi r^2) \Rightarrow$

$\Rightarrow S = \frac{1}{4}(64 - 16\pi) \Rightarrow$

$\Rightarrow S = 4(4 - \pi)\ \text{m}^2$

e)



$\ell = 12\ \text{m} \Rightarrow r = \frac{\ell\sqrt{3}}{2} \Rightarrow r = 6\sqrt{3}\,\text{m}$

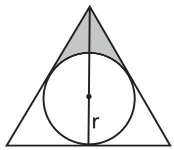
$S = \frac{1}{6}(S_{\text{hex}} - S_{\text{círc}}) \Rightarrow$

$\Rightarrow S = \frac{1}{6}\left(\frac{3\sqrt{3}\ \ell^2}{2}\ \pi r^2\right) \Rightarrow$

$\Rightarrow S = \frac{1}{6}\left(216\sqrt{3} - 108\pi\right) \Rightarrow$

$\Rightarrow S = 18(2\sqrt{3} - \pi)\ \text{m}^2$

f)



$\ell = 6\,\text{m} \Rightarrow h = \frac{\ell\sqrt{3}}{2} \Rightarrow h = 3\sqrt{3}\ \text{m}$

$r = \frac{1}{3}h \Rightarrow r = \sqrt{3}\ \text{m}$

$S = \frac{1}{3}(S_{\text{Tri}} - S_{\text{círc}}) \Rightarrow$

$\Rightarrow S = \frac{1}{3}\left(\frac{\ell^2\sqrt{3}}{4} - \pi r^2\right) \Rightarrow$

$\Rightarrow S = \frac{1}{3}(9\sqrt{3} - 3\pi) \Rightarrow$

$\Rightarrow S = (3\sqrt{3} - \pi)\ \text{m}^2$

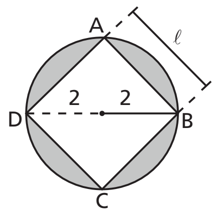
904. Seja *d* a diagonal do quadrado. Então:

$d = 4 \Rightarrow \ell\sqrt{2} = 4 \Rightarrow \ell = 2\sqrt{2}$.

$S = S_{\text{círc}} - S_{\text{qua}} \Rightarrow S = \pi r^2 - \ell^2 \Rightarrow$

$\Rightarrow S = \pi \cdot 2^2 - (2\sqrt{2})^2 \Rightarrow$

$\Rightarrow S = 4(\pi - 2)$.



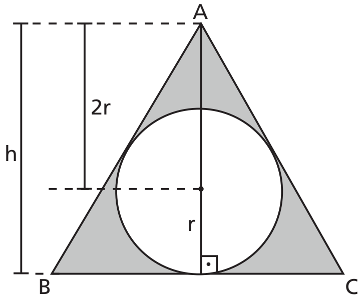
905. $h = 3r \Rightarrow \dfrac{\ell\sqrt{3}}{2} = 3r \Rightarrow \ell = 2\sqrt{3}r$

$S = S_{Tri} - S_{círc} \Rightarrow$

$\Rightarrow S = \dfrac{\ell^2\sqrt{3}}{4} - \pi r^2 \Rightarrow$

$\Rightarrow S = \dfrac{12 \cdot \sqrt{3}}{4} r^2 - \pi r^2 \Rightarrow$

$\Rightarrow S = (3\sqrt{3} - \pi)r^2$



906. $a = \dfrac{\ell\sqrt{3}}{2} = 5\sqrt{3} \Rightarrow \ell = 10 \text{ cm} \Rightarrow$

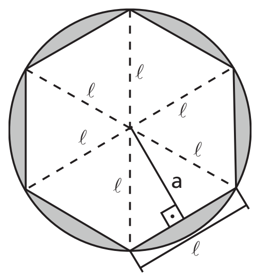
$\Rightarrow R = 10 \text{ cm}$

$S = S_{círc} - S_{hex} \Rightarrow$

$\Rightarrow S = \pi R^2 - \dfrac{3\sqrt{3}\ \ell^2}{2} \Rightarrow$

$\Rightarrow S = 100\pi - 150\sqrt{3} \Rightarrow$

$\Rightarrow S = 50(2\pi - 3\sqrt{3}) \text{ cm}^2$



907. $a = \sqrt{3} \text{ cm} \Rightarrow \dfrac{1}{3}h = \sqrt{3} \Rightarrow$

$\Rightarrow h = 3\sqrt{3} \text{ cm} \Rightarrow \dfrac{\ell\sqrt{3}}{2} = 3\sqrt{3} \Rightarrow$

$\Rightarrow \ell = 6 \text{ cm}$

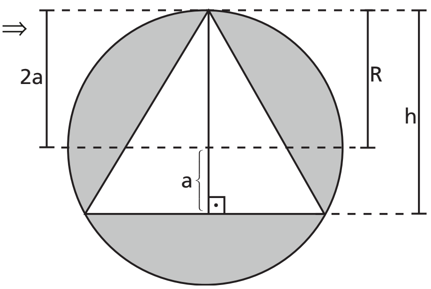
$R = 2a \Rightarrow R = 2\sqrt{3} \text{ cm}$

$S = S_{círc} - S_{Tri} \Rightarrow$

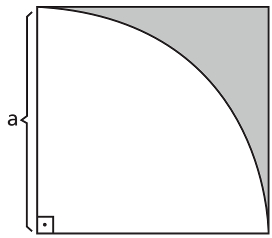
$\Rightarrow S = \pi R^2 - \dfrac{\ell^2\sqrt{3}}{4} \Rightarrow$

$\Rightarrow S = 12\pi - 9\sqrt{3} \Rightarrow$

$\Rightarrow S = 3(4\pi - 3\sqrt{3}) \text{ cm}^2$



908. a)

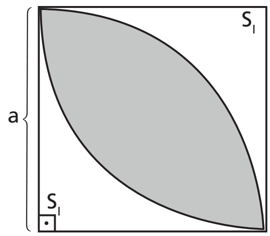


$S = S_{qua} - S_{setor} \Rightarrow$

$\Rightarrow S = a^2 - \dfrac{\pi a^2}{4} \Rightarrow$

$\Rightarrow S = \dfrac{4 - \pi}{4} a^2$

b)

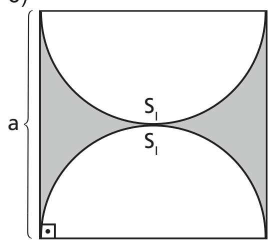


$S = S_{qua} - 2S_I \xRightarrow{\text{item } a}$

$\Rightarrow S = a^2 - 2 \cdot \dfrac{4 - \pi}{4} a^2 \Rightarrow$

$\Rightarrow S = \dfrac{\pi - 2}{2} \cdot a^2$

c)

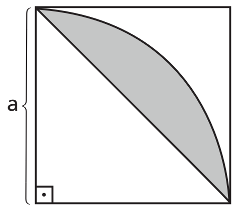


$S = S_{qua} - 2 \cdot S_I \Rightarrow$

$\Rightarrow S = a^2 - 2 \cdot \dfrac{\pi \left(\frac{a}{2}\right)^2}{2} \Rightarrow$

$\Rightarrow S = \dfrac{4 - \pi}{4} a^2$

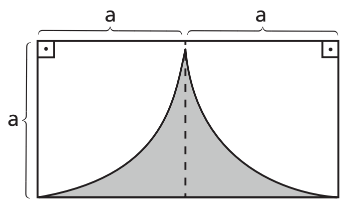
909. a)



Note que a área sombreada é metade da área sombreada do exercício 908, item *b*. Logo:

$S = \dfrac{\pi - 2}{4} a^2.$

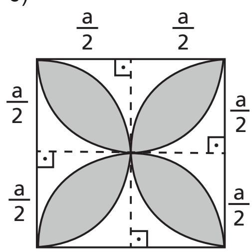
b)



Aqui a área sombreada é o dobro da área sombreada no exercício 908, item *a*. Logo:

$S = \dfrac{4 - \pi}{2} a^2.$

c)



$S_{pétala} \overset{\text{ex. 908}}{\underset{\text{item } b}{=}} \dfrac{\pi - 2}{4} \cdot \left(\dfrac{a}{2}\right)^2$

$S = 4 \cdot S_{pétala} \Rightarrow$

$\Rightarrow S = 4 \cdot \dfrac{\pi - 2}{8} a^2 \Rightarrow$

$\Rightarrow S = \dfrac{\pi - 2}{2} a^2$

910. a) $2p = 16$ cm $\Rightarrow a = 4$ cm

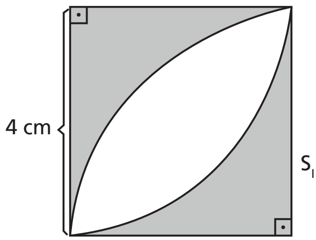
Exercício 908 item *a* $\Rightarrow$

$\Rightarrow S_I = \dfrac{4 - \pi}{4} \cdot a^2 \Rightarrow$

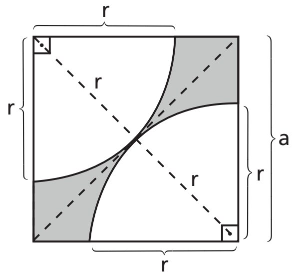
$\Rightarrow S_I = 4(4 - \pi)$

$S = 2 \cdot S_I \Rightarrow S = 2 \cdot 4(4 - \pi) \Rightarrow$

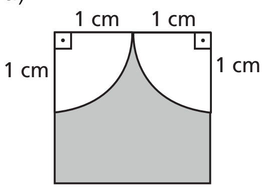
$\Rightarrow S = 8(4 - \pi)$ cm^2



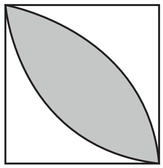
b) $2p = 16\text{ cm} \Rightarrow a = 4\text{ cm} \Rightarrow$
$\Rightarrow d = 4\sqrt{2}\text{ cm}$
$r = \dfrac{d}{2} \Rightarrow r = 2\sqrt{2}\text{ cm}$
$S = S_{qua} - 2 \cdot \dfrac{\pi r^2}{4} \Rightarrow$
$\Rightarrow S = 4^2 - 2 \cdot \dfrac{\pi \cdot (2\sqrt{2})^2}{4} \Rightarrow$
$\Rightarrow S = 4(4 - \pi)\text{ cm}^2$



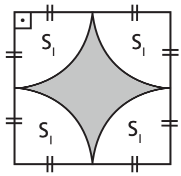
911. a)



b)



c)

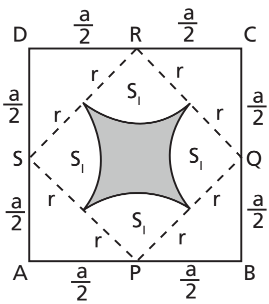


a)
$S = S_{qua} - \dfrac{S_{círc}}{2} \Rightarrow$
$\Rightarrow S = \ell^2 - \dfrac{\pi r^2}{2} \Rightarrow$
$\Rightarrow S = 2^2 - \dfrac{\pi \cdot 1^2}{2} \Rightarrow$
$\Rightarrow S = \dfrac{8 - \pi}{2}\text{ cm}^2$

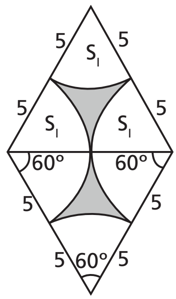
b)
Ex. 908 item *b* $\Rightarrow$
$\Rightarrow S = \dfrac{(\pi - 2)}{2} \cdot a^2 \Rightarrow$
$\Rightarrow S = \dfrac{(\pi - 2)}{2} \cdot 2^2 \Rightarrow$
$\Rightarrow S = 2(\pi - 2)\text{ cm}^2$

c)
$S_I = \dfrac{\pi \cdot 1^2}{4} \Rightarrow$
$\Rightarrow S_I = \dfrac{\pi}{4}\text{ cm}^2 \Rightarrow$
$\Rightarrow S = S_{qua} - 4S_I \Rightarrow$
$\Rightarrow S = 2^2 - 4 \cdot \dfrac{\pi}{4} \Rightarrow$
$\Rightarrow S = (4 - \pi)\text{ cm}^2$

912. $\triangle DRS \Rightarrow SR = \dfrac{a\sqrt{2}}{2} \Rightarrow$
$\Rightarrow r = \dfrac{a\sqrt{2}}{4}$
$S = S_{PQRS} - 4 \cdot S_I \Rightarrow$
$\Rightarrow S = \left(\dfrac{a\sqrt{2}}{2}\right)^2 - \dfrac{4 \cdot \pi\left(\dfrac{a\sqrt{2}}{4}\right)^2}{4} \Rightarrow$
$\Rightarrow S = \dfrac{4 - \pi}{8} \cdot a^2$



914. $S = 2 \cdot (S_{Tri} - 3 \cdot S_I) \Rightarrow$
$\Rightarrow S = 2\left(\dfrac{\ell^2\sqrt{3}}{4} - 3 \cdot \dfrac{\pi \cdot r^2}{6}\right) \Rightarrow$
$\Rightarrow S = 2 \cdot \left(\dfrac{10^2\sqrt{3}}{4} - \dfrac{3 \cdot \pi \cdot 5^2}{6}\right) \Rightarrow$
$\Rightarrow S = 25(2\sqrt{3} - \pi)\text{ cm}^2$



915. a) $AC + CB = AB \Rightarrow$

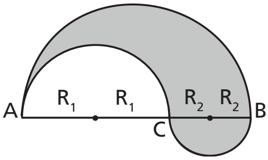
$\Rightarrow 3CB + CB = 32 \Rightarrow CB = 8$

$CB = 8$ cm $\Rightarrow AC = 24$ cm

$$S = \frac{\pi(R_1 + R_2)^2}{2} - \frac{\pi R_1^2}{2} + \frac{\pi R_2^2}{2} \Rightarrow$$

$$\Rightarrow S = \frac{\pi(16)^2}{2} - \frac{\pi \cdot 12^2}{2} + \frac{\pi \cdot 4^2}{2} \Rightarrow$$

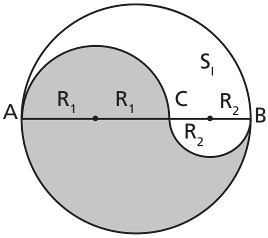
$\Rightarrow S = 64\pi$ cm^2



b) $S = \pi(R_1 + R_2)^2 - S_1 \Rightarrow$

$\Rightarrow S = \pi \cdot 16^2 - 64\pi \Rightarrow$

$\Rightarrow S = 192\pi$ cm^2



917. b) $AC + CO + OD + DB = 20 \Rightarrow$

$\Rightarrow 4AC = 20 \Rightarrow AC = 5$ cm

$CD = 2AC \Rightarrow CD = 10$ cm

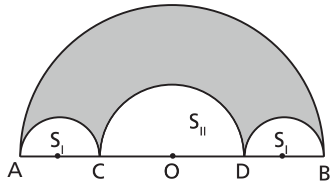
$$S_I = \pi \cdot \left(\frac{5}{2}\right)^2 \cdot \frac{1}{2} \Rightarrow$$

$$\Rightarrow S_I = \frac{25\pi}{8} \text{ cm}^2$$

$$S_{II} = \frac{\pi \cdot 5^2}{2} \Rightarrow S_{II} = \frac{25\pi}{2} \text{ cm}^2$$

$$S = \frac{\pi \cdot 10^2}{2} - 2 \cdot S_I - S_{II} \Rightarrow$$

$$\Rightarrow S = \frac{100\pi}{2} - 2 \cdot \frac{25\pi}{8} - \frac{25\pi}{2} \Rightarrow S = \frac{125\pi}{4} \text{ cm}^2$$



918. $AM + MB + BC + OC + OD + DA = 42 \Rightarrow$

$\Rightarrow 6r = 42 \Rightarrow r = 7$ cm

$S_I = S_{ABCD} - 2 \cdot S_{II} \Rightarrow$

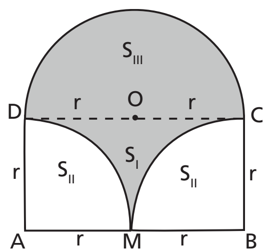
$$\Rightarrow S_I = 14 \cdot 7 - 2 \cdot \frac{\pi \cdot 7^2}{4} \Rightarrow$$

$$\Rightarrow S_I = \frac{196 - 49\pi}{2} \text{ cm}^2$$

$$S_{III} = \frac{\pi \cdot r^2}{2} \Rightarrow S_{III} = \frac{\pi \cdot 7^2}{2} \Rightarrow$$

$$\Rightarrow S_{III} = \frac{49\pi}{2}$$

$S = S_I + S_{III} \Rightarrow$

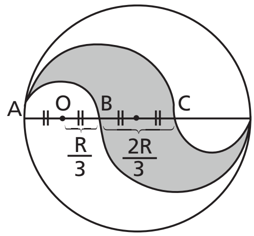


$$\Rightarrow S = \frac{196 - 49\pi}{2} + \frac{49\pi}{2} \Rightarrow$$
$$\Rightarrow S = 98 \text{ cm}^2$$

919. a) Análogo ao exercício 914.

b) $OA = OB = \dfrac{R}{3}$

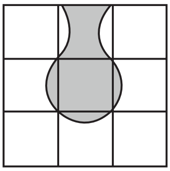
$BC = \dfrac{2R}{3}$

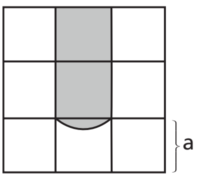


$$S = 2(S_{\text{setor AC}} - S_{\text{setor AB}}) \Rightarrow$$
$$\Rightarrow S = 2 \cdot \left(\frac{\pi \cdot BC^2}{2} - \frac{\pi \cdot OA^2}{2}\right) \Rightarrow$$
$$\Rightarrow S = 2 \cdot \left(\frac{\pi \cdot \left(\frac{2R}{3}\right)^2}{2} - \frac{\pi \cdot \left(\frac{R}{3}\right)^2}{2}\right) \Rightarrow$$
$$\Rightarrow S = \frac{\pi R^2}{3}$$

922. Note que as duas regiões sombreadas, nas figuras ao lado, são equivalentes.

Determinemos o valor do menor segmento circular determinado pelo lado de um quadrado de medida *a* e raio de circunferência circunscrita igual a R.





$$\ell = R\sqrt{2} \Rightarrow a = R\sqrt{2} \Rightarrow R = \frac{\sqrt{2}}{2}a$$
$$S_{seg} = \frac{1}{4} \cdot (S_{\text{círc}} - S_{qua}) \Rightarrow$$
$$\Rightarrow S_{seg} = \frac{1}{4}(\pi R^2 - \ell^2) \Rightarrow$$
$$\Rightarrow S_{seg} = \frac{1}{4}\left(\frac{\pi \cdot a^2}{2} - a^2\right) \Rightarrow S_{seg} = \frac{(\pi - 2)}{8}a^2$$

Sendo S a área sombreada, temos:

$$S = 2 \cdot S_{qua} + S_{seg} \Rightarrow S = 2 \cdot a^2 + \frac{\pi - 2}{8} \cdot a^2 \Rightarrow$$
$$\Rightarrow S = \frac{\pi + 14}{8} \cdot a^2.$$

923. Note o $\triangle ABO$, equilátero.

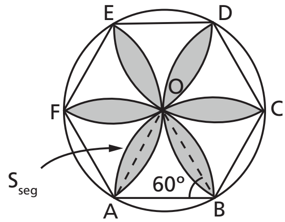
$$S_{seg} = S_{\underset{ABO}{setor}} - S_{\underset{ABC}{Tri}} \Rightarrow$$
$$\Rightarrow S_{seg} = \left(\frac{\pi \cdot 1^2}{6} - \frac{1^2\sqrt{3}}{4}\right) \text{cm}^2$$

Sendo S a área sombreada, temos:

$S = 12 \cdot S_{seg} \Rightarrow$

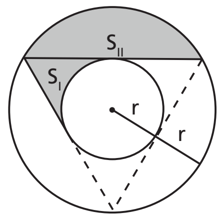
$\Rightarrow S = 12\left(\dfrac{\pi}{6} - \dfrac{\sqrt{3}}{4}\right) \Rightarrow$

$\Rightarrow S = \left(2\pi - 3\sqrt{3}\right)\ \text{cm}^2.$



925. Note que AC é o lado de um triângulo equilátero inscrito numa circunferência de raio 2r.

Sejam ℓ o lado do triângulo, S_C a área do círculo maior e S_c a área do círculo menor. Daí:



$\ell = (2r)\sqrt{3} \Rightarrow S_{tri} = \dfrac{(2r\sqrt{3})^2\sqrt{3}}{4} \Rightarrow$

$\Rightarrow S_{tri} = 3\sqrt{3}r^2$

$S_C = \pi(2r)^2 \Rightarrow S_C = 4\pi r^2;\ S_c = \pi r^2$

$S_I = \dfrac{S_{tri} - S_c}{3} \Rightarrow S_I = \dfrac{3\sqrt{3}r^2 - \pi r^2}{3}$

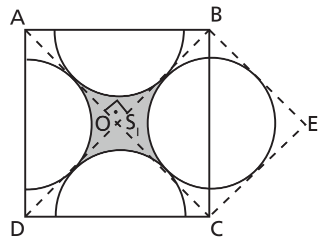
$S_{II} = \dfrac{S_C - S_{tri}}{3} \Rightarrow S_{II} = \dfrac{4\pi r^2 - 3\sqrt{3}r^2}{3}$

$S = S_I + S_{II} \Rightarrow S = \dfrac{3\sqrt{3}r^2 - \pi r^2 + 4\pi r^2 - 3\sqrt{3}r^2}{3} \Rightarrow S = \pi r^2$

926. $2p = 16 \Rightarrow \ell = 4$ cm

Considere $\overline{BE} \parallel \overline{OC}$ e $\overline{CE} \parallel \overline{OB}$.

Temos:



$OB = \dfrac{d}{2} \Rightarrow OB = \dfrac{\ell\sqrt{2}}{2} \Rightarrow$

$\Rightarrow OB = 2\sqrt{2}$ cm.

$S_I = \dfrac{1}{4}(S_{BECO} - S_{círc}) \Rightarrow$

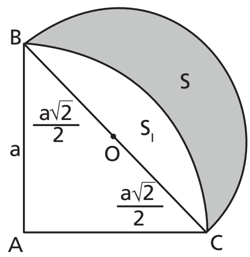
$\Rightarrow S_I = \dfrac{1}{4}\left[OB^2 - \pi\left(\dfrac{OB}{2}\right)^2\right] \Rightarrow$

$\Rightarrow S_I = \dfrac{1}{4} \cdot \left[(2\sqrt{2})^2 - \pi\left(\dfrac{2\sqrt{2}}{2}\right)^2\right] \Rightarrow S_I = \dfrac{4 - \pi}{2}\ \text{cm}^2$

$S = 4 \cdot S_I \Rightarrow S = 4 \cdot \left(\dfrac{4 - \pi}{2}\right) \Rightarrow S = 2(4 - \pi)\ \text{cm}^2$

927. a) $S_I = \dfrac{\pi a^2}{4} - \dfrac{a \cdot a}{2} \Rightarrow$

$\Rightarrow S_I = \dfrac{(\pi - 2)}{4} \cdot a^2$



$BC = a\sqrt{2} \Rightarrow OC = \dfrac{a\sqrt{2}}{2}$

$S = S_{\text{setor } BC} - S_I \Rightarrow$

$\Rightarrow S = \dfrac{\pi \cdot OC^2}{2} - \dfrac{(\pi - 2)a^2}{4} \Rightarrow$

$\Rightarrow S = \dfrac{\pi\left(\dfrac{a\sqrt{2}}{2}\right)^2}{2} - \dfrac{(\pi - 2)a^2}{4} \Rightarrow S = \dfrac{a^2}{2}$

b)

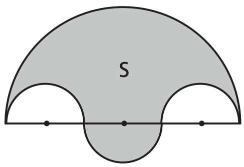


Figura 1

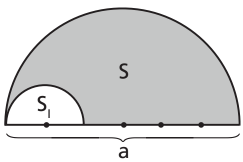
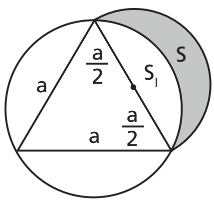


Figura 2

Note que a área da região sombreada na figura 1 é equivalente à área sombreada na figura 2.

$S = \dfrac{\pi\left(\dfrac{a}{2}\right)^2}{2} - \dfrac{\pi\left(\dfrac{a}{6}\right)^2}{2} \Rightarrow S = \dfrac{\pi a^2}{9}$

c) $\ell = a \Rightarrow R\sqrt{3} = a \Rightarrow R = \dfrac{a}{\sqrt{3}}$



$S_I = \dfrac{1}{3}(S_{\text{círc}} - S_{\text{tri}}) \Rightarrow$

$\Rightarrow S_I = \dfrac{1}{3}\left(\pi R^2 - \dfrac{\ell^2\sqrt{3}}{4}\right) \Rightarrow$

$\Rightarrow S_I = \dfrac{1}{3}\left[\pi\left(\dfrac{a}{\sqrt{3}}\right)^2 - \dfrac{(a)^2\sqrt{3}}{4}\right] \Rightarrow$

$\Rightarrow S_I = \left(\dfrac{\pi}{9} - \dfrac{\sqrt{3}}{12}\right) a^2$

$S = \dfrac{\pi\left(\dfrac{a}{2}\right)^2}{2} - S_I \Rightarrow$

$\Rightarrow S = \dfrac{\pi a^2}{8} - \dfrac{\pi a^2}{9} + \dfrac{\sqrt{3}a^2}{12} \Rightarrow S = \dfrac{\pi + 6\sqrt{3}}{72}\, a^2$

928. $AB = BC = r\sqrt{2}$

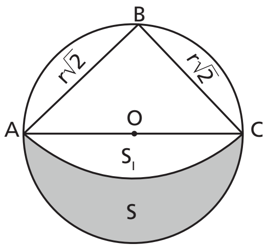
$S_I = \dfrac{\pi \cdot (r\sqrt{2})^2}{4} - \dfrac{(r\sqrt{2})(r\sqrt{2})}{2} \Rightarrow$

$\Rightarrow S_I = \dfrac{\pi r^2}{2} - r^2$

$S = \dfrac{\pi r^2}{2} - S_I \Rightarrow$

$\Rightarrow S = \dfrac{\pi r^2}{2} - \left(\dfrac{\pi r^2}{2} - r^2\right) \Rightarrow$

$\Rightarrow S = r^2$

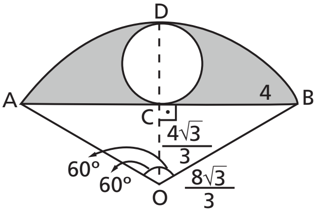


929. $\triangle OCB \Rightarrow \begin{cases} \text{sen } 60° = \dfrac{4}{OB} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{4}{OB} \Rightarrow OB = \dfrac{8\sqrt{3}}{3} \text{ cm} = OD \\ \text{tg } 60° = \dfrac{4}{OC} \Rightarrow \sqrt{3} = \dfrac{4}{OC} \Rightarrow OC = \dfrac{4\sqrt{3}}{3} \text{ cm} \end{cases}$

$CD = OD - CD \Rightarrow CD = \dfrac{8\sqrt{3}}{3} - \dfrac{4\sqrt{3}}{3} \Rightarrow CD = \dfrac{4\sqrt{3}}{3} \text{ cm} \Rightarrow$

$S = S_{setor} - S_{círculo} - S_{tri} \Rightarrow S = \dfrac{\pi \cdot OB^2}{3} - \pi\left(\dfrac{CD}{2}\right)^2 - \dfrac{(OB)(OB) \text{ sen } 120°}{2} \Rightarrow$

$\Rightarrow S = \pi \cdot \dfrac{64}{9} - \dfrac{\pi \cdot 16}{12} - \dfrac{64}{6} \cdot \dfrac{\sqrt{3}}{2} \Rightarrow S = \dfrac{4}{9}(13\pi - 12\sqrt{3}) \text{ cm}^2$



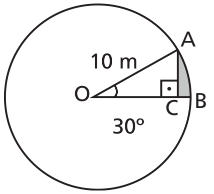
930. $\left.\begin{array}{l} \triangle AOC \Rightarrow \text{sen } 30° = \dfrac{AC}{OA} \Rightarrow \dfrac{1}{2} = \dfrac{AC}{10} \Rightarrow AC = 5 \text{ m} \\ \cos 30° = \dfrac{OC}{OA} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{OC}{10} \Rightarrow OC = 5\sqrt{3} \text{ m} \end{array}\right\} \Rightarrow$

$\Rightarrow S_{\triangle AOC} = \dfrac{5 \cdot 5\sqrt{3}}{2} = \dfrac{25\sqrt{3}}{2} \text{ m}^2$

$S = S_{setor} - S_{\triangle AOC} \Rightarrow$

$\Rightarrow S = \dfrac{\pi \cdot 10^2}{12} - \dfrac{25\sqrt{3}}{2} \Rightarrow$

$\Rightarrow S = \dfrac{25}{6}(2\pi - 3\sqrt{3}) \text{ m}^2$



931. ABC é triângulo equilátero de lado *a*.

$S_{seg} = S_{setor} - S_{\triangle ABC} \Rightarrow$

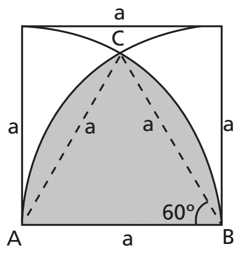
$\Rightarrow S_{seg} = \dfrac{\pi \cdot a^2}{6} - \dfrac{a^2\sqrt{3}}{4} \Rightarrow$

$\Rightarrow S_{seg} = \dfrac{2\pi - 3\sqrt{3}}{12} \cdot a^2$

$S = S_{\triangle ABC} + 2 \cdot S_{seg} \Rightarrow$

$\Rightarrow S = \dfrac{a^2\sqrt{3}}{4} + 2 \cdot \dfrac{(2\pi - 3\sqrt{3})}{12} a^2 \Rightarrow$

$\Rightarrow S = \dfrac{4\pi - 3\sqrt{3}}{12} a^2$

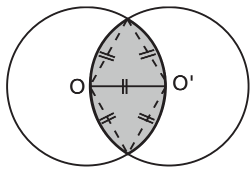


932. Note que este exercício é análogo ao anterior, bastando considerar $a = OO' = 26$ cm.

$S = 2 \cdot \dfrac{(4\pi - 3\sqrt{3})}{12} \cdot a^2 \Rightarrow$

$\Rightarrow S = 2 \cdot \dfrac{(4\pi - 3\sqrt{3})}{12} \cdot 26^2 \Rightarrow$

$\Rightarrow S = \dfrac{338(4\pi - 3\sqrt{3})}{3}$ cm^2



933. $\text{sen } 60° = \dfrac{AB}{AC} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{AB}{10} \Rightarrow$

$\Rightarrow AB = 5\sqrt{3}$ cm

$\cos 60° = \dfrac{BC}{AC} \Rightarrow \dfrac{1}{2} = \dfrac{BC}{10} \Rightarrow$

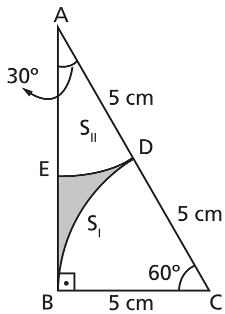
$\Rightarrow BC = 5 \text{ cm} \Rightarrow CD = 5 \text{ cm} \Rightarrow AD = 5$ cm

$S = S_{\triangle ABC} - S_{I} - S_{II} \Rightarrow$

$\Rightarrow S = \dfrac{(AB)(BC)}{2} - \dfrac{\pi(BC)^2}{6} - \dfrac{\pi(AD)^2}{12} \Rightarrow$

$\Rightarrow S = \dfrac{5\sqrt{3} \cdot 5}{2} - \dfrac{\pi \cdot 5^2}{6} - \dfrac{\pi \cdot 5^2}{12} \Rightarrow$

$\Rightarrow S = \dfrac{25}{4}(2\sqrt{3} - \pi)$ cm^2



934. $\text{sen } 45° = \dfrac{AB}{AC} \Rightarrow \dfrac{\sqrt{2}}{2} = \dfrac{AB}{10} \Rightarrow$

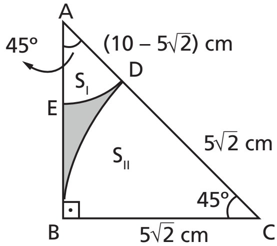
$\Rightarrow AB = 5\sqrt{2} \text{ cm} = CD = AB$

$(CD = 5\sqrt{2} \text{ cm}, AC = 10 \text{ cm}) \Rightarrow$

$\Rightarrow AD = \left(10 - 5\sqrt{2}\right)$ cm $= AE$

$\left(AE = \left(10 - 5\sqrt{2}\right) \text{ cm}, AB = 5\sqrt{2} \text{ cm}\right) \Rightarrow$

$\Rightarrow BE = \left(10\sqrt{2} - 10\right)$ cm



$$\widehat{BD} = \frac{45°}{360°} \cdot 2\pi \cdot (BC) \Rightarrow$$

$$\Rightarrow \widehat{BD} = \frac{1}{8} \cdot 2\pi \cdot 5\sqrt{2} \Rightarrow$$

$$\Rightarrow \widehat{BD} = \frac{5\pi\sqrt{2}}{4} \text{ cm}$$

$$\widehat{DE} = \frac{45°}{360°} \cdot 2\pi \cdot (AD) \Rightarrow \widehat{DE} = \frac{1}{8} \cdot 2\pi \cdot \left(10 - 5\sqrt{2}\right) \Rightarrow$$

$$\Rightarrow \widehat{DE} = \frac{\pi}{4}\left(10 - 5\sqrt{2}\right) \text{ cm}$$

$$2p = BE + \widehat{BD} + \widehat{DE} \Rightarrow 2p = 10\sqrt{2} - 10 + \frac{5\pi\sqrt{2}}{4} + \frac{\pi\left(10 - 5\sqrt{2}\right)}{4} \Rightarrow$$

$$\Rightarrow 2p = \frac{5}{2}\left(4\sqrt{2} + \pi - 4\right) \text{ cm}$$

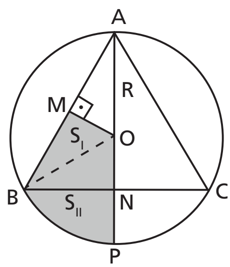
$$S = S_{\triangle ABC} - S_I - S_{II} \Rightarrow S = \frac{(AB)(BC)}{2} - \frac{1}{8}\pi \cdot (AD)^2 - \frac{1}{8}\pi \cdot (BC)^2 \Rightarrow$$

$$\Rightarrow S = \frac{(5\sqrt{2})(5\sqrt{2})}{2} - \frac{\pi\left(10 - 5\sqrt{2}\right)^2}{8} - \frac{\pi\left(5\sqrt{2}\right)^2}{8} \Rightarrow$$

$$\Rightarrow S = \frac{25}{2}\left(2 + \sqrt{2}\pi - 2\pi\right) \text{ cm}^2$$

935. Seja ℓ o lado do triângulo. Temos:

$\ell = R\sqrt{3}$; $OM = \dfrac{R}{2}$; $B\hat{O}P = 60°$.



$$S = S_I + S_{II} \Rightarrow$$

$$\Rightarrow S = \frac{(BM) \cdot (OM)}{2} + \frac{60°}{360°} \cdot \pi \cdot R^2 \Rightarrow$$

$$\Rightarrow S = \frac{\left(\frac{R\sqrt{3}}{2}\right) \cdot \frac{R}{2}}{2} + \frac{\pi R^2}{6} \Rightarrow$$

$$\Rightarrow S = \frac{\left(3\sqrt{3} + 4\pi\right)}{24} R^2$$

936. $\triangle ABC$ é isósceles e retângulo $\Rightarrow$

$\Rightarrow \hat{B} = \hat{C} = 45°$.

Note que $AP = r\sqrt{2} + r$.

($\triangle APB$ é retângulo, $\hat{B} = 45°$) $\Rightarrow$

$\Rightarrow AP = PB = r\sqrt{2} + r$

Analogamente, $AP = PC = r\sqrt{2} + r$.

Exercício 879 $\Rightarrow$

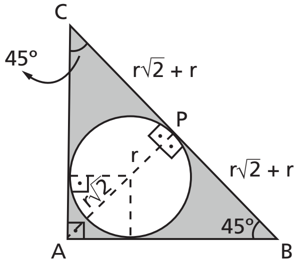
$\Rightarrow S_{ABC} = (BP)(PC) = (r\sqrt{2} + r)^2$

Logo:

$S = S_{ABC} - S_{círc} \Rightarrow$

$\Rightarrow S = (r\sqrt{2} + r)^2 - \pi r^2 \Rightarrow$

$\Rightarrow S = r^2(3 + 2\sqrt{2} - \pi).$



937. $\left.\begin{array}{l} R_A + R_B = 10 \ (1) \\ R_A + R_C = 14 \ (2) \\ R_B + R_C = 18 \ (3) \end{array}\right\} \xRightarrow{+}$

$\Rightarrow R_A + R_B + R_C = 21 \ (4)$

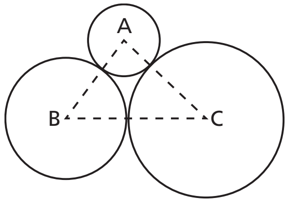
$(4) - (1)$: $R_C = 11$

$(4) - (2)$: $R_B = 7$

$(4) - (3)$: $R_A = 3$

Daí:

$S_A = 9\pi \text{ cm}^2$; $S_B = 49\pi \text{ cm}^2$; $S_C = 121 \text{ cm}^2$.



941. $\left\{\begin{array}{l} S_{setor} = \dfrac{\ell \cdot R}{8} \\ \ell = 2\pi, S_{setor} = 6\pi \end{array}\right. \Rightarrow 6\pi = \dfrac{2\pi R}{2} \Rightarrow R = 6 \text{ cm}$

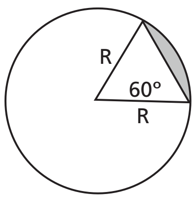
Na figura ao lado, temos:

$S_{seg} = S_{setor} - S_{tri} \Rightarrow$

$\Rightarrow S_{seg} = \dfrac{\pi \cdot R^2}{6} - \dfrac{R \cdot R \cdot \text{sen } 60°}{2} \Rightarrow$

$\Rightarrow S_{seg} = \dfrac{\pi \cdot 6^2}{6} - \dfrac{6^2 \cdot \left(\dfrac{\sqrt{3}}{2}\right)}{2} \Rightarrow$

$\Rightarrow S_{seg} = 3(2\pi - 3\sqrt{3}) \text{ cm}^2$



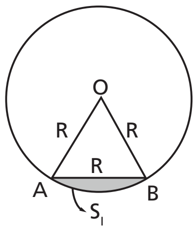
942. $AB = OA \Rightarrow \triangle OAB$ é equilátero $\Rightarrow$

$\Rightarrow A\hat{O}B = 60°$

$S_I = S_{setor} - S_{tri} \Rightarrow$

$\Rightarrow S_I = \dfrac{\pi R^2}{6} - \dfrac{R^2\sqrt{3}}{4} \Rightarrow$

$\Rightarrow S_I = \dfrac{2 - 3\sqrt{3}}{12} \cdot R^2$



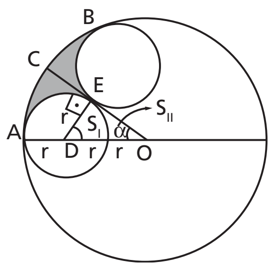
$S_{II} = S_{círc} - S_I \Rightarrow$

$\Rightarrow S_{II} = \pi R^2 - \dfrac{2 - 3\sqrt{3}}{12} R^2 \Rightarrow$

$\Rightarrow S_{II} = \dfrac{10\pi + 3\sqrt{3}}{12} R^2$

$$\frac{S_I}{S_{II}} = \frac{\dfrac{(2\pi - 3\sqrt{3})R^2}{12}}{\dfrac{(10\pi - 3\sqrt{3})R^2}{12}} = \frac{2\pi - 3\sqrt{3}}{10\pi + 3\sqrt{3}} \left(\text{ou } \frac{S_{II}}{S_I} = \frac{10\pi + 3\sqrt{3}}{2\pi - 3\sqrt{3}}\right)$$

943. Considerando as medidas indicadas na figura, temos:



$\text{sen } \alpha = \dfrac{r}{2r} \Rightarrow \text{sen } \alpha = \dfrac{1}{2} \Rightarrow \alpha = 30°$

S_I é o setor de 60° do círculo menor $\Rightarrow$

$\Rightarrow S_I = \dfrac{\pi r^2}{6}.$

$\triangle ODE \Rightarrow OE^2 + DE^2 = OD^2 \Rightarrow$

$\Rightarrow OE^2 + r^2 = (2r)^2 \Rightarrow OE = r\sqrt{3}$

$S_{II} = S_{\triangle ODE} - S_I \Rightarrow$

$\Rightarrow S_{II} = \dfrac{r\sqrt{3} \cdot r}{2} - \dfrac{\pi r^2}{6} \Rightarrow S_{II} = \dfrac{3\sqrt{3} - \pi}{6} \cdot r^2$

S_{III} é o setor de 30° do círculo maior.

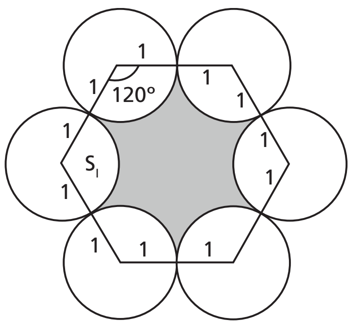
(arco $\overset{\frown}{AC}$) $\Rightarrow S_{III} = \dfrac{\pi(3r)^2}{12} \Rightarrow S_{III} = \dfrac{3\pi r^2}{4}.$

S é a área pedida. Então:

$S = 2\left(S_{III} - \dfrac{\pi r^2}{2} - S_{II}\right) \Rightarrow S = 2\left(\dfrac{3\pi r^2}{4} - \dfrac{\pi r^2}{2} - \dfrac{3\sqrt{3} - \pi}{6} r^2\right) \Rightarrow$

$\Rightarrow S = \dfrac{5\pi - 6\sqrt{3}}{6} r^2$

944. $S = S_{hex} - 6S_I \Rightarrow$

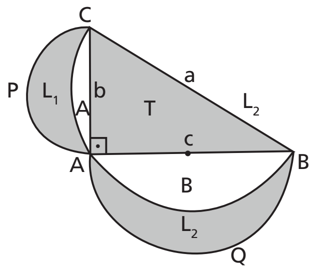


$\Rightarrow S = \dfrac{3\sqrt{3}\,\ell^2}{2} - 6 \cdot \dfrac{120°}{360°} \cdot \pi R^2 \Rightarrow$

$\Rightarrow S = \dfrac{3\sqrt{3}}{2} \cdot 2^2 - 2 \cdot \pi \cdot 1^2 \Rightarrow$

$\Rightarrow S = 2(3\sqrt{3} - \pi)$

945. Sejam L_1 e L_2 as áreas das lúnulas e T a área do triângulo.
A área S da superfície CPAQB pode ser calculada de dois modos:
1º) lúnula 1 + lúnula 2 + semicírculo de diâmetro *a*
2º) triângulo + semicírculo de diâmetro *b* + semicírculo de diâmetro *c*
Então:

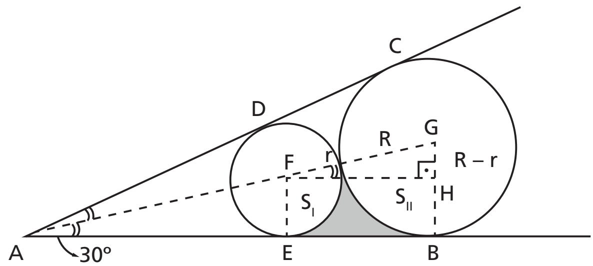


$$\left.\begin{array}{l} S = L_1 + L_2 + \dfrac{\pi a^2}{4} \\ S = T + \dfrac{\pi b^2}{4} + \dfrac{\pi a^2}{4} \end{array}\right\} \Rightarrow$$

$$\Rightarrow L_1 + L_2 + \frac{\pi a^2}{4} = T + \frac{\pi}{4}(b + c^2)$$

Logo, $L_1 + L_2 = T$.

947.



$\overrightarrow{AG}$ é bissetriz de $B\hat{A}C \Rightarrow F\hat{A}E = 30° = G\hat{F}H$ ($F\hat{A}E$ e $G\hat{F}H$ são correspondentes). Note que $G\hat{F}E = 120°$ e $F\hat{G}B = 60°$. Além disso, temos $EB = 2\sqrt{Rr}$ (exercício 563).

$$\text{sen } 30° = \frac{R - r}{R + r} \Rightarrow \frac{1}{2} = \frac{R - r}{R + r} \Rightarrow r = \frac{R}{3}$$

EBGF é trapézio de bases R e *r* e altura $2\sqrt{Rr} \Rightarrow$

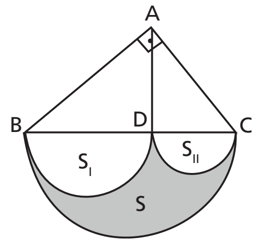
$$\Rightarrow S_{EBGF} = \frac{(R + r)2\sqrt{Rr}}{2} \Rightarrow S_{EBGF} = (R + r)\sqrt{Rr} \Rightarrow$$

$$\Rightarrow S_{EBGF} = \left(R + \frac{R}{3}\right)\sqrt{R \cdot \frac{R}{3}} \Rightarrow S_{EBGF} = \frac{4R^2\sqrt{3}}{9}$$

$$\left.\begin{array}{l} S_I = \dfrac{\pi r^2}{3} \Rightarrow S_I = \dfrac{\pi R^2}{27} \\ S_{II} = \dfrac{\pi R^2}{27} \end{array}\right\} \Rightarrow S_I + S_{II} = \frac{11\pi R^2}{54}$$

$$S = S_{EBGF} - (S_I + S_{II}) = \frac{4R^2\sqrt{3}}{9} - \frac{11\pi R^2}{54} = \frac{(24\sqrt{3} - 11\pi)R^2}{54}$$

948. $BC^2 = (1,5)^2 + 2^2 \Rightarrow BC = 2,5$ cm



$AB^2 = (BC)(BD) \Rightarrow$

$\Rightarrow (1,5)^2 = 2,5 \cdot BD \Rightarrow$

$\Rightarrow BD = \dfrac{9}{10}$ cm

$BD + DC = 2,5 \Rightarrow$

$\Rightarrow \dfrac{9}{10} + DC = 2,5 \Rightarrow$

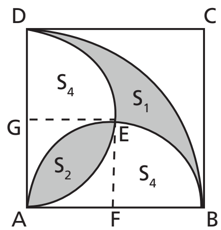
$\Rightarrow DC = \dfrac{8}{5}$ cm

$$S_I + S_{II} + S = \frac{\pi\left(\frac{BC}{2}\right)^2}{2} \Rightarrow \frac{\pi\left(\frac{BD}{2}\right)^2}{2} + \frac{\pi\left(\frac{DC}{2}\right)^2}{2} + S = \frac{\pi\left(\frac{BC}{2}\right)^2}{2} \Rightarrow$$

$$\Rightarrow \frac{\pi\left(\frac{9}{20}\right)^2}{2} + \frac{\pi\left(\frac{8}{10}\right)^2}{2} + S = \frac{\pi\left(\frac{5}{4}\right)^2}{2} \Rightarrow S = \frac{9\pi}{25}\ \text{cm}^2$$

949. Seja o lado do quadrado de medida *a*. S_3 é a área do setor determinado pelo arco $\overset{\frown}{BD}$.

$S_1 = S_3 - 2 \cdot S_4 - S_{AFEG} \Rightarrow$



$$\Rightarrow S_1 = \frac{\pi a^2}{4} - 2 \cdot \frac{\pi\left(\frac{a}{2}\right)^2}{4} - \left(\frac{a}{2}\right)^2 \Rightarrow$$

$$\Rightarrow S_1 = \frac{\pi - 2}{8} a^2 \ (1)$$

Exercício 908, item *b* $\Rightarrow$

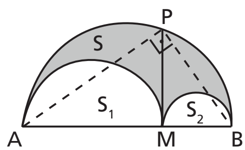
$$\Rightarrow S_2 = \frac{\pi - 2}{2} \cdot \left(\frac{a}{2}\right)^2 \Rightarrow$$

$$\Rightarrow S_2 = \frac{\pi - 2}{8} a^2 \ (2)$$

(1) e (2) $\Rightarrow S_1 = S_2$

950. Relações métricas $\Rightarrow$

$\Rightarrow PM^2 = (AM)(MB)$ (1)



$$S + S_1 + S_2 = \frac{\pi\left(\frac{AM + MB}{2}\right)^2}{2} \Rightarrow$$

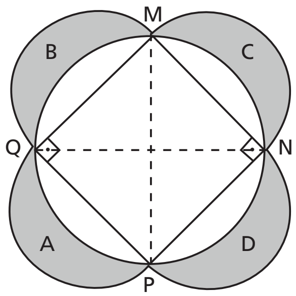
$$\Rightarrow S + \frac{\pi\left(\frac{AM}{2}\right)^2}{2} + \frac{\pi\left(\frac{MB}{2}\right)^2}{2} =$$

$$= \frac{\pi(AM + MB)^2}{8} \Rightarrow$$

$$\Rightarrow S = \frac{\pi(AM + MB)^2}{8} - \frac{\pi AM^2}{8} - \frac{\pi MB^2}{8} \Rightarrow$$

$$\Rightarrow S = \frac{\pi \cdot (AM)(MB)}{4} \overset{(1)}{\Longrightarrow} S = \frac{\pi \cdot PM^2}{4}$$

952. Exercício 945 $\Rightarrow A + B = S_{\triangle PQM} \Rightarrow$
Exercício 945 $\Rightarrow C + D = S_{\triangle PNM}$
$\Rightarrow A + B + C + D = S_{\triangle PQM} + S_{\triangle PNM} \Rightarrow$
$\Rightarrow A + B + C + D = S_{PQMN}$



953. Sejam R e *r* o raio do círculo circunscrito e o do círculo inscrito, respectivamente. Temos:

$$\left.\begin{array}{l} S = p \cdot r \Rightarrow r = \dfrac{S}{p} \\ S = \dfrac{abc}{4R} \Rightarrow R = \dfrac{abc}{4S} \end{array}\right\} \Rightarrow \frac{R}{r} = \frac{abc \cdot p}{4S^2} \Rightarrow$$

$$\Rightarrow \frac{R}{r} = \frac{abc \cdot p}{4 \cdot p \cdot (p - a)(p - b)(p - c)} \Rightarrow \frac{R}{r} = \frac{a \cdot b \cdot c}{4(p - a)(p - b)(p - c)}$$

954. Seja ℓ a medida dos lados congruentes. Temos:

$$p = \frac{\ell + \ell + 18}{2} \Rightarrow p = \ell + 9;\ r = 6 \text{ cm}$$

$$S = \sqrt{p(p - \ell)(p - \ell)(p - 18)} \Rightarrow S = \sqrt{(\ell + 9) \cdot 9 \cdot 9 \cdot (\ell - 9)} \Rightarrow$$

$$\Rightarrow S = 9\sqrt{\ell^2 - 81}$$

$$S = p \cdot r \Rightarrow 9\sqrt{\ell^2 - 81} = (\ell + 9) \cdot 6 \Rightarrow 5\ell^2 - 72\ell - 1053 = 0 \Rightarrow$$

$$\Rightarrow \ell = -9 \text{ (não serve) ou } \ell = \frac{117}{5} \text{ cm}$$

$$S = 9\sqrt{\ell^2 - 81} \Rightarrow S = 9\sqrt{\frac{13689}{25} - 81} \Rightarrow S = 9\sqrt{\frac{11664}{25}} \Rightarrow$$

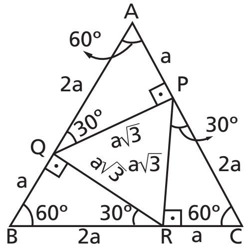
$$\Rightarrow S = \frac{9 \cdot 108}{5} \Rightarrow S = \frac{972}{5} \text{ cm}^2$$

$$S = \frac{abc}{4R} \Rightarrow R = \frac{abc}{4S} \Rightarrow R = \frac{18 \cdot \frac{117}{5} \cdot \frac{117}{5}}{4 \cdot \frac{972}{5}} \Rightarrow R = \frac{507}{40} \text{ cm}$$

955. Seja AP = a. Por trigonometria, obtemos as medidas indicadas na figura. Sendo *k* a razão de semelhança entre os dois triângulos, temos:

$$\frac{S_{ABC}}{S_{PQR}} = k^2 \Rightarrow$$

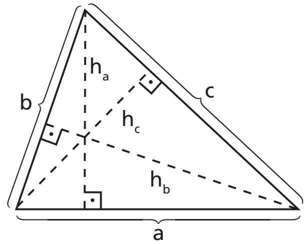
$$\Rightarrow \frac{S_{ABC}}{S_{PQR}} = \left(\frac{3a}{a\sqrt{3}}\right)^2 \Rightarrow \frac{S_{ABC}}{S_{PQR}} = 3.$$



956. Sendo S a área do triângulo, temos:

$$S = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2} = \frac{c \cdot h_c}{2} \Rightarrow$$

$$\Rightarrow ah_a = bh_b = ch_c$$



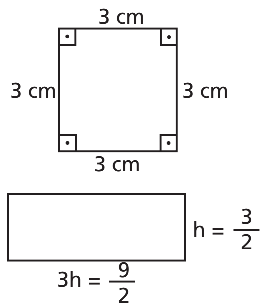
960. No tablete do Carlos, temos:

$$2(3h + h) = 12 \Rightarrow h = \frac{3}{2}\text{ cm}$$

$$S_{Paulo} = 3^2 = 9\text{ cm}^2$$

$$S_{Carlos} = \frac{9}{2} \cdot \frac{3}{2} = \frac{27}{4}\text{ cm}^2$$

Como $S_{Carlos} < S_{Paulo}$, é vantajoso para Carlos aceitar a troca.



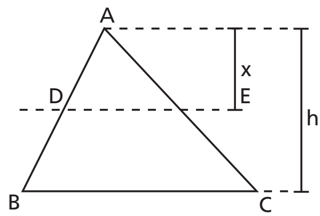
961. Os triângulos ADE e ABC serão semelhantes e

$$S_{ADE} + S_{BCDE} + S_{ABC} \Rightarrow$$

$$S_{ADE} + 3\,S_{ADE} = S_{ABC} \Rightarrow$$

$$\Rightarrow \frac{S_{ADE}}{S_{ABC}} = \frac{1}{4} \Rightarrow k^2 = \frac{1}{4} \Rightarrow$$

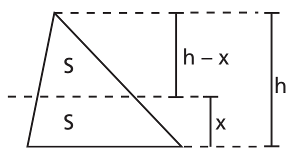
$$\Rightarrow \left(\frac{x}{h}\right)^2 = \frac{1}{4} \Rightarrow x = \frac{h}{2}.$$



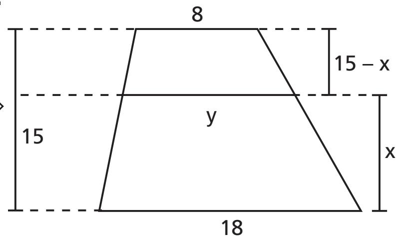
962. Seja S_T a área total do $\triangle$ABC. Temos:

$$\frac{S}{S_T} = \frac{1}{2} \Rightarrow k^2 = \frac{1}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{h - x}{x}\right)^2 = \frac{1}{2} \Rightarrow x = \frac{2 - \sqrt{2}}{2}\,h.$$



963. "A razão entre as áreas é igual ao quadrado da razão de semelhança."
Então:



$$\frac{\frac{(18+y)x}{2}}{\frac{(8+y)(15-x)}{2}} = \left(\frac{x}{15-x}\right)^2 = \left(\frac{18}{y}\right)^2 \Rightarrow$$

$$\Rightarrow \frac{x}{15-x} = \frac{18}{y} \ (1)$$

$$\frac{(18+y)x}{(8+y)(15-x)} = \frac{x^2}{(15-x)^2} \Rightarrow \frac{18+y}{(8+y)} = \frac{x}{(15-x)} \ (2)$$

(1) e (2) $\Rightarrow$ ($y = 12$ m, $x = 9$ m)

964. $\ell_1 = 8$ m, $\ell_2 = 15$ m, $S_3 = S_1 + S_2$.
Temos:

$$\frac{S_1}{S_2} = \left(\frac{\ell_1}{\ell_2}\right)^2 \Rightarrow \frac{S_1}{S_2} = \frac{8^2}{15^2} \Rightarrow S_1 = \frac{64}{225} S_2.$$

Devemos ter

$$S_3 = S_1 + S_2 \Rightarrow S_3 = S_1 + \frac{64}{225} \cdot S_2 \Rightarrow \frac{S_3}{S_2} = \frac{289}{225} \Rightarrow$$

$$\Rightarrow \left(\frac{\ell_3}{\ell_2}\right)^2 = \frac{289}{225} \Rightarrow \left(\frac{\ell_3}{15}\right)^2 = \frac{289}{225} \Rightarrow \ell_3 = 17 \text{ m}.$$

967. Sejam ℓ o lado do quadrado cuja área é procurada e ℓ_i o lado do quadrado inscrito num círculo de raio 10 cm. Temos:

$$\ell_i = R\sqrt{2} \Rightarrow \ell_i = 10\sqrt{2} \text{ cm}.$$

$$\ell = \frac{\sqrt{5}-1}{2} \cdot \ell_i \Rightarrow \ell = \frac{\sqrt{5}-1}{2} \cdot 10\sqrt{2} \text{ cm}$$

$$S = \ell^2 \Rightarrow S = \left(\frac{\sqrt{5}-1}{2} \cdot 10\sqrt{2}\right)^2 \Rightarrow S = 100(3-\sqrt{5}) \text{ cm}^2$$

970.

$$S_{dec} = 10 \cdot \frac{\ell_{10} \cdot a_{10}}{2} \Rightarrow S_{dec} = 5 \cdot \frac{\sqrt{5}-1}{2} \cdot R \cdot \frac{R}{4}\sqrt{10+2\sqrt{5}} \Rightarrow$$

$$\Rightarrow S_{dec} = \frac{5}{8} \cdot (\sqrt{5}-1)\sqrt{10+2\sqrt{5}}\, R^2$$

$$S_{pent} = 5 \cdot \frac{\ell_5 \cdot a_5}{2} \Rightarrow S_{pent} = \frac{5}{2} \cdot \frac{R}{2}\sqrt{10-2\sqrt{5}} \cdot \frac{R}{4}(\sqrt{5}+1) \Rightarrow$$

$$\Rightarrow S_{pent} = \frac{5}{16}(\sqrt{5}+1) \cdot \sqrt{10-2\sqrt{5}}$$

$$\frac{S_{dec}}{S_{pent}} = 2 \cdot \frac{(\sqrt{5}-1)\sqrt{10+2\sqrt{5}}}{(\sqrt{5}+1)\sqrt{10-2\sqrt{5}}} \Rightarrow \frac{S_{dec}}{S_{pent}} = \sqrt{5} - 1$$

971. $2p = 80 \Rightarrow \ell_8 = 10$ cm

Exercício 725 $\Rightarrow \ell_8 = R\sqrt{2-\sqrt{2}} \Rightarrow$

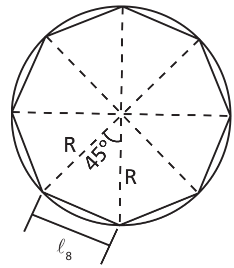
$\Rightarrow R\sqrt{2-\sqrt{2}} = 10 \Rightarrow$

$\Rightarrow R = \dfrac{10}{\sqrt{2-\sqrt{2}}}$.

$S = \cancel{8} \cdot \dfrac{R \cdot R \cdot \text{sen } 45°}{\cancel{2}} \Rightarrow$

$\Rightarrow S = 4 \cdot \left(\dfrac{10}{\sqrt{2-\sqrt{2}}}\right)^2 \cdot \dfrac{\sqrt{2}}{2} \Rightarrow$

$\Rightarrow S = 200(\sqrt{2}+1)$ cm^2



972. $S = 8 \cdot \dfrac{R \cdot R \cdot \text{sen } 45°}{2} \Rightarrow S = 8 \cdot \dfrac{6 \cdot 6 \cdot \sqrt{2}}{4} \Rightarrow S = 72\sqrt{2}$ cm^2

973. $OM + MC = OC \Rightarrow$

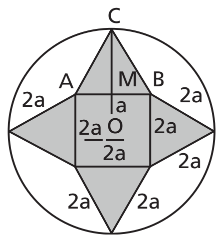
$\dfrac{2a}{2} + \dfrac{2a\sqrt{3}}{2} = R \Rightarrow$

$\Rightarrow a = \dfrac{\sqrt{3}-1}{2} \cdot R$

$S_{qua} = (2a)^2 \Rightarrow$

$\Rightarrow S_{qua} = \left[2 \cdot \dfrac{(\sqrt{3}-1)}{2} \cdot R\right]^2 \Rightarrow$

$\Rightarrow S_{qua} = (4-2\sqrt{3})R^2$



$S_{Tri} = \dfrac{(2a)^2\sqrt{3}}{4} \Rightarrow S_{Tri} = a^2\sqrt{3} \Rightarrow S_{Tri} = \left(\dfrac{\sqrt{3}-1}{2} \cdot R\right)^2 \cdot \sqrt{3} \Rightarrow$

$\Rightarrow S_{Tri} = \dfrac{2\sqrt{3}-3}{2} \cdot R^2$

$S_{Fig} = S_{qua} + 4 \cdot S_{Tri} \Rightarrow S_{Fig} = (4-2\sqrt{3})R^2 + 4 \cdot \left(\dfrac{2\sqrt{3}-3}{2} \cdot R^2\right) \Rightarrow$

$\Rightarrow S_{Fig} = 2(\sqrt{3}-1)R^2$

974. $\overline{AB} \perp \overline{CD} \Rightarrow S_{ACBD} = \dfrac{(AB)(CD)}{2} = \dfrac{34 \cdot 17\sqrt{3}}{2} = 289\sqrt{3}$ cm^2

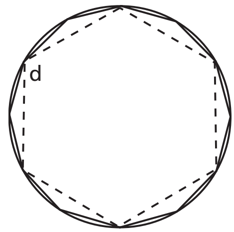
975. Sendo R o raio do círculo, note que a diagonal menor do dodecágono é igual ao ℓ_6, que é igual a R. Assim:

$d = \ell_6 = R$.

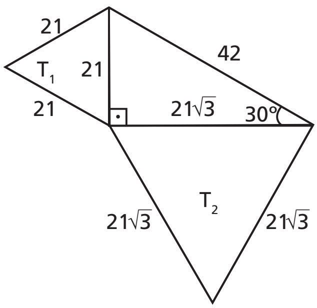
$\ell_4 = R\sqrt{2} \Rightarrow \ell_4 = d\sqrt{2} \Rightarrow$

$\Rightarrow S = \ell_4^2 \Rightarrow S = (d\sqrt{2})^2 \Rightarrow$

$\Rightarrow S = 2d^2$



976. A medida da hipotenusa é 42 cm e a do outro cateto é $21\sqrt{3}$ cm. Temos:



$T_1 = \dfrac{(21)^2\sqrt{3}}{4} \Rightarrow T_1 = \dfrac{441\sqrt{3}}{4}$ cm^2

$T_2 = \dfrac{(21\sqrt{3})^2\sqrt{3}}{4} \Rightarrow$

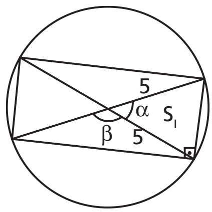
$\Rightarrow T_2 = \dfrac{1\,323\sqrt{3}}{4}$ cm^2

$S_{qua} = (42)^2 \Rightarrow S_{qua} = 1\,764$ cm^2

$\dfrac{T_1 + T_2}{S_{qua}} = \dfrac{441\sqrt{3} + 1\,323\sqrt{3}}{4 \cdot 1\,764} \Rightarrow$

$\Rightarrow \dfrac{T_1 + T_2}{S_{qua}} = \dfrac{\sqrt{3}}{4}$

977. $4\,S_1 = 25 \Rightarrow$



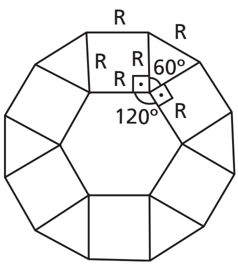
$\Rightarrow 4 \cdot \dfrac{5 \cdot 5 \cdot \text{sen}\,\alpha}{2} = 25 \Rightarrow$

$\Rightarrow \text{sen}\,\alpha = \dfrac{1}{2} \Rightarrow \alpha = 30° \Rightarrow$

$\Rightarrow \beta = 150°$

Resposta: 30° ou 150°.

978. $S_{dod} = S_{hex} + 6 \cdot S_{qua} + 6 \cdot S_{tri} \Rightarrow$



$\Rightarrow S_{dod} = \dfrac{3\sqrt{3}\,R^2}{2} + 6 \cdot R^2 + 6 \cdot \dfrac{R^2\sqrt{3}}{4} \Rightarrow$

$\Rightarrow S_{dod} = 3\sqrt{3}R^2 + 6\,R^2 \Rightarrow$

$\Rightarrow S_{dod} = 3(\sqrt{3} + 2)R^2$

979. $S = p \cdot r$ (1) $S = (p - a) \cdot r_a$ (2) $S = (p - b) \cdot r_b$ (3) $S = (p - c) \cdot r_c$ (4)

Multiplicando membro a membro (1), (2), (3) e (4), vem:

$S \cdot S \cdot S \cdot S = p \cdot r \cdot (p - a)r_a \cdot (p - b)r_b \cdot (p - c)r_c$

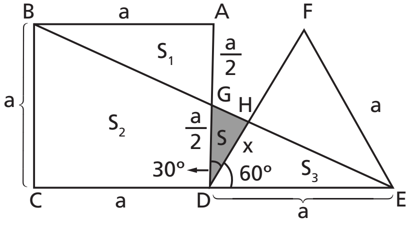
e como $p(p - a)(p - b)(p - c) = S^2$, vem:

$S^4 = p(p - a)(p - b)(p - c) \cdot r \cdot r_a \cdot r_b \cdot r_c \Rightarrow S^2 = r \cdot r_a \cdot r_b \cdot r_c \Rightarrow$

$\Rightarrow S = \sqrt{r \cdot r_a \cdot r_b \cdot r_c}$

980. $\triangle ABG \equiv \triangle DEG\ (LAA_o) \Rightarrow$

$\Rightarrow DG = \dfrac{a}{2}$



$$\left.\begin{array}{l} S_1 + S_2 = a^2 \\ S_2 + S + S_3 = \dfrac{2a \cdot a}{2} = a^2 \end{array}\right\} \Rightarrow$$

$$\Rightarrow S_1 + S_2 = S_2 + S + S_3 \Rightarrow$$

$$\Rightarrow S = S_1 - S_3 \Rightarrow$$

$$\Rightarrow S = \frac{a^2}{4} - \frac{a \cdot x \cdot \text{sen } 60°}{2} \ (1)$$

$$S = \frac{\frac{a}{2} \cdot x \cdot \text{sen } 30°}{2} \Rightarrow x = \frac{4S}{a \text{ sen } 30°} \ (2)$$

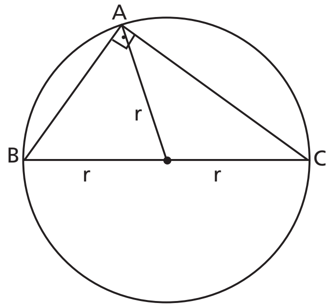
Substituindo (2) em (1):

$$S = \frac{a^2}{4} - \frac{a \cdot \frac{4S}{a \text{ sen } 30°} \cdot \text{sen } 60°}{2} \Rightarrow S = \frac{a^2}{4} - 2\sqrt{3}S \Rightarrow$$

$$\Rightarrow S\left(1 + 2\sqrt{3}\right) = \frac{a^2}{4} \Rightarrow$$

$$\Rightarrow S = \frac{2\sqrt{3} - 1}{44} \cdot a^2$$

983. $AB = 3k$, $AC = 4k$, $BC = 5k$



Note que $r = \dfrac{5}{2} k \Rightarrow k = \dfrac{2}{5} r$ (1)

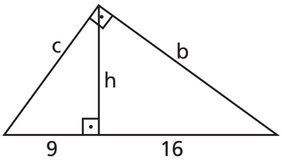
$$S = \frac{abc}{4r} \Rightarrow S = \frac{3 \cdot 4 \cdot 5 \cdot k^3}{4 \cdot \frac{5}{2} k} \Rightarrow$$

$$\Rightarrow S = 6k^2 \ (2)$$

$$(1) \text{ em } (2) \Rightarrow S = 6 \cdot \left(\frac{2}{5} r\right)^2 \Rightarrow$$

$$\Rightarrow S = \frac{24}{25} r^2$$

984. Sendo *h* a altura relativa à hipotenusa, temos:



$$h^2 = 16 \cdot 9 \Rightarrow h = 12 \text{ cm} \Rightarrow$$

$$\Rightarrow S = \frac{(16 + 9) \cdot 12}{2} \Rightarrow$$

$$\Rightarrow S = 150 \text{ cm}^2$$

$$a = 16 + 9 \Rightarrow a = 25 \text{ cm}$$

$$\text{Relações métricas} \Rightarrow \begin{cases} b^2 = 25 \cdot 16 \Rightarrow b = 20 \text{ cm} \\ c^2 = 25 \cdot 9 \Rightarrow c = 15 \text{ cm} \end{cases}$$

$$p = \frac{a+b+c}{2} \Rightarrow p = \frac{25+20+15}{2} \Rightarrow$$

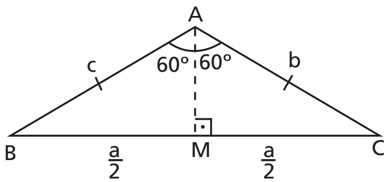
$$\Rightarrow p = 30 \text{ cm}$$

$$\left.\begin{array}{l} S = p \cdot r \Rightarrow 150 = 30 \cdot r \Rightarrow r = 5 \text{ cm} \Rightarrow S_I = 25\pi \text{ cm}^2 \\ S = \dfrac{abc}{4R} \Rightarrow 150 = \dfrac{25 \cdot 20 \cdot 15}{4R} \Rightarrow R = \dfrac{25}{2} \Rightarrow S_{II} = \dfrac{625}{4}\pi \text{ cm}^2 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \frac{S_I}{S_{II}} = \frac{4}{25}$$

985. $\text{sen } 60° = \dfrac{MC}{AC} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{\frac{a}{2}}{AC} \Rightarrow$

$$\Rightarrow AC = \frac{a\sqrt{3}}{3} = AB$$



$$S = \frac{(AB) \cdot (AC) \cdot \text{sen } 120°}{2} \Rightarrow$$

$$\Rightarrow S = \frac{\left(\frac{a\sqrt{3}}{3}\right)^2 \cdot \frac{\sqrt{3}}{2}}{2} \Rightarrow S = \frac{a^2\sqrt{3}}{12}$$

$$p = \left(\frac{a\sqrt{3}}{3} + \frac{a\sqrt{3}}{3} + a\right) \cdot \frac{1}{2} \Rightarrow p = \frac{2\sqrt{3}+3}{6} \cdot a$$

$$\left.\begin{array}{l} S = \dfrac{a \cdot b \cdot c}{4R} \Rightarrow \dfrac{a^2\sqrt{3}}{12} = \dfrac{a \cdot \frac{a\sqrt{3}}{3} \cdot \frac{a\sqrt{3}}{3}}{4R} \Rightarrow R = \dfrac{a\sqrt{3}}{3} \\ S = p \cdot r \Rightarrow \dfrac{a^2\sqrt{3}}{12} = \dfrac{2\sqrt{3}+3}{6} \cdot a \cdot r \Rightarrow r = \dfrac{2-\sqrt{3}}{2} \end{array}\right\} \Rightarrow \frac{R}{r} = \frac{2(2\sqrt{3}+3)}{3}$$

987. $\ell_1 = 2$ cm, $\ell_2 = 3$ cm

Os dois eneágonos, por serem regulares e convexos, são semelhantes. Então:

$$\frac{S_1}{S_2} = \left(\frac{\ell_1}{\ell_2}\right)^2 \Rightarrow \frac{S_1}{S_2} = \left(\frac{2}{3}\right)^2 \Rightarrow S_1 = \frac{4}{9}S_2.$$

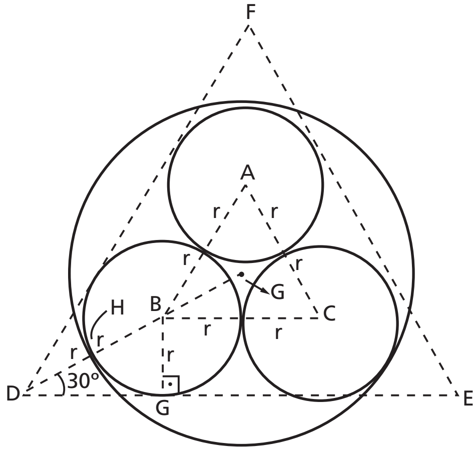
Seja ℓ o lado do eneágono que queremos determinar e S a sua área. Temos:

$$S = S_1 + S_2 \Rightarrow S = \frac{4}{9}S_2 + S_2 \Rightarrow S = \frac{13}{9}S_2 \Rightarrow \frac{S}{S_2} = \frac{13}{9} \Rightarrow$$

$$\Rightarrow \left(\frac{\ell_1}{\ell_2}\right)^2 = \frac{13}{9} \Rightarrow \left(\frac{\ell}{3}\right)^2 = \frac{13}{9} \Rightarrow \ell = \sqrt{13} \text{ cm}.$$

988. $\triangle ABC$ é equilátero de lado 2r. Sendo *h* sua altura, temos:

$h = \dfrac{(2r)\sqrt{3}}{2} \Rightarrow h = r\sqrt{3}$.



Note que G é baricentro do $\triangle ABC$. Daí, $BG = \dfrac{2}{3} \cdot h \Rightarrow BG = \dfrac{2}{3} r\sqrt{3}$.

Note também que $\overrightarrow{DB}$ é bissetriz de $E\hat{D}F$. Daí, $B\hat{D}G = 30°$.

$\triangle BDG \Rightarrow \text{sen } 30° = \dfrac{BG}{BD} \Rightarrow \dfrac{1}{2} = \dfrac{r}{BD} \Rightarrow BD = 2r$

$(BD = 2r, BH = r) \Rightarrow BH = r$

Sendo R o círculo do raio maior, temos:

$R = BH + BG \Rightarrow R = r + \dfrac{2}{3} r\sqrt{3}$

Logo:

$S = \pi R^2 - 3\pi r^2 \Rightarrow S = \pi\left(r + \dfrac{2}{3} r\sqrt{3}\right)^2 - 3\pi r^2 \Rightarrow S = \dfrac{2(2\sqrt{3} - 1)\,\pi r^2}{3}$.

990. $S_1 = \dfrac{\pi a^2}{2}$

Exercício 931 $\Rightarrow$

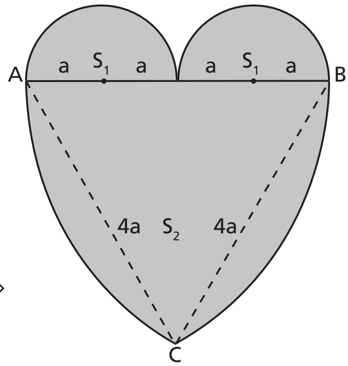
$S_2 = \dfrac{4\pi - 3\sqrt{3}}{12}(4a)^2 \Rightarrow$

$\Rightarrow S_2 = \dfrac{(16\pi - 12\sqrt{3})a^2}{3} \Rightarrow$

$\Rightarrow S = 2\,S_1 + S_2 \Rightarrow$

$\Rightarrow S = 2 \cdot \dfrac{\pi a^2}{2} + \dfrac{(16\pi - 12\sqrt{3})a^2}{3} \Rightarrow$

$\Rightarrow S = \dfrac{(19\pi - 12\sqrt{3})a^2}{3}$



991. $\triangle CDB \Rightarrow DB^2 + 12^2 = 13^2 \Rightarrow DB = 5$ cm

$(DB = 5 \text{ cm}, AB = 14 \text{ cm}) \Rightarrow AD = 9$ cm

$S_{ABC} = \dfrac{14 \cdot 12}{2} \Rightarrow S_{ABC} = 84 \text{ cm}^2$

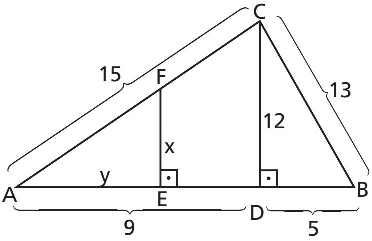
$\triangle AEF \sim \triangle ADC \Rightarrow \dfrac{CD}{EF} = \dfrac{AD}{AE} \Rightarrow$

$\Rightarrow \dfrac{12}{x} = \dfrac{9}{y} \Rightarrow x = \dfrac{4}{3}y \ (1)$

$S_{AEF} = \dfrac{1}{2} \cdot S_{ABC} \Rightarrow$

$\Rightarrow \dfrac{xy}{2} = \dfrac{1}{2} \cdot 84 \Rightarrow xy = 84 \ (2)$

(1) em (2) $\Rightarrow \dfrac{4}{3}y \cdot y \ \ 84 \Rightarrow y = 3\sqrt{7}$ cm



992. $\triangle XYZ \Rightarrow \text{tg } 30^\circ = \dfrac{r}{XZ} \Rightarrow$

$\Rightarrow \dfrac{\sqrt{3}}{3} = \dfrac{r}{XZ} \Rightarrow XZ = r\sqrt{3}$

Temos:

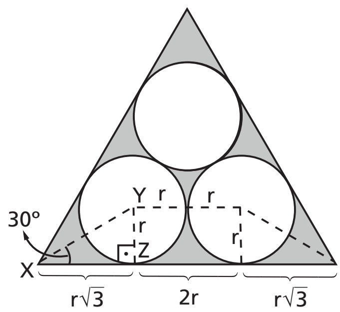
$r\sqrt{3} + 2r + r\sqrt{3} = a \Rightarrow$

$\Rightarrow r = \dfrac{(\sqrt{3} - 1)a}{4}$

$S = S_{tri} - 3\, S_{círc} \Rightarrow$

$\Rightarrow S = \dfrac{a^2\sqrt{3}}{4} - 3\pi \cdot r^2 \Rightarrow$

$\Rightarrow S = \dfrac{a^2\sqrt{3}}{4} - 3 \cdot \pi \cdot \dfrac{(\sqrt{3} - 1)^2 \cdot a^2}{4^2} \Rightarrow S = \dfrac{2\sqrt{3} - 3(2 - \sqrt{3})\pi}{8} \cdot a^2$



993. $(AB)^2 = 256 \Rightarrow$

$\Rightarrow AB = 16 \text{ cm} = BC = CD = AD$

$\left.\begin{array}{l} B\hat{C}E + B\hat{C}F = 90^\circ \\ F\hat{C}D + B\hat{C}F = 90^\circ \end{array}\right\} \Rightarrow$

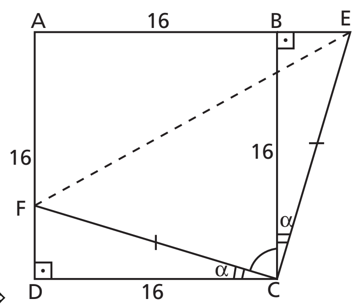
$\Rightarrow B\hat{C}E = F\hat{C}D = \alpha$

$\left.\begin{array}{l} \triangle BEC\text{: tg } \alpha = \dfrac{BE}{16} \\ \triangle FDC\text{: tg } \alpha = \dfrac{FD}{16} \end{array}\right\} \Rightarrow$

$\Rightarrow BE = FD \Rightarrow$

$\Rightarrow \triangle BEC \equiv \triangle DFC \Rightarrow FC \equiv EC$

$S_{\triangle ECF} = 200 \Rightarrow \dfrac{(FC)(CE)}{2} = 200 \Rightarrow$



$\Rightarrow (FC) \cdot (FC) = 400 \Rightarrow FC = EC = 20$ cm
$\triangle BCE$: $BE^2 + BC^2 = EC^2 \Rightarrow BE^2 + 16^2 = 20^2 \Rightarrow BE = 12$ cm

994. $\ell = \dfrac{1}{6} 2\pi R \Rightarrow 12\pi = \dfrac{1}{6} \cdot 2\pi \cdot R \Rightarrow$

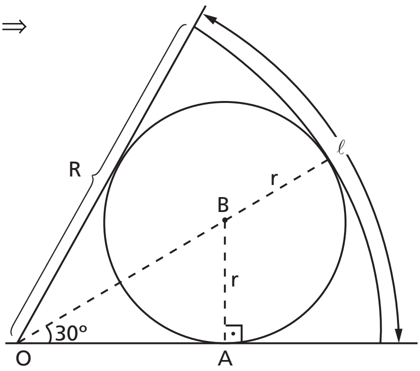
$\Rightarrow R = 36$ cm

$\triangle OAB \Rightarrow \text{sen } 30° = \dfrac{r}{OB} \Rightarrow$

$\Rightarrow \dfrac{1}{2} = \dfrac{r}{R - r} \Rightarrow r = \dfrac{R}{3} \Rightarrow$

$r = \dfrac{36}{3} \Rightarrow r = 12 \Rightarrow S = \pi r^2 \Rightarrow$

$\Rightarrow S = 144\pi$ cm^2



996. $EC = b \Rightarrow DE = a - b$

$\overline{CE} \sim \overline{CF} \Rightarrow EF = b\sqrt{2} = AE = AF$

$\triangle ADE \Rightarrow (a - b)^2 + a^2 = (b\sqrt{2})^2 \Rightarrow$

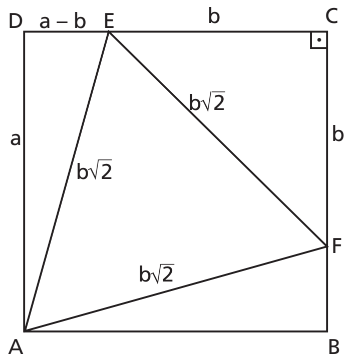
$\Rightarrow b^2 + 2ab - 2a^2 = 0 \Rightarrow$

$\Rightarrow b = (\sqrt{3} - 1)a$

$S = \dfrac{(b\sqrt{2})^2\sqrt{3}}{4} \Rightarrow$

$\Rightarrow S = \dfrac{[(\sqrt{3} - 1)a \cdot \sqrt{2}]^2\sqrt{3}}{4} \Rightarrow$

$\Rightarrow S = (2\sqrt{3} - 3)a^2$



997. $AD = \dfrac{\ell\sqrt{3}}{2} \Rightarrow AD = \dfrac{8\sqrt{3} \cdot \sqrt{3}}{2} \Rightarrow$

$\Rightarrow AD = 12$ cm $\Rightarrow (AS = SD = 6$ cm$)$

$\triangle AMD$ é retângulo em D, MS é mediana $\Rightarrow$

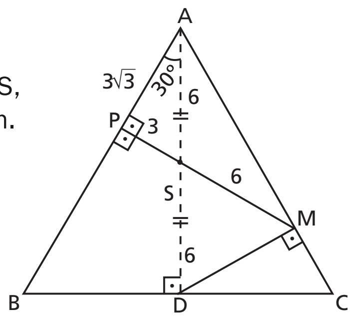
$\Rightarrow MS = \dfrac{AD}{2} \Rightarrow MS = 6$ cm

Aplicando a Trigonometria no $\triangle APS$, obtemos $PS = 3$ cm, $AP = 3\sqrt{3}$ cm.

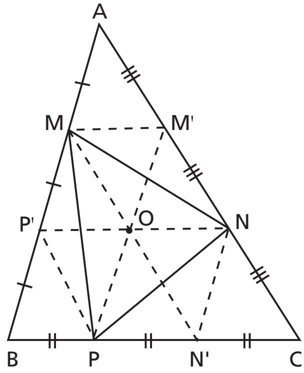
$S_{\triangle APM} = \dfrac{(AP) \cdot (PM)}{2} \Rightarrow$

$\Rightarrow S_{\triangle APM} = \dfrac{3\sqrt{3} \cdot 9}{2} \Rightarrow$

$\Rightarrow S_{\triangle APM} = \dfrac{27\sqrt{3}}{2}$ cm^2



998. Usando base média de triângulo (AP'N) e de trapézio (MN'CB) é fácil concluir que o $\triangle ABC$ é formado por 9 triângulos equivalentes.
Os lados do $\triangle MNP$ são diagonais dos paralelogramos MN'NO, MOPP' e PONN'. Logo, a área de MPN é equivalente a 3 dos triângulos que formam o $\triangle ABC$.
Então:
$\dfrac{S_{ABC}}{S_{MNP}} = \dfrac{9}{3} \Rightarrow \dfrac{S_{ABC}}{S_{MNP}} = 3.$

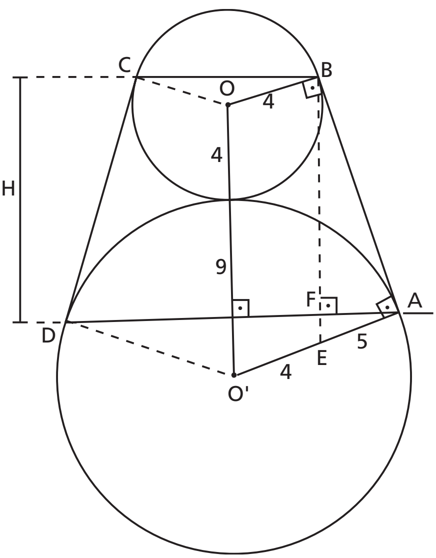


999. Traçamos $\overline{BE}$, com $\overline{BE} \perp \overline{AD} \Rightarrow \overline{BE} \ // \ \overline{OO'}$ (1)
$(\overline{OB} \perp \overline{AB}, \overline{O'A} \perp \overline{AB}) \Rightarrow \overline{OB} \ // \ \overline{O'A}$ (2)
(1) e (2) $\Rightarrow$ EBOO' é paralelogramo $\Rightarrow BE = 13$ cm
$\triangle ABE \Rightarrow AB^2 + AE^2 = BE^2 \Rightarrow AB^2 + 5^2 = 13^2 \Rightarrow AB = 12$ cm
Rel. métricas $\Rightarrow AB^2 = (BE) \cdot (BF) \Rightarrow 12^2 = 13 \cdot H \Rightarrow$

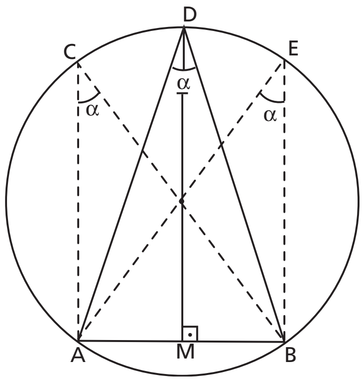
$$\Rightarrow H = \frac{144}{13} \text{ cm}$$

$$S_{ABCD} = \frac{(AD + BC) \cdot H}{2} \Rightarrow S_{ABCD} = 24 \cdot \frac{144}{13} \cdot \frac{1}{2} \Rightarrow$$

$$\Rightarrow S_{ABCD} = \frac{1\,728}{13} \text{ cm}^2$$



1000. Note na figura ao lado os triângulos ABC, ABD e ABE, de mesma base $\overline{AB}$ e mesmo ângulo (α) opostos a essa base.



No $\triangle$ABD:

$$\left.\begin{array}{l}\overline{DM} \text{ passa pelo centro}\\ \overline{DM} \perp \overline{AB} \Rightarrow AM = MD\end{array}\right\} \xrightarrow{LAL}$$

$\Rightarrow \triangle AMD \equiv \triangle BMD \Rightarrow$

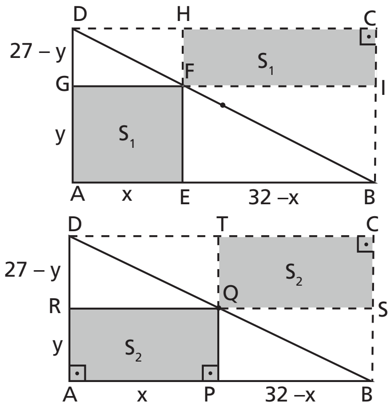
$\Rightarrow AD \equiv BD \Rightarrow \triangle ABD$ é isósceles.

Como $\overline{DM}$ passa pelo centro, $\overline{DM}$ é a maior altura relativa à base $\overline{AB}$. Logo, o $\triangle$ABD isósceles é o que tem maior área.

1001. Exercício 784 $\Rightarrow$

$$\Rightarrow \begin{cases} S_{AEFG} \equiv S_{CHFI} \\ S_{APQR} \equiv S_{CSQT} \end{cases}$$

Das figuras ao lado é imediato concluir que a área será a maior possível quando a base e a altura forem iguais à metade dos catetos correspondentes. Isto é, as dimensões do retângulo devem ser 13,5 e 16.

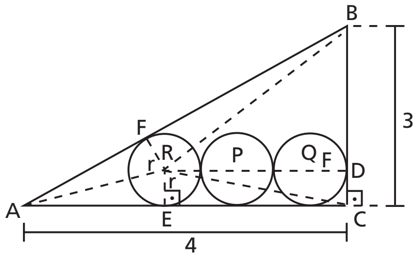


1004. BC = 3 cm, AC = 4 cm $\Rightarrow$ AB = 5 cm

$S_{ABC} = S_{BCR} + S_{ACR} + S_{ABR} \Rightarrow$

$$\Rightarrow \frac{(AC) \cdot (BC)}{2} = \frac{(BC)(RD)}{2} + \frac{(AC)(ER)}{2} + \frac{(AB)(FR)}{2} \Rightarrow$$

$$\Rightarrow 4 \cdot 3 = 3 \cdot 5 \cdot r + 4 \cdot r + 5 \cdot r \Rightarrow r = \frac{1}{2} \text{ cm}$$



1005. diâmetro $= 8$ cm $\Rightarrow R = 4$ cm

$B = \ell_3 \Rightarrow B = R\sqrt{3} \Rightarrow B = 4\sqrt{3}$ cm

$b = \ell_6 \Rightarrow b = R \Rightarrow b = 4$ cm

$h = a_6 - a_3 \Rightarrow$

$\Rightarrow h = \dfrac{R\sqrt{3}}{2} - \dfrac{R}{2} \Rightarrow$

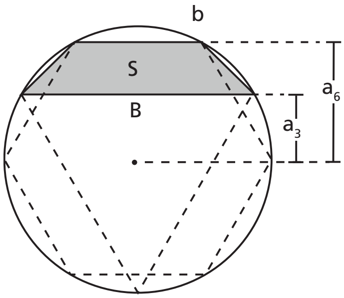
$\Rightarrow h = \dfrac{4\sqrt{3}}{2} - \dfrac{4}{2} \Rightarrow$

$\Rightarrow h = (2\sqrt{3} - 2)$ cm

$S = \dfrac{(B + b)h}{2} \Rightarrow$

$\Rightarrow S = \dfrac{(4\sqrt{3} + 4)(2\sqrt{3} - 2)}{2} \Rightarrow$

$\Rightarrow S = 8 \text{ cm}^2$



1006. Sejam $k - 1, k, k, + 1$ as medidas dos lados; h_1, h_2 e h_3 suas respectivas alturas. Temos:

$2p = k - 1 + k + k + 1 \Rightarrow 2p = 3k \Rightarrow \begin{cases} S = 6k \\ p = \dfrac{3k}{2} \end{cases}$

$S = \sqrt{p(p - a)(p - b)(p - c)} \Rightarrow$

$$\Rightarrow 6k = \sqrt{\frac{3k}{2}\left(\frac{3k}{2} - k + 1\right)\left(\frac{3k}{2} - k\right)\left(\frac{3k}{2} - k - 1\right)} \Rightarrow$$

$\Rightarrow k^2 = 196 \Rightarrow k = 14$

$\dfrac{(k - 1)h_1}{2} = 6k \Rightarrow \dfrac{(14 - 1)h_1}{2} = 6 \cdot 14 \Rightarrow h_1 = \dfrac{168}{13}$

$\dfrac{k\,h_2}{2} = 6k \Rightarrow h_2 = 12$

$\dfrac{(k - 1)h_3}{2} = 6k \Rightarrow \dfrac{(14 + 1)h_3}{2} = 6 \cdot 14 \Rightarrow h_3 = \dfrac{56}{5}$

1007. $\begin{cases} xy = a^2 \\ x^2 + y^2 = d^2 \end{cases} \Rightarrow \begin{cases} \pm\, 2xy = \pm 2a^2 \\ x^2 + y^2 = d^2 \end{cases}$

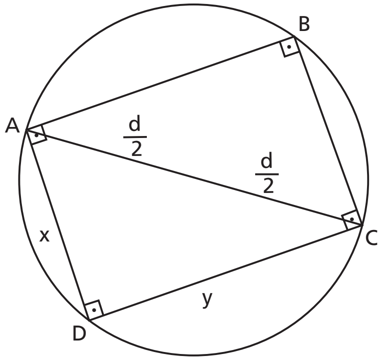
Somando membro a membro, temos:

$\begin{cases} (x + y)^2 = (d^2 + 2a^2) \\ (x - y)^2 = (d^2 - 2a^2) \end{cases} \Rightarrow \begin{cases} x + y = \sqrt{d^2 + 2a^2} \\ x - y = \sqrt{d^2 - 2a^2} \end{cases}$

Resolvendo o último sistema, encontramos:

$x = \dfrac{\sqrt{d^2 + 2a^2} + \sqrt{d^2 - 2a^2}}{2}$; $y = \dfrac{\sqrt{d^2 + 2a^2} + \sqrt{d^2 - 2a^2}}{2}$.

Devemos ter $d^2 - 2a^2 \geqslant 0 \Rightarrow d \geqslant a\sqrt{2}$.
Note:
$d = a\sqrt{2} \Rightarrow x = y = a \Rightarrow$ ABCD é quadrado.



1008. Sejam *b* e *c* os catetos. Temos:

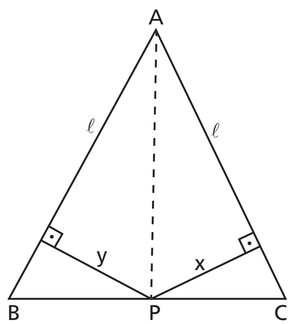
$$\begin{cases} \dfrac{bc}{2} = 120 \\ b^2 + c^2 = a^2 \end{cases} \Rightarrow \begin{cases} bc = 240 \\ b^2 + c^2 = a^2 \end{cases} \Rightarrow \begin{cases} 2bc = 480 \\ b^2 + c^2 = a^2 \end{cases}$$

Somando membro a membro as equações do último sistema:
$(b + c)^2 = 480 + a^2 \Rightarrow b + c = \sqrt{480 + a^2}$.
Então:
$(bc = 240;\ b + c = \sqrt{480 + a^2}) \Rightarrow$
$\Rightarrow$ *b* e *c* são raízes da equação $x^2 - \sqrt{480 + a^2}\ x + 240 = 0$.
Resolvendo esta equação, encontramos os valores de *b* e *c*:

$$b = \frac{\sqrt{a^2 + 480} + \sqrt{a^2 - 480}}{2} \text{ cm};\ c = \frac{\sqrt{a^2 + 480} - \sqrt{a^2 - 480}}{2} \text{ cm}.$$

Além disso, devemos ter:
$a^2 - 480 \geqslant 0 \Rightarrow a \geqslant 4\sqrt{30}$ cm.

1009. Sejam S a área do triângulo ABC e ℓ a medida dos lados congruentes.
Temos:
$S = S_{ABP} + S_{ACP} \Rightarrow$
$\Rightarrow S = \dfrac{\ell \cdot y}{2} + \dfrac{\ell x}{2} \Rightarrow$
$\Rightarrow (x + y) = \dfrac{25}{\ell}$.



1010. $\triangle OAB \Rightarrow \text{sen } 30° = \dfrac{r}{18 - r} \Rightarrow$
$\Rightarrow \dfrac{1}{2} = \dfrac{r}{18 - r} \Rightarrow r = 6$ m

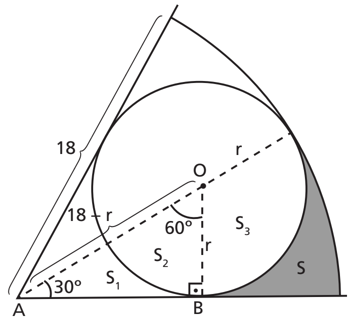
$$S_{\triangle OAB} = \frac{OA \cdot OB \cdot \text{sen } 60^\circ}{2} \Rightarrow$$

$$\Rightarrow S_{\triangle OAB} = \frac{12 \cdot 6 \cdot \left(\frac{\sqrt{3}}{2}\right)}{2} \Rightarrow$$

$$\Rightarrow S_{\triangle OAB} = 18\sqrt{3}\ m^2$$

$$S_1 = S_{\triangle OAB} - S_2 \Rightarrow$$

$$\Rightarrow S_1 = 18\sqrt{3} - \frac{\pi \cdot r^2}{6} \Rightarrow$$



$$\Rightarrow S_1 = 18\sqrt{3} - \frac{\pi \cdot 6^2}{6} \Rightarrow S_1 = \left(18\sqrt{3} - 6\pi\right) m^2$$

$$S + S_1 + S_2 + S_3 = \frac{\pi R^2}{12} \Rightarrow S + 18\sqrt{3} - 6\pi + \frac{\pi r^2}{2} = \frac{\pi \cdot 18^2}{12} \Rightarrow$$

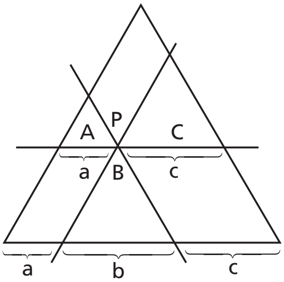
$$\Rightarrow S = 3(5\pi - 6\sqrt{3})\ m^2$$

1011. Note que o triângulo original e os triângulos de áreas A, B e C são semelhantes. Sendo S a área do triângulo original, temos:

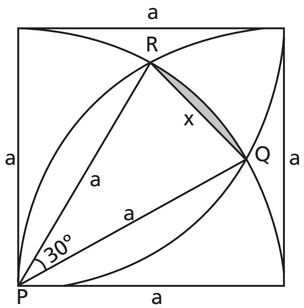
$$\frac{S}{(a + b + c)^2} = \frac{A}{a^2} = \frac{B}{b^2} = \frac{C}{c^2} \Rightarrow$$

$$\Rightarrow \frac{\sqrt{S}}{a + b + c} = \frac{\sqrt{A}}{a} = \frac{\sqrt{B}}{b} = \frac{\sqrt{C}}{c} \Rightarrow$$

$$\Rightarrow S = (\sqrt{A} + \sqrt{B} + \sqrt{C})^2.$$



1012. A área procurada é igual à área de um quadrado de lado *x* mais 4 vezes a área do segmento circular sombreado na figura ao lado.



1º) Cálculo de x^2:

$$x^2 = a^2 + a^2 - 2 \cdot a \cdot a \cdot \cos 30^\circ \Rightarrow$$

$$\Rightarrow x^2 = 2a^2 - 2a^2 \cdot \frac{\sqrt{3}}{2} \Rightarrow$$

$$\Rightarrow x^2 = \left(2 - \sqrt{3}\right)a^2$$

2º) Cálculo da área do segmento circular:

$$S_{seg} = S_{setor} - S_{\triangle PQR} \Rightarrow S_{seg} = \frac{\pi a^2}{12} - \frac{a \cdot a \cdot \text{sen } 30^\circ}{2} \Rightarrow$$

$$\Rightarrow S_{seg} = \left(\frac{\pi}{12} - \frac{1}{4}\right) a^2$$

3º) Área da região sombreada:

$$S = x^2 + 4 \cdot S_{seg} \Rightarrow S = (2 - \sqrt{3})\ a^2 + 4\left(\frac{\pi}{12} - \frac{1}{4}\right)a^2 \Rightarrow$$

$$\Rightarrow S = \left(2 - \sqrt{3} + \frac{\pi}{3} - 1\right) a^2 \Rightarrow S = \frac{\left(\pi + 3 - 3\sqrt{3}\right) a^2}{3}$$

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Atual Editora

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