Untangling the Decibel Dilemma

By H. Ward Silver, NØAX

If decibel values seem like words from a foreign language, take heart. Decibel measurements are easier to understand than you may realize. This article is expanded from the QST version, published in January 2020.

You'll find references to decibels throughout amateur radio in particular and electronics in general. For instance, you may hear someone say that their antenna has 4 dB (decibels) of gain, or that a particular type and length of coaxial cable has 1 dB of matched loss at a given frequency.

This tutorial does include mathematics because the decibel is a mathematical construct. If you want to dive head first into the math, that's great, and your understanding will be better for doing so. On the other hand, if math just isn't your thing, skip the equations and concentrate on the explanations and tables.

Let's start with five quick points:

● The decibel is a ratio of two power values. It is computed using logarithms, so that very large and small ratios result in numbers that are easy to work with.

● A positive decibel value indicates a ratio greater than one, and a negative decibel value indicates a ratio of less than one. Zero decibels indicates a ratio of exactly one. See the Table 1 for a list of easily remembered decibel values for common ratios.

● A letter following dB, such as dBm, indicates a specific reference value. See Table 2.

● If given in dB, the gains (or losses) of a series of stages in a radio or communications system can be added together:

$$System\,Gain\,(\text{dB}) = Gain_1 + Gain_2 + \ldots + Gain_n$$

● Losses and attenuation are equivalent to negative values of gain; i.e. a loss of 3 dB is equivalent to a gain of –3 dB and 10 dB of attenuation is equivalent to a gain of –10 dB

Table 1 – Common Decibel Values

Power Ratio	*Decibel Value (dB)*	*Voltage Ratio*	*Decibel Value (dB)*
0.001	–30	0.001	–60
0.01	–20	0.01	–40
0.1	–10	0.1	–20
0.125	–9	0.125	–18
0.25	–6	0.25	–12
0.5	–3	0.5	–6
0.79	–1	0.707	–3
1	0	1	0
1.26	1	1.414	3
2	3	2	6
4	6	4	12
5	7	5	14
8	9	8	18
20	13	20	26
50	17	50	34
10	10	10	20
100	20	100	40
1000	30	1,000	60

Table 2 – Decibel Reference Abbreviations

Abbreviation	*Reference value*
dBm	one milliwatt (1 mW)
dBW	one watt (1 W)
dBV	one volt (1 V)
dBμV	one microvolt (1 μV)
dBi	gain of an isotropic antenna
dBd	maximum gain of a half-wave dipole in free space
dBFS	full-scale value
dBc	carrier power

Decibels — The History

The need for a consistent way to compare signal strengths and how they change under various conditions is as old as telecommunications itself. The original unit of measurement was the "mile of standard cable." It was devised by the telephone companies and represented the signal loss that would occur in a mile of standard telephone cable (roughly #19 AWG copper wire) at a frequency of around 800 Hz. If you were measuring loss in a telephone line in the early 20th century, you might say that it amounted to "5 miles of standard cable." Because everyone knew how much signal was lost in 1 mile of cable, the effect of a 5-mile loss was easy to understand.

In the 1920s, this unit of measure was replaced by the *Bel (B)* in honor of Alexander Graham Bell, inventor of the telephone and founder of the Bell Telephone Company. One Bel represented a 10-fold gain or loss of power. This turned out to be too much change for most measurements and calculations, so the *decibel (dB)*, or 1⁄10 of a Bel, became the widely used measure of signal change. (The metric prefix *deci-* or *d-* represents 1/10 or multiplication by 0.1.)

Uses of Decibels

Sound intensity or *sound pressure level (SPL)* is also specified in decibels. In this case, the reference level of 0 dB corresponds to a pressure of 0.0002 microbars which is the standard threshold for being able to hear a sound. As the sounds get louder, the value of SPL in dB also increases, indicating an increase with respect to the reference level.

SPL in the average home is about 50 dB above the 0-dB threshold that serves as the SPL reference. When a vacuum cleaner 1 meter away is on, SPL increases to 70 dB. A chainsaw 1 meter away produces a SPL of 110 dB, and the threshold of discomfort from sound intensity is 120 dB.

Because each 10 dB (or 1 Bel) represents difference by a factor of 10, 120 dB (12 Bels) represents a pressure 10^{12} times greater than the reference threshold level — a change of a million-million. Our ears respond logarithmically to changes in sound level, which makes the decibel a very useful tool of comparison.

Radio and electronic circuits also deal with signal levels that change by many orders of magnitude. Thus, the decibel is a common feature of the technical side of amateur radio. For example, received signal strengths on the HF bands are usually reported in S-units. Each S-unit represents a change in strength of 5 to 6 dB. Although most receiver S meters are not accurately calibrated, it is useful to consider that a change in signal strength of one S-unit is a change in signal power of approximately four.

Here are some other places you'll find the ubiquitous decibel:

- Filter bandwidth is the width of the frequency range over which signals are attenuated less than 3 dB, or where the filter output is no less than half of the input power.
- Feed line loss is specified in decibels per some length (100 feet or 100 meters is common) at a particular frequency.
- Antenna gain is given in decibels, usually compared to an isotropic or dipole antenna.
- Power amplifier and preamplifier gain is usually given in dB.

How to Calculate Decibels

The *log* of a number is short for *logarithm* and is the answer to the question, "To what value does the logarithm's base value need to be raised in order to equal the number in question?" When calculating decibels, we use the *common logarithm*, written as *log*, with its base value of 10. (The *natural logarithm* written as *ln*, uses a base value of *e*, which is 2.71828.)

For example, if the number in question is 100, the base value of 10 would have to be raised to the power of 2 to equal 100. In other words, $10^2 = 100$. Thus, the common logarithm of 100 is 2. Similarly, $\log(1{,}000) = \log(10^3) = 3$ and $\log(1/10) = \log(10^{-1}) = -1$. For all decibel calculations, use the common logarithm.

$$\text{dB} = 10 \log\left(\frac{power}{reference\ power}\right)$$

$$\text{dB} = 20 \log\left(\frac{voltage}{reference\ voltage}\right) = 20 \log\left(\frac{current}{reference\ current}\right)$$

Adding Decibels Together

Another useful characteristic of decibels is that gains and losses of stages in a radio system can be added together if they are specified in decibels. For example, if you have an antenna with 8 dB of gain connected to a preamplifier with 15 dB of gain, the total gain is simply 8 + 15 = 23 dB. Similarly, if a power amplifier with 12 dB of gain is connected to a feed line with 1 dB of loss and then to an antenna with 4 dB of gain, the total gain of that combination is 12 – 1 + 4 = 15 dB. Losses are treated as negative gains.

Rounding Decibels

You have to be careful when rounding off values in dB! If you calculate a gain of 100 (not in dB) and round down to 99, you're only off by 1%. But if you calculate a gain of 100 dB and round down to 99 dB, that change of 1 dB represents a 27% difference because dB are logarithmic! It's worth using an extra decimal place or two in order to avoid big errors.

Converting Decibels to Power Ratios

If you are given a ratio in decibels and asked to calculate the power or voltage ratio, here are the formulas to use:

$$\textit{power ratio} = \log^{-1}\left(\frac{dB}{10}\right) \text{ and } \textit{voltage or current ratio} = \log^{-1}\left(\frac{dB}{20}\right)$$

This can also be written as:

$$\textit{power ratio} = 10^{(dB/10)} \text{ and } \textit{voltage or current ratio} = 10^{(dB/20)}$$

Voltage and Current Ratios

It may seem confusing that the logarithm of voltage and current ratios are multiplied by 20 instead of 10. First, decibels are always about power ratios, so don't think there is a "voltage decibel" and a "current decibel" that is different from a "power decibel." A decibel is a decibel is a decibel. Using the equations $P = V^2/R$ and $P = I^2R$ to substitute for the power values, you'll see that the ratios inside the parentheses of the decibel equation become V^2/V_{ref}^2 and I^2/I_{ref}^2. These ratios can also be written as $(V/V_{ref})^2$ and $(I/I_{ref})^2$.

Logarithms treat exponents specially: log (value^{Exp}) = Exp × log (value). So, in the case of the voltage and current ratios, $\log [(V/V_{ref})^2] = 2 \log (V/V_{ref})$. Thus, when using voltage and current ratios, dB = 10 × 2 log (ratio) = 20 log (ratio).

In the preceding explanation, we assumed that both voltages or currents were measured across or through the same value of resistance, R. If R is not the same, for example, when gain in dB is being measured for an amplifier with high input impedance and low output impedance, we have to take that into account.

When computing the gain from two voltage measurements, the formula looks like this:

$$\text{dB} = 20 \log\left(\frac{\textit{output voltage} / \textit{output impedance}}{\textit{input voltage} / \textit{input impedance}}\right) = 20 \log\left(\frac{V_{OUT} / Z_{OUT}}{V_{IN} / Z_{IN}}\right)$$

And when calculating the voltage ratio:

$$voltage\ ratio = \sqrt{\frac{Z_{OUT}}{Z_{IN}}} \log^{-1}\left(\frac{dB}{20}\right)$$

Decibel Shortcuts

You don't necessarily need to carry a calculator around with you all the time to work with decibels. You'll find that, most of the time, you can estimate the decibel equivalent of a ratio or the ratio represented by a value in decibels. Remembering a few values corresponding to common ratios and some powers of ten from the table of common decibel values will satisfy many ham radio needs.

Decibel values for ratios not shown in Table 1 can often be calculated by using the property (a × b) in dB = (a) in dB + (b) in dB. Here are some examples:

- dB value of 25 = dB value of (5 × 5) = dB value of 5 + dB value of 5 = 7 + 7 = 14 dB
- dB value of 40 = dB value of (20 × 2) = dB value of 20 + dB value of 2 = 13 + 3 = 16 dB
- dB value of 0.2 = dB value of (0.1 × 2) = dB value of 0.1 + dB value of 2 = –10 + 3 = –7 dB
- dB value of 0.005 = dB value of (0.01 × 0.5) = dB value of 0.01 + dB value of 0.5 = –20 + (–3) = –23 dB

Special Decibel Abbreviations

You will often see the abbreviation dB followed by a letter. That means the value was calculated using a specific reference value. The letter indicates that the value is "decibels with respect to…" followed by the reference value. For example, you will frequently see power levels given in dBm. The lower case "m" stands for milliwatt (mW), with 0 dBm corresponding to the reference power of 1 mW. For example, 10 dBm would be 10 times that or 10 mW, and –6 dBm would be ¼ mW. In other words, dBm is another way of referring to power. It can make life a bit easier if you're doing system calculations. There are a number of other common abbreviations that specify certain reference levels, and several are listed in Table 2.

An example of calculating *Effective Radiated Power* or *ERP* helps explain how dBm is used. Say we have a transmitter that supplies 100 W, a feed line that has 3 dB loss, and an antenna gain of 6 dB. What is our ERP? Instead of having to use the decibel formula three times: convert the power to dBm, do the additions and subtractions, then convert back.

Start by converting 100 W to dBm: 100 W = 100,000 mW = 10^5 mW, so the *transmitter power output* (*TPO*) is +50 dBm. Then we lose 3 dB in the coax, so we are down to +47 dBm (+50 minus the 3 dB loss). Finally, we gain 6 dB at the antenna for a net result of +53 dBm (+47 plus 6 dB gain). Because 3 dB represents a doubling of power, the resulting ERP is 50 dBm + 3 dB = 100 W × 2 = 200 W.

It is important to stress that, while decibels represent change, dBm represents a particular power level. It's like saying, "I have so many watts of power." All the same rules of decibels apply when using dBm. How many dBm is 5 W of power? First, 1 W is 1,000 mW or +30 dBm and 10 times that is 10 W, or +40 dBm. Half of 10 W = 5 W which is the same as +40 dBm – 3 dB or +37 dBm. Thus, 5 W equals +37 dBm. You can find online calculators that convert between dBm and watts, such as the one at **www.everythingrf.com/rf-calculators/watt-to-dbm**.

Decibels and Power Examples

Let's suppose you have an amateur transmitter that has an output power of 10 W, but you would like a little more power to contact a distant station. As illustrated in Figure 1, after adjusting your transmitter, you measure the output power again and find it is now generating 20 W. How many dB increase is this?

Step 1: Use the 10 W signal as the reference. Divide 20 W by 10 W to find the power ratio of 20 W / 10 W = 2.

Step 2: Find the logarithm of the power ratio to get log (2) = 0.3

Step 3: Multiply this result by 10 = 10 × 0.3 = 3 dB

Your adjustments increased the power of your signal by 3 dB. When power is doubled, there is a 3 dB increase. This is true no matter what the actual power levels are. For example, increasing power from 50 to 100 W is a ratio of 2 and a 3 dB increase.

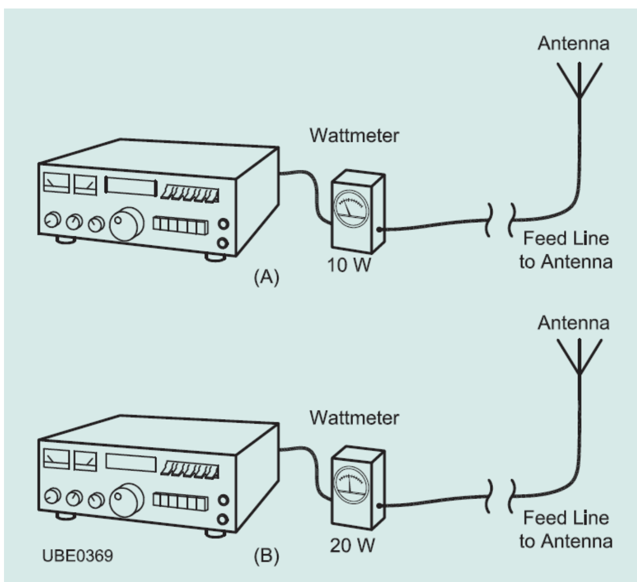

Figure 1 — The output power from a transmitter (A) is 10 W. After making some adjustments to the transmitter tuning, you measure the power and find it has increased to 20 W (B). The text describes how to calculate the decibel increase that occurred.

Now suppose you use an amplifier to increase your output power to 1,000 W. Choose the reference power to be 10 W again, and divide the new power by the reference.

Step 1: Use the 10 W signal as the reference. Divide 1,000 W by 10 W to find the power ratio of 1,000 W / 10 W = 100.

Step 2: Find the logarithm of the power ratio to get log (100) = 2

Step 3: Multiply this result by 10 = 10 × 2 = 20 dB

Your amplifier has increased the power of your signal by 20 dB.

Whenever you multiply or divide the reference power by a factor of 2, you will have a 3 dB change in power. You might guess, then, that if you multiplied the power by four, it would be a 6 dB increase. If you multiplied the power by eight, it would be a 9 dB increase. You would be right in both cases.

Suppose the power in part of a circuit, such as the one shown in Figure 2, measures 5 milliwatts, and in another part of the circuit, it measures 40 mW. Using the 5 mW value as the reference power, how many decibels greater is the 40 mW power?

dB = 10 log (40 / 5) = 10 log (8) = 10 × 0.9 = 9 dB

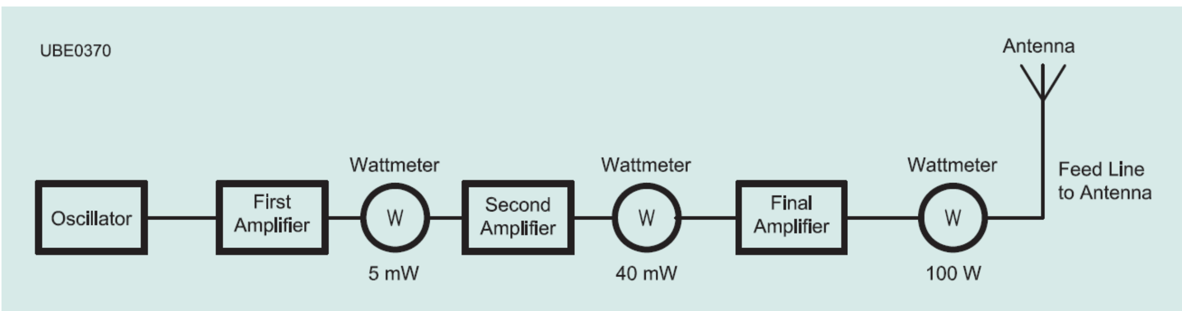

Figure 2 — A simple amateur transmitter amplifies the signal from an oscillator and then feeds that signal to an antenna. It uses several amplifier stages. The input power to one of those stages is 5 milliwatts, and the output from that stage is 40 milliwatts. The text describes how to calculate the gain of that amplifier stage.

To determine what happens if the power decreases, we can continue with the problem above, and measure the actual power arriving at the antenna. In this station, the final amplifier supplies 100 W to a long length of coaxial cable connecting the transmitter to the antenna. Because some power is lost in this cable, we measure only 75 W at the antenna. This time, we'll use the 100 W amplifier output as our reference. We want to compare the power at the antenna with the transmitter output power.

dB = 10 log (75 / 100) = 10 log (0.75) = 10 × –0.125 = –1.25 dB

The negative sign tells us that we have less power than our reference due to the feed line loss.

Converting Decibels and Percentage

Sometimes, we need to know percentage instead of a decibel value. Here are the formulas to make those conversions. First, to calculate the decibel equivalent of a power or voltage percentage:

$$\text{dB} = 10 \log\left(\frac{\text{Percentage Power}}{100\%}\right)$$

$$\text{dB} = 20 \log\left(\frac{\text{Percentage Voltage}}{100\%}\right)$$

And to convert from percentages to decibels:

$$\text{Percentage Power} = 100\% \times \log^{-1}\left(\frac{\text{dB}}{10}\right)$$

$$\text{Percentage Voltage} = 100\% \times \log^{-1}\left(\frac{\text{dB}}{20}\right)$$

Here's a practical application. Suppose you are using an antenna feed line that has a signal loss of 1 dB. You can calculate the amount of transmitter power that's actually reaching your antenna and how much is lost in the feed line.

$$\text{Percentage Power} = 100\% \times \log^{-1}\left(\frac{-1}{10}\right) = 100\% \times \log^{-1}(-0.1) = 79.4\%$$

That means 79.4% of your power is reaching the antenna and 20.6% is lost in the feed line. Here are some other examples of converting between percentages and decibels:

- A power ratio of 20% = 10 log (20% / 100%) = 10 log (0.2) = –7 dB
- A voltage ratio of 150% = 20 log (150% / 100%) = 20 log (1.5) = 3.52 dB
- –3 dB represents a percentage power = 100% × $\log^{-1}$ (–3 / 10) = 50%
- 4 dB represents a percentage voltage = 100% × $\log^{-1}$ (4 / 20) = 158%

Using a Calculator with Decibels

You will need a calculator that includes the *log* and *inverse log* function to work with decibel values. The inverse log (written as $\log_{10}^{-1}$ or just $\log^{-1}$) is sometimes referred to as "antilog." Most calculators use the inverse log notation. On scientific calculators, the inverse log key may be labeled LOG^{-1}, ALOG, or 10^{x}, which means "raise 10 to the power of this value." Some calculators require a two-button sequence, such as INV then LOG. (Read your calculator's manual if you are not clear about how to use these functions.) Be sure that your calculator is set to calculate common logarithms and not natural logs.

Here are step-by-step instructions to use the scientific calculator that comes with the *Windows* operating system to calculate the ratio of 20 W to 10 W in decibels:

Step 1: If necessary, click C to clear the calculator, then enter 20.

Step 2: Click / to start the division, then enter 10, and click =. The display will show a value of 2.

Step 3: Click LOG. The display will show a value of 0.301…

Step 4: Click *, then enter 10, and click =. The display will show a value of 3.01…This is the value of the ratio 20/10 = 2 in dB.

Similarly, to convert the value of 3 dB back to a power ratio, follow these steps:

Step 1: Enter 3, then click /, enter 10, and click =. The display will show a value of 0.3.

Step 2: Click 10^x. The display will show a value of 1.995… This is the value of the ratio with a decibel value of 3.

There are also many online converters that calculate decibels, such as this useful web page at Crown Audio: **www.crownaudio.com/en-US/tools/calculators#db**.

To convert 9 dB to a power ratio using LOG^{-1}:

Step 1: Enter 9, then click /, enter 10, and click =. The display will show a value of 0.9.

Step 2: Click LOG^{-1}. The display will show a value of 7.94. This is the ratio with a decibel value of 9.

To use the 10^X method:

Step 1: Enter 9, then click /, enter 10, and click =. The display will show a value of 0.9.

Step 2: Click 10^x. The display will show a value of 7.94.

There is a shortcut that uses the X^Y function of a calculator to find the power ratio for any value of dB. Raise the value 1.26 to the decibel value. For example, to find the power ratio corresponding to a decibel value of 12 dB, enter 1.26 into the calculator, press the XY key, enter 12, and press the = key. The result is a power ratio of 16.01. This is easy to do with most calculators, and you do not have to use the INV LOG or LOG^{-1} function.

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